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K1 AS A DAWSON MODELLING OF A. R. ANDERSON'S SENSE OF "OUGHT"

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Alan Ross Anderson once wrote "from a formal point of view we may regard deontic logic simply as a branch of alethic logic." This claim from p. 178 of [1], a reprinting of his "The Formal Analysis of Normative Systems," is not strictly correct.*

To use Anderson's sense of "ought" is to define the deontic formulae: $O(p)$ and $P(p)$, by one of the following patterns.

$$\text{Pattern I:} \quad \begin{aligned} O(p) &=_{df} L(\sim p \supset \mathbf{S}) \\ P(p) &=_{df} \sim O(\sim p) \end{aligned}$$

$$\text{Pattern II:} \quad \begin{aligned} P(p) &=_{df} M(p . \sim \mathbf{S}) \\ O(p) &=_{df} \sim P(\sim p) \end{aligned}$$

Of course, $O(p)$ symbolizes "It ought to be that p" while $P(p)$ symbolizes "It is permitted that p." $L(p)$ symbolizes "It is necessary that p" and $M(p)$ symbolizes "It is possible that p." In these patterns, $\mathbf{S}$ represents a contingent proposition saying that a sanction has been incurred. I say that Anderson's sense of "ought" is given by definition patterns rather than by definitions because they give only a recipe for defining "ought." We do not have a definition until we have a logic for $L(\ )$ and $M(\ )$ and specify exactly what $\mathbf{S}$ says. In this essay I shall not discuss the adequacy of using such a pattern for defining "ought."

Anderson uses the second pattern in [1] when he investigates ways of developing deontic logic within different systems of alethic logic. However, he uses the first pattern when he discusses and defends his patterns for defining "ought" and "permitted." Anderson discusses and defends these patterns on pages 170-171 and 200-205 of [1] as well as in [2] and [3]. In 1967 in [5], Anderson continued to defend the basic idea behind his pattern for defining "ought," viz. to say that you ought to do something is to say

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that if you do not do it something bad will happen. Anderson's patterns for defining the deontic notions have been carefully criticized by E. J. Lemmon and P. H. Nowell-Smith in [12], by von Wright in [19], but especially by H. Castañeda in [8]. I should note that Anderson in [4] and [5], and L. F. Goble in [10] modify Anderson's early approach to deontic logic. The requirement that "ought" be defined by use of strict implication has been dropped since they do not want to say that ought-statements are necessarily true if true. For the same reason Anderson avoids defining "ought" in terms of entailment between non-performance and incursion of a sanction. In [5], he studies using relevant implication.

Still, as interesting as these new developments are, I shall restrict my attention to Anderson's early strict implication analysis of deontic operators. My aim is to show how Anderson's development of deontic logic, based on use of strict implication, can be reduced completely to alethic logic and to draw some consequences about iteration of deontic and alethic operators.

Use of Pattern II enabled Anderson to develop deontic propositional logics simply by providing alethic logics plus some conditions on the propositional constant **S**. Anderson does require that the alethic logics be what he calls "normal." But these are very weak requirements. An alethic system **AS** is normal if it meets the following conditions.

a) All theorems of classical propositional logic are theorems of **AS**.
b) Equivalent formulae from the classical propositional calculus can be substituted for one another.
c) i) $p \supset M(p)$ is a theorem of **AS**.
   ii) $M(p \vee q) \equiv (M(p) \vee M(q))$ is a theorem of **AS**.
   iii) $L(p) \equiv {\sim}M({\sim}p)$ is a theorem of **AS**.
   iv) $L(p \supset p)$ is a theorem of **AS**.
   v) $M(p) \supset p$ *is not* a theorem of **AS**.

He also requires that the deontic logics developed be normal deontic logics. A deontic system **D** is normal if it meets conditions (a) and (b) of alethic normalcy plus those of (d) below.

d) i) $O(p) \supset P(p)$ is a theorem of **D**.
   ii) $P(p \vee q) \equiv (P(p) \vee P(q))$ is a theorem of **D**.
   iii) $O(p) \equiv {\sim}P({\sim}p)$ is a theorem of **D**.
   iv) $P(p) \supset p$ *is not* a theorem of **D**.
   v) $p \supset P(p)$ *is not* a theorem of **D**.
   vi) If **D** is an extension of an alethic system, $M(p) \supset P(p)$ *is not* a theorem of **D**.

Anderson finally requires that $M({\sim}\mathbf{S})$ be a theorem of the deontic logics developed. He lays down this last requirement because a punishment which is necessary, i.e., comes whatever may happen, is no punishment but is instead a universal disaster. But did he reduce deontic to alethic logic?

