

PEIRCE'S PARADOXICAL SOLUTION TO THE
 LIAR'S PARADOX

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1 *Peirce's analysis of the problem (1864-65)* In Lecture 1 and Lecture 3 of a series of lectures presented in 1864-65 (Lectures on the Philosophy of Science delivered at Harvard), Peirce discusses the Liar's Paradox. In Lecture 1 he discusses the sentence, "This very proposition is false.,"; in Lecture 3 he examines the sentence in the form "What is here written is not true." This sentence, as we know, leads to paradoxical conclusions. I will first consider Peirce's analysis of the problem and then his solution to it.

1.1 The Problem Stated

- S1 This very proposition is false.
 S2 What is here written is not true.

Peirce argues that the problem with this sentence is that it is logically meaningless or logically nonsense, where nonsense is defined as "that which has a certain resemblance to a symbol without being a symbol."¹ Each genuine symbol is subject to three systems of formal laws; these are the laws of (1) grammar, (2) logic, and (3) rhetoric. Each symbol to be meaningful must satisfy the formal conditions of grammar, of logic, and of the intelligibility of symbols.² This symbol is grammatically correct but fails to be a genuine symbol because it does not satisfy the formal conditions of logic.

In the case of the above sentence, S1, a logical law, the law of the excluded middle, does not apply. Peirce says,

This is a proposition to which the principle of the excluded middle, namely

1. Ms 340, 1864.

2. Ms 340; Ms 347, 1864-65.

that every symbol must be false or true, does not apply. For if it is false, it is thereby true. And if not false, it is thereby not true.³

A logically meaningful sentence will satisfy the laws of logic. Peirce argues that this logical law does not apply to S1 because this symbol has no object. Logic, Peirce says, is concerned with assertoric propositions.⁴ He says of assertoric propositions, "Propositions which assert always assert something of an object, which is the subject of the proposition."⁵ In the case of S1, however, the proposition "does itself state that it has no object. It talks of itself and only of itself and has no external relation whatever."⁶ That is, the subject of the proposition being the proposition itself, the predicate makes no assertion of an object to which the proposition refers. An assertoric proposition, then, makes reference to an external object, but this proposition "talks of itself and only of itself and has no external relation whatever." "Logical laws," however, "only hold good as conditions of a symbol having an object."⁷

Similarly concerning S2 Peirce says that we get an infinite number of propositions:

What is here written
The statement that that is false
The statement that that is false
The statement that that is false

and so on to infinity.

What are each of these statements about. Each one, it would seem, if about anything at all is about the last statement of this infinite series. However, it being an infinite series, there is no last statement; as such the whole set of statements are about nothing—they have no meaning whatever.⁸

Thus, again the symbol has no object, but having an object is a necessary condition of a symbol if the proposition is to fulfill all logical laws and the code of logical laws must be satisfied for a symbol to be logically meaningful. This symbol having no object then is logically meaningless. Peirce argues in Lecture 6 of the 1864-65 lecture series that in an affirmative proposition the object referred to must be an existent class of things, not a null class.⁹ He explains that contradictory predicates can be asserted of a null class; thus such of a proposition fails to satisfy the code of logical laws and is logically meaningless. The Liar's Paradox is analyzed, then,

3. Ms 340.

4. Ms 743, "Rules of Logic Logically Deduced," June 23, 1860.

5. *Ibid.*

6. Ms 340.

7. *Ibid.*

8. Ms 342, Lecture 3, 1864-65 lecture series.

9. Ms 344, Lecture 6, 1864-65 lecture series.

like an affirmative proposition the subject of which is a null class. During this period he maintains that such logically meaningless propositions are false.¹⁰

In an unpublished paper (Ms 726), Peirce essentially repeats this analysis when considering the proposition, "This very proposition is true." He says,

Why is this absurd? Because it has no reference to an object mediate or immediate. Its subject being the proposition itself, it has no subject except itself. And since by its predicate it only refers to the reference of itself to an object, that object being in turn a reference to an object and so on ad infinitum and it has no object. This shows that if a symbol can have no object it is absurd, which is the same as illogical . . .¹¹

2 *Peirce's solution to the problem (1864-65)* Concerning S1 and S2, it may be argued:

- (1) The statement is logically meaningless, in having no object and thereby failing to satisfy all the laws of logic. As such it is *neither* true nor false.
- (2) The statement is logically meaningless and as such it is not true. But it asserts that it is not true, and thus it is true. The statement then is *both* true and not true.¹²

2.1 Peirce rejects (1) above, maintaining that the view that S1 and S2 are neither true nor false is self-contradictory. He says, "The statement as meaningless is not true; but then it is true after all for it says that it is not true."¹³ It is self-contradictory, it would seem, to say that it is neither true nor false for every proposition considered as an assertion has a truth value according to Peirce. As logically meaningless, the propositions S1 and S2 are at least not true from the point of view of logic. S2 presents a unique case, Peirce argues, for in that the content of the proposition is an assertion that it is not true, it is also true. This solution is stated only in reference to S2. In Lecture 1, where S1 is considered, the problem is similarly analyzed but no solution is presented. Peirce is not committed to equating 'not true' with 'false' on the basis of his statement of S2, but would seem nonetheless to do so in the following, written in support of (2) above:

The fact is that in this proposition truth and not truth—affirmative and negative—this and other—coincide. It stands upon the boundary of the true and the false; and is therefore in both.¹⁴

10. *Ibid.* In 1868 Peirce revised his view to maintain that particular propositions have existential import, universal propositions do not. Also meaningless propositions are said to be true rather than false.

