Notre Dame Journal of Formal Logic
Volume XVI, Number 3, July 1975
NDJFAM

THE CONFIRMATION OF SENTENCES BY INSTANCES WITH DIFFERENT TRUTH-VALUES OF ITS ATOMS

W. A. VERLOREN van THEMAAT

For the last few decades a long discussion has been carried on that questions whether general (quantified) sentences G are confirmed (or, as the case may be, refuted) by all individual sentences asserting the truth or falsity of all predicates of G for some individual. Formally: if G contains the one-place predicates $P_1, \ldots, P_n$, and only these predicates, then any sentence $\bigcap_{i=1}^{n} t_i P_i(a)$, in which t_i is $\neg$ or the absence of any symbol, is an individual sentence relevant for G. Any $t_i P_i(a)$ is called a *conjunct* in it. Many people have been puzzled by the fact that, in many instances, some individual sentences i_1 relevant for G intuitively seem to confirm G and other relevant sentences i_2 intuitively do not seem to confirm G, though according to formal logic both i_1 and i_2 are in agreement with G.

In this paper a *quantitative* concept of confirmation is employed, but I shall not attempt to give criteria for the calculation of a *numerical* value of the degree of confirmation. I shall only explore whether one given sentence is more or less confirmed by several molecular sentences. A calculus for the assignment of numerical values to confirmations may then be evaluated against these findings.

A much-cited example is "All ravens are black." It seems that this sentence is confirmed by instances of black ravens, but that instances of not-black non-ravens are irrelevant for this sentence. Some authors, e.g., Janina Hosiasson-Lindenbaum, *cf.* [2], have tried to escape this paradox by the assumption that, in principle, not-black non-ravens and black non-ravens also confirm the sentence "All ravens are black," but in a far lower degree, in virtue of the fact that, among "all things in the Universe," there are far more non-ravens than ravens, and far more not-black things than black things. This, in turn, is due to the fact that speakers generally introduce words in a language for such predicates, and that there are far more things to which they do not apply than things to which they do apply.

Most publications restrict the discussion to sentences with two predicates. Though they do not pronounce this principle explicitly, the

Received January 18, 1974

authors seem to be led by the heuristic principle that they should start the exploration of a field with relatively simple cases and pass to more complex cases only when the problems for these simple cases have been solved. But this is a misapplication of this principle.

Calculations for formulas with three predicates are indeed slightly more complicated than those for formulas with two predicates, and the number of combinations of signs of conjuncts in the individual sentences is greater. But those calculations raise no real problems. The true difficulty of the problem lies in the semantics of the confirmation relation. So there is good reason to take formulas with three predicates into consideration, if the semantics of the confirmation relation is more clearly elucidated by this. And this is actually the case.

As stated above, the apparent small confirmation of general sentences with two predicates by relevant individual sentences, in which one (or both) conjuncts have a negation, is due to the fact that most predicates divide the universe into a relatively small class of things to which it applies and a relatively large class to which it does not apply. The situation changes completely for general sentences with three predicates. Then, in the class of things to which one of the predicates applies, a second predicate does not divide "Universe" into two sub-classes, but into a perhaps extensive, but clearly demarcated class of things. It is not difficult to find instances in which these sub-classes have comparable size and so individual sentences in which some conjuncts are negative tend to confirm the general sentences. Black expresses similar thoughts in [1], especially p. 179.

Let the predicates be: M (Mammal), B (Bear Children), and F (Female) and the general sentence G:

$$(x)\ \{M(x) \wedge B(x)\} \rightarrow F(x),$$

or, in colloquial language: "All mammals which bear children are female." Then the classes of relevant individual sentences are:

I1) $M(a) \wedge B(a) \wedge F(a)$

"This female mammal bears children." Confirmation.

I2) $M(a) \wedge B(a) \wedge \neg F(a)$

"This non-female mammal bears children." Falsification; the only falsifying case. This case does not occur.

I3) $M(a) \wedge \neg B(a) \wedge F(a)$

"This female mammal does not bear children." Confirmation (not very relevant).

I4) $M(a) \wedge \neg B(a) \wedge \neg F(a)$

"This non-female mammal does not bear children." Strong confirmation.

I5) $\neg M(a) \wedge B(a) \wedge F(a)$

"This female being, not a mammal, bears children." Confirmation; actually some female serpents bear children.

I6) $\neg M(a) \wedge B(a) \wedge \neg F(a)$

"This non-female being, not a mammal, bears children." Confirmation; probably no instances, but this is an empirical matter.

I7) $\neg M(a) \wedge \neg B(a) \wedge F(a)$

"This female being, not a mammal, does not bear children." Much weaker confirmation than I1 and I4; still some relevance, because a female being must be living.

I8) $\neg M(a) \wedge \neg B(a) \wedge \neg F(a)$

"This non-female being, not a mammal, does not bear children." Very weak confirmation.

Among the 8 cases only I2 falsifies *G*. I6 is a hypothetical case, so our intuition regarding the force of its confirmation is not very reliable. I3 and I5 are analogous to the observation of black non-ravens in the case of "All ravens are black." Only a refined quantitative concept of confirmation could decide in which degree they confirm *G*.

So the cases remaining for closer examination are: I1, I4, I7, I8. I1 is most analogous to the observation of a black raven in the case of "All ravens are black." But, unlike the raven case, I4 confirms *G* almost as strongly as I1, in agreement with the fact that the number of non-female (in practice mostly male) mammals is about equal to that of the female ones. In practice such cases are actually considered for the examination of *G*.

I7 confirms *G* far more weakly, because the class for which it holds is far larger: only living beings may be female, but a considerable part of the species of living beings has a sex dichotomy, and mammals are only a small fraction of them. But since it restricts the universe of discourse to living beings, it has still some relevance.

I8, lastly, confirms *G* most weakly and is analogous to the observation of a not-black non-raven in the raven case. It hardly restricts the universe of discourse: a non-female being may not only be a male living being, but also may be a living being of a species without sex dichotomy, or a non-living being.

So we see a clear degradation of $\mathsf{C}(G, \mathrm{I})$ (the confirmation of *G* with respect to I), if I is consecutively I1 or I4, I7, I8. But the analogy of the relation of I8 to I7 to that of I7 to I1 justifies the assumption that $\mathsf{C}(G, \mathrm{I7}) < \mathsf{C}(G, \mathrm{I1})$ and $\mathsf{C}(G, \mathrm{I8}) < \mathsf{C}(G, \mathrm{I7})$, and that even $\mathsf{C}(G, \mathrm{I8})$, however small, is not zero. Analogously C (All ravens are black, This is not black and not a raven) must also have a non-zero, though very low, value.

Black [1], p. 185, has observed a discrepancy between the confirmation concept of Hempel, for example, according to which, in principle, everything in the universe is relevant for every hypothesis, and the actual practice of investigation. This discrepancy can be eliminated by saying that, in practice, explorers seek instances which confirm (or refute) their hypotheses as strongly as possible.

REFERENCES

[1] Black, M., "Notes on the 'Paradoxes of confirmation'," in *Aspects of Inductive Logic*, edited by J. Hintikka and P. Suppes, North-Holland, Amsterdam (1966).

[2] Hosiasson-Lindenbaum, "On confirmation," *The Journal of Symbolic Logic*, vol. 5 (1940), pp. 133-148.

Amsterdam-West
The Netherlands