

THE DEPENDENCE OF A MEREOLOGICAL AXIOM

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In this note we show that in the standard axiom system for mereology which follows, the reflexive axiom, $M2$, is dependent on $M3$, DM , $M4$, and $M5$.

- $M1$. $[AB]: A \varepsilon \text{el}(B) \supset B \varepsilon B$.
- $M2$. $[A]: A \varepsilon A \supset A \varepsilon \text{el}(A)$.
- $M3$. $[ABC]: A \varepsilon \text{el}(B) \cdot B \varepsilon \text{el}(C) \supset A \varepsilon \text{el}(C)$.
- DM . $[Aa]: A \varepsilon \text{KI}(a) \equiv A \varepsilon A : [D]: D \varepsilon a \supset D \varepsilon \text{el}(A) : [D]: D \varepsilon \text{el}(A) \supset [\exists EF]. E \varepsilon a \cdot F \varepsilon \text{el}(D) \cdot F \varepsilon \text{el}(E)$.
- $M4$. $[Aa]: A \varepsilon a \supset [\exists B]. B \varepsilon \text{KI}(a)$.
- $M5$. $[ABa]: A \varepsilon \text{KI}(a) \cdot B \varepsilon \text{KI}(a) \supset A \varepsilon B$.
- $P1$. $[AD]: A \varepsilon A \cdot D \varepsilon \text{el}(A) \supset [\exists F]. F \varepsilon \text{el}(D)$
- PF $[AD]: \text{Hyp}(2) \supset$
 $[\exists B].$
 3) $B \varepsilon \text{KI}(A)$ [$M4$, 1]
 4) $A \varepsilon \text{el}(B)$ [DM , 3, 1]
 5) $D \varepsilon \text{el}(B)$ [$M3$, 2, 4]
 $[\exists F]. F \varepsilon \text{el}(D)$ [DM , 3, 5]
- $P2$. $[A]: A \varepsilon A \supset A \varepsilon \text{KI}(\text{el}(A))$ [DM , $a/\text{el}(A)$, E/D , $P1$]
- $P3$. $[ABD]: A \varepsilon A \cdot B \varepsilon \text{KI}(A) \cdot D \varepsilon \text{el}(B) \supset [\exists EF]. E \varepsilon \text{el}(A) \cdot F \varepsilon \text{el}(D) \cdot F \varepsilon \text{el}(E)$
- PF $[ABD]: \text{Hyp}(3) \supset$
 4) $A \varepsilon \text{el}(B)$ [DM , 2, 1]
 $[\exists EF].$
 5) $E \varepsilon A$ }
 6) $F \varepsilon \text{el}(D)$ } [DM , 2, 3]
 7) $F \varepsilon \text{el}(E)$ }
 8) $E = A$ [5, 1]
 9) $F \varepsilon \text{el}(A)$ [7, 8]
 10) $F \varepsilon \text{el}(B)$ [$M3$, 6, 3]
 $[\exists G].$
 11) $G \varepsilon \text{el}(F)$ [DM , 2, 10]
 12) $G \varepsilon \text{el}(D)$ [$M3$, 11, 6]
 $[\exists EF]. E \varepsilon \text{el}(A) \cdot F \varepsilon \text{el}(D) \cdot F \varepsilon \text{el}(E)$ [9, 12, 11]

P4. $[A]: A \varepsilon A. \supset. A \varepsilon \mathbf{el}(A)$

PF $[A]. \therefore A \varepsilon A. \supset:$

$[\exists B]:$

2) $B \varepsilon \mathbf{Kl}(A).$

[M4, 1]

3) $A \varepsilon \mathbf{el}(B):$

[DM, 2, 1]

4) $[D]: D \varepsilon \mathbf{el}(A). \supset. D \varepsilon \mathbf{el}(B):$

[M3, 3]

5) $[D]: D \varepsilon \mathbf{el}(B). \supset. [\exists EF]. E \varepsilon \mathbf{el}(A). F \varepsilon \mathbf{el}(D).$

$F \varepsilon \mathbf{el}(E):$

[P3, 1, 2]

6) $B \varepsilon \mathbf{Kl}(\mathbf{el}(A)).$

[DM, 2, 4, 5]

7) $A \varepsilon \mathbf{Kl}(\mathbf{el}(A)).$

[P2, 1]

8) $A = B:$

[M5, 6, 7]

9) $A \varepsilon \mathbf{Kl}(A).$

[2, 8]

$A \varepsilon \mathbf{el}(A)$

[DM, 9, 1]

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