Notre Dame Journal of Formal Logic
Volume XVI, Number 4, October 1975
NDJFAM

ON A CONSERVATIVE ATTITUDE TOWARD SOME NAIVE SEMANTIC PRINCIPLES

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1* Underlying much of the literature on the Liar and related paradoxes is a deep-seated conservatism about naive semantic principles. It is commonplace to observe that the paradoxes reveal an inadequacy in those principles. On the other hand, any semantic theory that departs from them too radically is open to the charge of implausibility. Naive principles may indeed be inadequate; in certain crucial instances they may yield actual contradiction. But they maintain a hold on us nonetheless. Indeed, the principles are on the whole good ones; in most contexts they work well enough. Some revision is demanded by the paradoxes, to be sure, but we ought not to revise irresponsibly. Ideally, we should be able to capture the paradoxes exactly, deal with them in some appropriate way, and then leave naive semantics otherwise alone. Accordingly, we have:

(1) Exceptions are to be allowed to naive semantics only where naive semantics is inconsistent.

As a methodological guideline, (1) seems to have motivated much of the recent work on the paradoxes. (See, e.g., Fitch [2], p. 77; Martin [6], p. 279; Sommers [8], pp. 259f. See also Anderson's observation [1], p. 9.) The conservative ideal it expresses is independent of any agreement on a list of naive semantic principles or on which principle or combination of principles is to suffer exception in the face of the paradoxes.

Nevertheless, there is general agreement that, whatever else is involved, the following two principles form at least part of our naive and precritical semantics:

a) A sentence is true just in case it corresponds to the facts, to reality. (*The correspondence principle*)

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*This paper is based in a loose way on work done for my Ph.D. dissertation under Prof. Hans G. Herzberger at the University of Toronto. I am indebted to Prof. Herzberger for many points that have found their way into this paper. The usual disclaimer of responsibility applies.

Received December 18, 1972

b) The predicate 'true' applies just to the true sentences. (*The semantic closure principle*)

Let us call the conjunction of these two principles "(**N**)", reminiscent of "naive". It is common practice to locate the exceptions to naive semantics at (**N**). Thus, the so-called "levels of language" view takes the position that the correspondence principle is correct, but sacrifices semantic closure in such a way that our global truth-concept is shattered into infinitely many hierarchically arranged fragmentary concepts. Alternatively, it has been argued that this position pays too high a price. What the paradoxes show, it has been said, is that the correspondence principle does not hold across the board. (See Herzberger [4], p. 35 and [5], pp. 25f.) We might extract what is common to these views as follows:

(2) The exceptions to naive semantics are confined to (**N**).

Theses (1) and (2) together suggest—but do not entail—the following initially attractive thesis:

(3) Exceptions are to be allowed to (**N**) only where (**N**) is inconsistent.

I shall argue that (3) is implausible. What attractiveness it has comes, I suggest, from our concentrating too much on situations in which a sentence refers directly to itself, as with the simple Liar paradox. Once more complicated situations are considered, the implausibility of (3) is more apparent. I base my argument on a consideration of indirect self-reference, referential "cycles" containing two or more sentences. But my point could be made just as well by considerations of other kinds (e.g., of infinite, non-cyclic chains of semantic descent—see section **3**, below). My purpose here is not to catalogue the kinds of contexts in which (3) is implausible, but only to argue that it is.

2 Consider a syntax **SYN** whose vocabulary consists of:

a) denumerable singular terms,
b) two predicates of degree one, P and T,
c) a negation sign,

and whose formation rules are just the ones one would expect with such a vocabulary. Let 'x' and 's' (with or without subscripts) range over the singular terms and the sentences of **SYN**, respectively. If s has an atomic constituent of the form Tx, we shall call s a *semantic sentence*. Let $n(s)$ be s preceded by a negation sign. This allows us to write '$n^2(s)$' for the double negation of s, and in general '$n^i(s)$' for the i-fold negation of s. If i is odd and s is of the form Tx, we call $n^i(s)$ an *odd semantic sentence*.

Let a *model* for **SYN** be a triple $\mathfrak{M} = \langle X_{\mathfrak{M}}, f_{\mathfrak{M}}, g_{\mathfrak{M}}\rangle$ such that

a) $X_{\mathfrak{M}}$ is a set including the set of all sentences of **SYN**,
b) $f_{\mathfrak{M}}$ is a one-one denotation function whose domain is the set of singular terms of **SYN** and whose range is $X_{\mathfrak{M}}$,
c) $g_{\mathfrak{M}}$ is a function assigning a subset of $X_{\mathfrak{M}}$ to P.

The restrictions on $X_{\mathfrak{M}}$ and $f_{\mathfrak{M}}$ are simply for our own convenience. For they allow us to define a function $t_{\mathfrak{M}}$ assigning to each s its unique *truth sentence*, the Tx such that $f_{\mathfrak{M}}(x) = s$.

