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TERMINAL FUNCTORS PERMISSIBLE WITH SYLLOGISTIC

C. A. MEREDITH

(Notes of a paper delivered in July 1957, on the possibilities of enriching syllogistic as axiomatised, e.g. in Łukasiewicz's *Aristotle's Syllogistic*, with complex terms, with appropriate axioms).

I. General conclusions *

(i) The *logical product* of a pair of terms, *kab*, does not lend itself for use in this way, since one would expect it to have $\vdash Akaba$ and $\vdash Akabb$, which would yield $\vdash Iab$ by Darapti, making all *I* propositions true.

(ii) Similarly with the *null term* $\wedge$; one would expect this to have $\vdash A \wedge a$, but $\vdash A \wedge a$ and $\vdash A \wedge b$ (obtained from $\vdash A \wedge a$ by substitution) would yield *Iab*, again by Darapti.

(iii) *Disjunction* and *implication* are possible, so long as they are not combined with negation (when of course conjunction and the null term would be definable).

(iv) The *modal* λ and μ ("necessarily", "possibly") seem alone to combine with negation, *n*.

II. Postulates for non-modal terminal functors

(i) If we take the syllogistic *E* as undefined (defining *Aab* as *Eanb* and *I* as *NE*), syllogistic with terminal negation may be axiomatised by 1. *Eana*, 2. *NEaa* and 3. Camenes.

(ii) For terminal disjunction, *v*, we may subjoin to ordinary syllogistic (without *n*) first of all the four axioms

1. *Aavab*	3. *CAacCAbcAvabc*
2. *Abvab*	4. *CEacCEbcEvabc*

Where *t* and *s* are terms, we write $t = s$ where we have both $\vdash Ats$ and $\vdash Ast$; and where *p* and *q* are propositions, we write $p = q$ where we have both $\vdash Cpq$ and $\vdash Cqp$. The above for *v* will give $vaa = a$, $Avabc = KAacAbc$, and $Evabc = KEacEbc = Ecvab$. The addition of

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$$5.\ CAavbcCAavbdCOabIcd$$

easily gives $CAavbcCEabAac$, which is independent of 1-4 but intuitively desirable.

(iii) If we use fab for the implicative term "b if a", we may define vab as $ffabb$, and the "total term" $\vee$ as faa. In syllogistic with implicative terms we would need to be able to establish $ffaba = a$ (cf. the Peircean propositional equation $CCpqp = p$), $fafbc = fbfac$ (cf. the propositional law of commutation $CpCqr = CqCpr$) and $ffabb = ffbaa$. One possible axiom is $CAafbcCAabAac$ (cf. Frege's law $CCpCqrCCpqCpr$); also $CAcaCEbcEfabc$ (cf. the fourth axiom for v).

III. Modal syllogistic Aristotle combined the assertions

1. $\vdash CLAbcCLAabLAac$
2. $\vdash CLAbcCAabLAac$

with the rejection

3. $\dashv CAbcCLAabLAac$.

If we treat L and M as "pseudo-functors", equating $LAbc$ with $Ab\lambda c$, we may obtain Aristotle's results—Barbara will give $CAb\lambda cCAabAa\lambda c$, and so 2; $A\lambda aa$ and sorites will give $CAb\lambda cCAa\lambda bAa\lambda c$, and so 1; and rejection of $CAbcA\lambda b\lambda c$ will reject 3.

Given $LAab = Aa\lambda b$, $MAab = Aa\mu b$, $LIab = Ia\lambda b$, $MIab = Ia\mu b$, $\mu = n\lambda n$ and $\lambda = n\mu n$, we can prove $LEab = Ea\mu b$, $MEab = Ea\lambda b$, $LOab = Oa\mu b$, $MOab = Oa\lambda b$.

We can, if we wish, add $\lambda\lambda = \lambda$ or $\mu\lambda = \lambda$ or $\mu\lambda = \mu$, analogous to the reduction theses of S4, S5 and the Ł-modal system.

