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CORRIGENDUM TO MY PAPER: "RECOGNIZABLE ALGEBRAS OF FORMULAS"

JOHN GRANT

By constructing a counterexample Martin Ziegler has shown that Theorems 1 and 2 are incorrect in the paper "Recognizable algebras of formulas," this Journal, Volume XIII (1972), pp. 521-526. The proof given in the paper breaks down unless only existential (or universal) formulas are considered about recognizable algebras of formulas.

We explain the problem where (as in the counterexample) $\mathcal{L} = \mathcal{L}_{\omega\omega}$ and there is one alternation of quantifiers. Suppose that we are given a recognizable algebra of formulas R, sentences $J_1 = (\exists y_1)(\exists y_2)J(y_1, y_2)$, $J_2 = (\forall y_1)(\exists y_2)J(y_1, y_2)$ in the language of R and $\mathfrak{A} \equiv \mathfrak{B}$. Let the translation of $J(y_1, y_2)$ be $J^*(Y_1, Y_2)$ in the proof of Theorem 1. Then it is claimed in the proof that $R(\mathfrak{A}) \models J_1$ iff there exists an allowable ψ_1 and there exists an allowable ψ_2 such that $\mathfrak{A} \models J^*(\psi_1, \psi_2)$. Also $R(\mathfrak{B}) \models J_1$ under the same condition. Thus $R(\mathfrak{A}) \models J_1$ iff $R(\mathfrak{B}) \models J_1$ and so the proof goes through for existential sentences. But it is also claimed in the proof that $R(\mathfrak{A}) \models J_2$ iff for every allowable ψ_1 there exists an allowable ψ_2 such that $\mathfrak{A} \models J^*(\psi_1, \psi_2)$. This claim is incorrent. For notice the implicit assumption that given a ψ_1 there exists a unique ψ_2 such that $\mathfrak{A} \models J^*(\psi_1, \psi_2)$. However ψ_1 by the use of different sequences of parameters of $\mathfrak{A}$ may define many elements of $R(\mathfrak{A})$. Then there need not exist a unique ψ_2 such that $\mathfrak{A} \models J^*(\psi_1, \psi_2)$ even though $R(\mathfrak{A}) \models J_2$.

The corrections in the statements are as follows: Page 523, line 25: change "$\mathcal{L}$-≡" to "$\mathcal{L}$-existentially equivalent to."
Page 524, line 26: insert "existential" between "an" and "$\mathcal{L}$-embedding".
Page 524, line 41: omit "and to".
Page 524, line 42: insert "to a category of algebras (the maps being existential $\mathcal{L}$-embeddings)" between ")" and ".".

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University of Florida
Gainesville, Florida