We are now in a position to state the correspondence principle and the semantic closure principle for the general context provided by **SYN** and its models. The correspondence principle states that s is true in a model if and only if it corresponds (to the facts, to reality) in that model:

$$V_{\mathfrak{M}}(s) = 1 \text{ iff } s \in \mathsf{CORR}_{\mathfrak{M}}.$$

This of course demands an explication of correspondence. Where 'ϕ' ranges over the two predicates of **SYN** and where $\mathsf{EXT}_{\mathfrak{M}}(\phi)$ is the extension of ϕ with respect to the model $\mathfrak{M}$, correspondence can be explicated for sentences of **SYN** as follows:

$$\phi x \in \mathsf{CORR}_{\mathfrak{M}} \text{ iff } f_{\mathfrak{M}}(x) \in \mathsf{EXT}_{\mathfrak{M}}(\phi)$$
$$n(s) \in \mathsf{CORR}_{\mathfrak{M}} \text{ iff } s \notin \mathsf{CORR}_{\mathfrak{M}}.$$

To complete the account, we need only let

$$\mathsf{EXT}_{\mathfrak{M}}(P) = g_{\mathfrak{M}}(P)$$
$$\mathsf{EXT}_{\mathfrak{M}}(T) = \{s\colon V_{\mathfrak{M}}(s) = 1\}.$$

The semantic closure principle is expressed by this last identity.

By this account, (**N**) stipulates that a language whose syntax is **SYN** and whose admissible models are models for **SYN** have among its rules of valuation the following:

1) $V_{\mathfrak{M}}(Px) = 1$ iff $f_{\mathfrak{M}}(x) \in g_{\mathfrak{M}}(P)$,
2) $V_{\mathfrak{M}}(t_{\mathfrak{M}}(s)) = 1$ iff $V_{\mathfrak{M}}(s) = 1$,
3) $V_{\mathfrak{M}}(n(s)) = 1$ iff $V_{\mathfrak{M}}(s) \neq 1$.

A valuation which satisfies these conditions we might call a *naive valuation*, in the sense that it satisfies the naive principles combined in (**N**). (There may of course be "naive" valuations in this sense that nevertheless violate other naive semantic principles not contained in (**N**).)

These rules are not by themselves sufficient to assign a value to each sentence in each model. For let $\mathfrak{M}$ be such that $s = t_{\mathfrak{M}}(s)$. Here the rules give us the tautology: $V_{\mathfrak{M}}(s) = 1$ iff $V_{\mathfrak{M}}(s) = 1$, and so are consistent with an assignment of any value whatever to s. ((**N**) does not presume bivalence.) This by itself would not be particularly significant. It might simply mean that the correspondence and semantic closure principles must be supplemented by additional ones.

3 But of course there are other cases in which (**N**) gives rise to actual contradiction. The classical example is the Liar paradox. Let $\mathfrak{M}$ be such that $s = n(t_{\mathfrak{M}}(s))$. A contradiction follows by valuation rules 2) and 3) of section **2**.

This is the simplest and most direct kind of case for which (**N**) yields a contradiction. In order to isolate such cases, let us introduce the auxiliary notion of the syntactic relation G which a semantic sentence bears to its grammatical subject. Then, using Quine's terminology ([7], p. 221), let $R_{\mathfrak{M}}$ be the proper ancestral of the relative product of G into $f_{\mathfrak{M}}$. We call $R_{\mathfrak{M}}$ the relation of "semantic descent" with respect to the model $\mathfrak{M}$. Intuitively, s_2 bears $R_{\mathfrak{M}}$ to s_1 just in case s_2 is a semantic sentence whose subject denotes in $\mathfrak{M}$ a semantic sentence whose subject denotes in $\mathfrak{M}$ . . . a semantic sentence whose subject denotes s_1 in $\mathfrak{M}$.

A sentence s_1 of **SYN** is semantically ungrounded (Herzberger [3]) in a model $\mathfrak{M}$ for **SYN** just in case one can start from s_1 and descend semantically with respect to $\mathfrak{M}$ without end. This can occur in three ways:

1) s_1 may be a member of an $R_{\mathfrak{M}}$-cycle: $R_{\mathfrak{M}}(s_1, s_1)$;
2) s_1 may lead into an $R_{\mathfrak{M}}$-cycle of which it is not itself a member: $\sim R_{\mathfrak{M}}(s_1, s_1)$, but for some $s_i \neq s_1$, $R_{\mathfrak{M}}(s_1, s_i)$ and $R_{\mathfrak{M}}(s_i, s_i)$;
3) s_1 may head an infinite, non-cyclic $R_{\mathfrak{M}}$-chain: for all $i, j \geqslant 1$ there is an s_{i+1} such that $R_{\mathfrak{M}}(s_i, s_{i+1})$ and $s_i = s_j$ only if $i = j$.

We restrict ourselves henceforth to models in which semantic ungroundedness of type 3) is ruled out. For any set Y of sentences of **SYN**, form the smallest set Y^* such that

a) $Y \subseteq Y^*$,
b) $n(s) \in Y^*$ only if $s \in Y^*$.