11. Ms 726, "Chapter I, Definition of Logic," 1865.

12. Ms 342.

13. *Ibid.*

14. *Ibid.*

2.2 In support of this view Peirce presents an argument by analogy. He instructs us to consider a sheet of paper, part red, part blue. Every point on this sheet paper is either red or blue. If this is the case, he says, then what color is the boundary line between the two colors? Peirce answers,

It is plainly as much either one as it is the other. We must therefore say that it is both or neither.¹⁵

He says we may argue that,

- (1) It is neither because color resides in a surface not a line. But this view will not suffice, Peirce says, for "as motion is not in any instant in that it is a relation between instants it nonetheless is at any given instant. Similarly, while color requires a surface, it is at every point of that surface and thus this line can be characterized as colored."
- (2) The boundary is both red and green. Peirce argues, "If that line is not red it lies without the red part of the sheet therefore, if I simply draw away the red portion, I cannot affect the color of that line which lies without it. Accordingly that line and whatever the moving boundary passes over is neither red nor green; but it may pass over the whole sheet and therefore the whole sheet is neither red nor green. But it clearly is green. It seems to me, therefore, that the proper answer is that the boundary is both red and green;—the distinction between them vanishing at this point."¹⁶

Analogously the above proposition is both true and false. To say that it is neither true nor false, we are told, is self-contradictory; but saying that it is both true and not-true, as a limiting case where the distinction between affirmation and negation vanishes, is not self-contradictory.

Since the proposition is logically meaningless, in referring to no object, it is analogous to a proposition the subject of which refers to a null class. Such a subject will admit of contradictory predicates; it is on these grounds that we say that the proposition represents no actual object, is logically meaningless, and thus is false. But in the case of the Liar's Paradox this proposition asserts that what is here written is false; as such, the proposition, having no object and thus being logically meaningless and false, what the proposition says of itself, i.e., what is here written is false, is seen to be true. That is, "this proposition so far as it is spoken about by itself is false but so far as it speaks about itself is true."¹⁷ Peirce adds,

But this is a distinction without a difference. The question is whether this proposition is in all respects true. If it is not in all respects true, then it is in all respects true; for two reasons 1st because what it says cannot in

15. *Ibid.*

16. *Ibid.*

17. *Ibid.*

that case be altogether true and 2nd because it is seen to accord precisely with what is in all respects true; namely, that it isn't in all respects true.¹⁸

From this viewpoint, Peirce argues that we must say that this proposition is both true and false.

3 *Peirce's revised analysis of the Liar's Paradox (1868ff.)* In 1868 and thereafter, Peirce argues (5.340) that

a proposition is true only if whatever is said in it is true, but is false if anything said in it is false.

Every proposition is said to be either true or false. If anything said in the proposition is false, then it is false. Otherwise it is true.

Concerning the Liar's Paradox, he argues in 1868 that this proposition signifies either more or less than is explicitly asserted.¹⁹ If it signifies less then it signifies nothing and is meaningless. But, Peirce argues, the proposition in question has a meaning. He rejects his argument of 1864-65, maintaining that this proposition means more than is explicitly stated.

Peirce argues that every proposition besides what it explicitly asserts also tacitly implies its own truth. The Liar's Paradox expressly asserts about itself that it is false. That is, the proposition, which we shall call S2, "What is here written is not true." explicitly asserts that S2 is not true. Further, Peirce argues, following the argument of Paul of Venice, that every proposition also tacitly implies its own truth. Part of the meaning of a proposition is that the proposition asserted is true. That is, every proposition p implies a proposition referring to p which states that p is true. A formal condition of any asserted proposition is

$$p \vdash 'p' \text{ is true.}$$

Thus the proposition means more than is explicitly asserted. It means both "The proposition ' p ' is not true." and "The proposition ' p ' is true.", considering the proposition in the light of what is explicitly asserted and tacitly implied by any proposition. A proposition explicitly asserts what is stated in the proposition and tacitly implies the truth of that assertion. In the case of S2 this results in a contradictory proposition. What is explicitly asserted is ' p ' is not true and what is tacitly implied is ' p ' is true. Peirce concludes,

The proposition in question, therefore, is true in all other respects but in its implication of its own truth.²⁰

In that the proposition thus involves a contradiction, it is false.

Peirce maintains this position in regard to the Liar's Paradox after 1868. He thus says (3.447, 1896),

18. *Ibid.*

19. 5.340.

20. 5.340.

'This proposition is false.', far from being meaningless, is self-contradictory. That is, it means two irreconcilable things. That it involves contradiction (that is, leads to contradiction if supposed true), is easily proved. For if it be true, it is true; while if it be true, it is false. Every proposition besides what it explicitly asserts, tacitly implies its own truth. The proposition is not true unless *both* what it explicitly asserts and what it tacitly implies, are true. This proposition, being self-contradictory, is false; and hence, what it explicitly asserts is true. But what it tacitly implies (its own truth) is false.

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