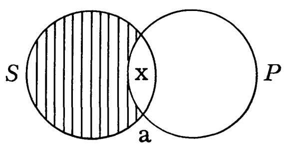
Notre Dame Journal of Formal Logic
Volume X, Number 3, July 1969

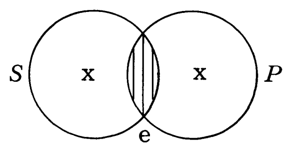
A THEORY OF CATEGORICAL SYLLOGISM

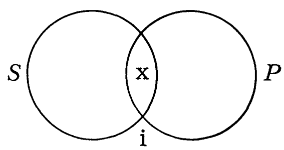
SETSUO SAITO

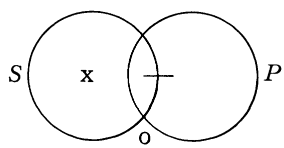
In this paper I shall try to present a theory which can take the place of the classical theory of categorical syllogism. It seems to me that this method has advantages, in particular for instructive purposes, that it is simpler and easier.

If we adapt the traditional interpretation of categorical propositions, Venn's diagrams must be modified as follows:





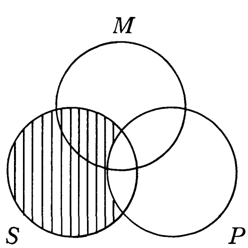


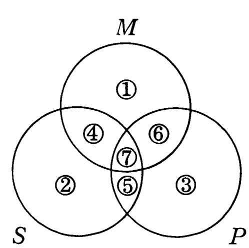


A bar indicates non-emptiness of a compound region in which the bar is contained.

Now we divide syllogisms into four cases where conclusions are respectively *S*a*P*, *S*e*P*, *S*i*P*, *S*o*P*, and in each case we search the necessary condition for the validity of the syllogisms, by means of modified Venn diagrams.

1) The case where the conclusion is *S*a*P*:
If the conclusion of a valid syllogism is *S*a*P*,



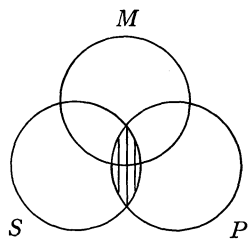


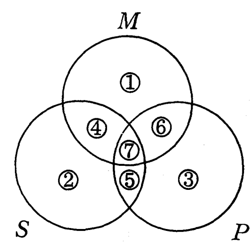
Received June 3, 1967

then ② + ④ is empty. If ② is empty, then the minor premise is *SaM*. If the minor premise is *SaM* and ④ is empty, then the major premise is *MaP*. A valid syllogism, therefore, whose conclusion is *SaP*, must be as follows:

MaP
SaM
∴ *SaP*

2) The case where the conclusion is *SeP*:

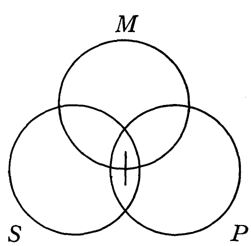


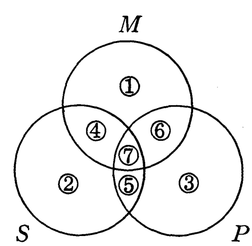


If the conclusion of a valid syllogism is *SeP*, then ⑤ + ⑦ is empty. If ⑤ is empty, then either the major premise is *PaM* or the minor premise is *SaM*. If the former case holds, then the minor premise is either *SeM* or *MeS*, since ⑦ is empty. If the latter case holds, then the major premise is either *PeM* or *MeP* since ⑦ is empty. Therefore, a valid syllogism whose conclusion is *SeP* must be one of the following:

PaM	*PaM*	*PeM*	*MeP*
SeM	*MeS*	*SaM*	*SaM*
∴ *SeP*	∴ *SeP*	∴ *SeP*	∴ *SeP*

3) The case where the conclusion is *SiP*:





If the conclusion of a valid syllogism is *SiP*, then one of the premises must assert that *X* - (⑤ + ⑦) is empty. Here *X* is some region asserted to be not empty by the other premise.