Then we shall say that a *naive valuation over a set Y* of sentences with respect to a model $\mathfrak{M}$ for **SYN** is a function $V_{\mathfrak{M}}$ whose domain is Y^* and whose range contains 1, and which satisfies valuation rules 1)-3) of section **2** above. To say that there is a naive valuation over Y is to say that if we consider only members of Y and their constituent sentences, it is possible to assign values without violating either the correspondence principle or semantic closure. Then we have:

Lemma *Where $i \geqslant 0$ and s_1 need not be distinct from s_2, let $s_1 = n^i(t_{\mathfrak{M}}(s_2))$. If $V_{\mathfrak{M}}$ is a naive valuation over a set containing s_1 and its semantic descendants with respect to $\mathfrak{M}$, then just in case i is odd, $V_{\mathfrak{M}}(s_1) = 1$ iff $V_{\mathfrak{M}}(s_2) \neq 1$.*

This follows from valuation rules 1)-3) of section **2** above.

A sentence s is a member of an $R_{\mathfrak{M}}$-cycle just in case it is a semantic descendant of itself with respect to $\mathfrak{M}$. We shall say that s is a member of an $R_{\mathfrak{M}}$-cycle *of length k* just in case the set of semantic descendants of s with respect to $\mathfrak{M}$ has exactly k members, one of which is s. Then from the Lemma we have:

Theorem 1 *An $R_{\mathfrak{M}}$-cycle of arbitrary length contains an odd number of odd semantic sentences just in case there is no naive valuation with respect to $\mathfrak{M}$ over the set of sentences in that cycle.*

We shall say that a set Y of sentences of **SYN** is *naively satisfiable* (*assailable*) with respect to a model $\mathfrak{M}$ just in case there is a naive valuation over Y which assigns truth (falsity) to each sentence in Y. Then from the Lemma we have also:

Theorem 2 *The set of sentences in an $R_{\mathfrak{M}}$-cycle is naively satisfiable (assailable) with respect to $\mathfrak{M}$ just in case the cycle contains no odd semantic sentences.*

4 Let us now restrict ourselves to models $\mathfrak{M}$ in which $R_{\mathfrak{M}}$ is anti-symmetric, thus ruling out $R_{\mathfrak{M}}$-cycles with more than a single member. The context provided by **SYN** and such models we shall call *Context* 1. In Context 1, a sentence is semantically ungrounded just in case it either

(i) is a semantic sentence which is an immediate semantic descendant of itself: $s = n^i(t_{\mathfrak{M}}(s))$ for some $i \geqslant 0$,

or

(ii) heads a chain of semantic descent which leads to a sentence of kind (i).

Thus in Context 1, the set of semantic descendants of a given semantically ungrounded sentence always contains a sentence of kind (i). If that sentence is an odd semantic one, there is no naive valuation over the set, by Theorem 1. Consistency demands an exception to (**N**) in such cases. Thesis (3) is workable and indeed perhaps plausible in Context 1: exceptions to (**N**) are allowed in just the cases described.

In *Context* 2 we shall broaden our consideration to include cycles of arbitrary length. It is obvious that (3) is implausible in this broader context for $R_{\mathfrak{M}}$-cycles that contain an *even* number of odd semantic sentences. There are naive valuations over such cycles by Theorem 1. Hence (3) demands that (**N**) apply without exception to such cycles. But by Theorem 2 such cycles are neither naively satisfiable nor naively assailable. Consider, for instance, a model $\mathfrak{M}$ such that $s_1 = n(t_{\mathfrak{M}}(s_2))$ and $s_2 = n(t_{\mathfrak{M}}(s_1))$. Some naive valuations over this cycle assign truth to s_1 but not to s_2, and some assign truth to s_2 but not to s_1. But by the Lemma, there are none which treat s_1 and s_2 the same way.

In such a case the problem with (3) becomes apparent. It requires us to distinguish and treat differently sentences that do not differ in any relevant way. It is not *inconsistent* to do so; it is *possible* to maintain (3). We might, for instance, adopt the policy that in cases such as the above the alphabetically first sentence is to be true. Or we might single out the alphabetically last. There are many ways to implement (3). The problem is that none of these ways seem to have a rationale. Thesis (3) thus conflicts with a methodological principle of sufficient reason by requiring us to choose arbitrarily among alternative implementations. Such a methodological principle might be expressed as follows:

The several features of our semantics must have rationales.

Just what qualifies as a legitimate "rationale" would of course ultimately have to be explained. Nevertheless, the present lack of such an account ought not to blind us to the fact that (3) is indeed implausible in cases such as the one above. It requires us to make a distinction without a relevant difference.

In short, (3) is not in general a tenable thesis. It assumes that (**N**) conflicts only with the demands of consistency. In fact, however, it conflicts also with the demand that our semantics avoid arbitrariness. The illusion of plausibility arises perhaps from concentrating too much on cases of direct self-reference, as in Context 1, where if there is any naive valuation at all over a cycle, that cycle is naively satisfiable and naively assailable, and where arbitrariness is thus not required.

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