If to Łukasiewicz's four axioms for syllogistic without negation we add

5. $A\lambda aa$
6. $Aa\mu a$
7. $A\mu\mu a\lambda\mu a$
8. $A\mu\lambda a\lambda\lambda a$,

we obtain all the S5-like equations $\lambda\lambda = \lambda = \mu\lambda$, $\mu\mu = \mu = \lambda\mu$. In this calculus the two rejections

9. $\dashv CAa\mu bIab$ (giving $\dashv A\mu aa$)
10. $\dashv CAabIa\lambda b$ (giving $\dashv Aa\lambda a$)

are independent of one another. Some rule is also needed to give such rejections as $\dashv CAabCAbaIa\lambda b$, with terms occurring more than twice.

Alternatively, to syllogism with n (with the three axioms given in *II* (i)), we could add $\mu = n\lambda n$ and

4. $A\lambda aa$
5. $A\lambda nna\lambda\lambda a$
6. $An\lambda n\lambda a\lambda nna$.

The previous 5-8 will then all follow, and $\dashv CEa\lambda bNEanb$ will suffice for both the rejections 9 and 10.

These are simpler systems than S5 proper, as only modal functors of one argument are involved.

Historical note on I. (i) added by A. N. Prior:

J. N. Keynes, in the first edition of his *Studies and Exercises in Formal Logic* (1884), p. 282, has the following exercise (numbered 274):

> "Whatever P and Q may stand for, we may shew *a priori* that some P is Q. For All PQ is Q by the law of identity, and similarly All PQ is P; therefore, by a syllogism in *Darapti*, some P is Q." How would you deal with this paradox?

The same question appears as Exercise 262 in the second edition (1887) of Keynes's work, as Exercise 294 in the third edition (1894) and as exercise 346 in the fourth (1906), and it is alluded to by Venn in the second edition of his *Symbolic Logic* (1894). p. 153n.

The answer Keynes wanted, I think, was that either *À*-propositions entail the existence of their subjects, in which case none of them—not *Akaba* or *Akabb*, and not even *Aaa*—is *a priori* true; or they do not, in which case Darapti is invalid (at all events if *I*-propositions do have this entailment). Meredith's view above appears to be that the satisfaction of *Akaba* and *Akabb* is so basic to the notion of a conjunctive term, and *Iab* so absurd as a logical thesis, that such terms can just have no place in a system containing Darapti. One might put it this way: In a system designed to apply to non-empty terms only, we can only introduce negative terms if we also confine the system to non-universal terms (since the negation of a universal term would be empty), and conjunctive terms only if we confine it further to terms such that every pair of them has a non-empty intersection; but this is too restrictive altogether.

This reasoning, however, presupposes that (i) if we introduce negative terms every term will have a negative, and that (ii) if we introduce conjunctive terms every pair of terms will have their conjunction. A. J. Baker, in "Non-empty complex terms", *Notre Dame Journal of Formal Logic*, vol. 7, no. 1, January 1966, pp. 48-56, rejects the second of these assumptions (though for some reason he retains the first, and so allows himself to be forced into excluding universal as well as empty terms). He says in effect that *kab* is only conditionally a well-formed term, namely on the condition that *Iab* is true. He does not discuss *Akaba* and *Akabb*, but on his principles it would seem that anything of either of these forms which is a proposition at all will be a true one. The same cannot be said, of course, for *Iab*. Yet *Iab* follows from *Akaba* and *Akabb* by Darapti, which Baker considers valid; thus Bakerian validity cannot be guaranteed to take us from laws to laws. For similar oddities in Strawson, whom Baker broadly follows, see T. J. Smiley's "Mr. Strawson on the traditional logic", *Mind* vol. 76, no. 301, January 1967, pp. 118-120.

Storrs McCall in "Connexive class logic", *The Journal of Symbolic Logic*, vol. 32, no. 1, March 1967, pp. 83-90, has a system for use with all sorts of terms (empty, non-empty, universal, non-universal), and with terminal conjunction, in which Darapti is accepted and *Akaba* and *Akabb* frankly abandoned. But unlike any systems known to Keynes, McCall's is subtle enough, while rejecting *Akaba* and *Akabb*, to retain *Aaa*, and indeed all the laws of Meredith's II(i) above.

Trinity College
Dublin, Ireland