

A NOTE ON INDEPENDENCE

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Acquaintance with [1] is presupposed and we shall use its numeration of the axioms. At the end of the paper Thacher Robinson states:

“It is interesting to note that the author has verified that *no normal* truth-table in ≤ 5 truth values will suffice for the independence of (2.2). Bernays constructed for the author a six-valued normal truth-table (subsequently lost) which also shows the independence of (2.2).”

The following (see p. 411) six-valued normal truth-table suffices for the purpose. The designated values are 0 and 1. (2.2) takes the value 2 if $p = 0$, $q = 4$ and $r = 2$. See also [2] and [3] where five and six-valued normal truth-tables are used to establish the independence of (2.1) and (2.2) respectively but in a different context, i.e., the connectives used besides implication to obtain formulations of the propositional calculus are not the same as those in [1].

REFERENCES

- [1] Robinson, T. Thacher, “Independence of two nice sets of axioms for the propositional calculus,” *The Journal of Symbolic Logic*, vol. 33 (1968), pp. 265-270.
- [2] Shukla, A., “A set of axioms for the propositional calculus with implication and converse non-implication,” *Notre Dame Journal of Formal Logic*, vol. 6 (1965), pp. 123-128.
- [3] Shukla, A., “A set of axioms for the propositional calculus with implication and non-equivalence,” *Notre Dame Journal of Formal Logic*, vol. 7 (1966), pp. 281-286.

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TRUTH TABLE

| <i>A</i> | <i>B</i> | $A \supset B$ | $A \vee B$ | $A \& B$ | $\sim A$ | <i>f</i> |
|----------|----------|---------------|------------|----------|----------|----------|
| 0 | 0 | 0 | 0 | 0 | 4 | 5 |
| | 1 | 0 | 0 | 0 | | |
| | 2 | 2 | 0 | 5 | | |
| | 3 | 3 | 0 | 5 | | |
| | 4 | 3 | 0 | 5 | | |
| | 5 | 5 | 0 | 5 | | |
| 1 | 0 | 0 | 0 | 0 | 4 | |
| | 1 | 0 | 0 | 0 | | |
| | 2 | 2 | 0 | 5 | | |
| | 3 | 3 | 0 | 5 | | |
| | 4 | 5 | 0 | 5 | | |
| | 5 | 5 | 0 | 5 | | |
| 2 | 0 | 0 | 0 | 5 | 3 | |
| | 1 | 0 | 0 | 5 | | |
| | 2 | 0 | 5 | 5 | | |
| | 3 | 3 | 5 | 5 | | |
| | 4 | 3 | 5 | 5 | | |
| | 5 | 0 | 5 | 5 | | |
| 3 | 0 | 0 | 0 | 5 | 2 | |
| | 1 | 0 | 0 | 5 | | |
| | 2 | 2 | 5 | 5 | | |
| | 3 | 0 | 5 | 5 | | |
| | 4 | 2 | 5 | 5 | | |
| | 5 | 0 | 5 | 5 | | |
| 4 | 0 | 1 | 0 | 5 | 0 | |
| | 1 | 1 | 0 | 5 | | |
| | 2 | 1 | 5 | 5 | | |
| | 3 | 1 | 5 | 5 | | |
| | 4 | 1 | 5 | 5 | | |
| | 5 | 1 | 5 | 5 | | |
| 5 | 0 | 0 | 0 | 5 | 0 | |
| | 1 | 0 | 0 | 5 | | |
| | 2 | 0 | 5 | 5 | | |
| | 3 | 0 | 5 | 5 | | |
| | 4 | 0 | 5 | 5 | | |
| | 5 | 0 | 5 | 5 | | |