For the case where the major premise asserts that *X* - (⑤ + ⑦) is empty, the following table is obtained:

X	X - (⑤ + ⑦)	The major premises which assert that X - (⑤ + ⑦) is empty.	The minor premises which assert that X is not empty.
① + ⑥	① + ⑥	none	
④ + ⑦	④	MaP	SaM, MaS, SiM, MiS
② + ⑤	②	none	
① + ④ + ⑥ + ⑦	① + ④ + ⑥	none	
② + ④ + ⑤ + ⑦	② + ④	none	

Therefore, a valid syllogism in this case must be one of the following:

MaP	MaP	MaP	MaP
SaM	MaS	SiM	MiS
∴ SiP	∴ SiP	∴ SiP	∴ SiP

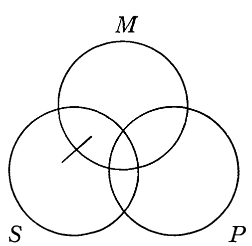
Since the major premise and the minor premise are symmetrical in the above discussion, a valid syllogism whose minor premise asserts that X - (⑤ + ⑦) is empty must be the following:

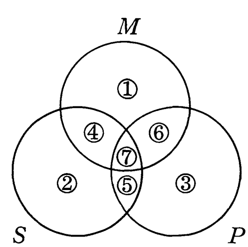
PaM	MaP	PiM	MiP
MaS	MaS	MaS	MaS
∴ SiP	∴ SiP	∴ SiP	∴ SiP

Therefore, a valid syllogism whose conclusion is SiP must be one of the following:

PaM	MaP	PiM	MiP	MaP	MaP	MaP
MaS	MaS	MaS	MaS	SaM	SiM	MiS
∴ SiP	∴ SiP	∴ SiP	∴ SiP	∴ SiP	∴ SiP	∴ SiP

4) The case where the conclusion is SoP:





Considering this case in the same way as in the case 3), the following table is obtained:

i) The case where the major premise asserts that X - (② + ④) is empty.

X	X - (②+④)	The major premises which assert that X - (②+④) is empty	The minor premises which assert that X is not empty
①+⑥	①+⑥	none	
④+⑦	⑦	*PeM*, *MeP*	*SaM*, *MaS*, *SiM*, *MiS*
②+⑤	⑤	*PaM*	*SoM*, *SeM*, *MeS*
①+④+⑥+⑦	①+⑥+⑦	none	
②+④+⑤+⑦	⑤+⑦	none	

ii) The case where the minor premise asserts that X - (②+④) is empty.

X	X - (②+④)	The minor premises which assert that X - (②+④) is empty	The major premises which assert that X is not empty
①+④	①	*MaS*	*PeM*, *MeP*, *MoP*
⑥+⑦	⑥+⑦	none	
③+⑤	③+⑤	none	
①+④+⑥+⑦	①+⑥+⑦	none	
③+⑤+⑥+⑦	③+⑤+⑥+⑦	none	

Therefore, a valid syllogism whose conclusion is *SoP* must be one of the following:

PeM	*PeM*	*PeM*	*PeM*	*MeP*	*MeP*
SaM	*MaS*	*SiM*	*MiS*	*SaM*	*MaS*
∴ *SoP*	∴ *SoP*	∴ *SoP*	∴ *SoP*	∴ *SoP*	∴ *SoP*
MeP	*MeP*	*PaM*	*PaM*	*PaM*	*MoS*
SiM	*MiS*	*SoM*	*SeM*	*MeS*	*MaS*
∴ *SoP*	∴ *SoP*	∴ *SoP*	∴ *SoP*	∴ *SoP*	∴ *SoP*

It is easy to show by means of the modified Venn diagrams that the above-stated 24 syllogisms are all valid.

Shibaura Institute of Technology
Tokyo, Japan