Section III of [1] is entitled "Reduction of Deontic Logic to Alethic Logic." In section II he showed that the sole addition of $M({\sim}\mathbf{S})$ as an axiom —axiom 15 on p. 171 of [1]— to a normal alethic system gives a normal deontic logic if $P(p)$ and $O(p)$ are defined by Pattern II. In section III he carries the reduction even further by showing that it is not necessary to add the axiom $M(\sim\mathbf{S})$. He shows that we can develop a normal deontic logic if we simply add a constant **B** to a normal alethic logic. (Upon interpretation the propositional constant **B** would say that a sanction has been incurred.) On line 15 of p. 176 of [1], he ends a proof that $M(\sim(M(\sim p).p))$ is a theorem of any normal alethic system. Hence, if **B** is in the system and if we have uniform substitution for propositional variables, as we always shall for systems in this essay, $M(\sim(M(\sim\mathbf{B}).\mathbf{B}))$ is also a theorem. Anderson then defines **S** as $M(\sim\mathbf{B}).\mathbf{B}$. (As defined **S** says that a sanction that need not be incurred is incurred.) With this definition of **S**, he of course has $M(\sim\mathbf{S})$ as a theorem. Hence, the results of his section II give him a normal deontic logic. It is at this point that he makes the claim quoted at the beginning of this essay. I contend, though, that the claim is not strictly correct because he needs to supplement alethic systems with the constant **B** to develop deontic systems. I grant that Anderson is correct in noting that for formal manipulations we do not need to pay attention to the intended interpretation of **B**. Still, we can see **B** when we do deontic logic but not when we do plain alethic logic. And from a formal point of view what we can and cannot see is extremely significant. So, as I see it, the problem of reducing deontic to alethic logic in an Anderson development of deontic logic is the problem of eliminating the constant **B**. To show how to eliminate the constant **B**, I shall introduce the notion of a Dawson modelling.

In [9], E. E. Dawson showed that if we abbreviated $ML(p)$ as $O(p)$ and $LM(p)$ as $P(p)$, S4.2 contained a normal deontic logic.[1] Since S4.2, as well as all other systems discussed in this essay, meet conditions (a), (b), and (c) of alethic and deontic normality we shall pay attention only to those conditions in item (d) of the definition of "deontic normalcy." For Dawson this involved showing that the following were theorems of S4.2: $ML(p) \supset LM(p)$, viz. $O(p) \supset P(p)$, $LM(p \vee q) \equiv (LM(p) \vee LM(q))$, viz. $P(p \vee q) \equiv (P(p) \vee P(q))$, and $ML(p) \equiv \sim LM(\sim p)$, viz. $O(p) \equiv \sim P(\sim p)$, but that the following were not theorems of S4.2: $LM(p) \supset p$, viz. $P(p) \supset p$, $p \supset LM(p)$, viz. $p \supset P(p)$, and $M(p) \supset LM(p)$, viz. $M(p) \supset P(p)$. In [6], Lennart Åqvist showed that S4 with $LM(p)$ abbreviated as $O(p)$ and $MLM(p)$ abbreviated as $P(p)$ contains a normal deontic logic. Åqvist also showed that S3 with $LLML(p)$ abbreviated as $O(p)$ and $MMLM(p)$ as $P(p)$ contains a normal deontic logic. Let us recall the notion of an irreducible modality and then, in light of these examples define a Dawson modelling. A modality is any unbroken sequence of zero or more of the unary operators: $\sim$, $L(\ )$, and $M(\ )$. A modality, mod.X, is irreducible in a system if it is not provable in the system that mod.X is equivalent to any other modality. If **AS** is a normal

1. Some information on all alethic systems referred to in this essay can be found in the excellent modal logic textbook [11].

alethic logic, mod.1 and mod.2 are irreducible modalities in **AS**, mod.1 is abbreviated as $O(p)$ and mod.2 as $P(p)$, then mod.1 and mod.2 provide *a Dawson modelling in* **AS** *for "ought" and permitted* if $O(p)$ and $P(p)$ meet the conditions of item (d) for deontic normalcy. A Dawson modelling is a complete reduction of deontic to alethic logic since in a Dawson modelling no signs appear save those of alethic logic or abbreviations of them.

Let us pose for ourselves this problem: Can we eliminate the constant **B** so that we can get Dawson modellings which can be called Dawson modellings for Anderson's way of defining "ought" and "permitted"? The answer will make the question clear. How could Anderson use his patterns for defining $O(p)$ and $P(p)$ without the propositional constant **B**? When he has only the constant **B** he uses Pattern II to define $P(p)$ as: $M(p . \sim(M(\sim \mathbf{B}) . \mathbf{B}))$, where $M(\sim \mathbf{B}) . \mathbf{B}$ is the unabbreviated form of **S**. If Anderson did not have **B** the defining formula for $P(p)$ would be: $M(p . \sim(M(\sim q) . q))$ where both p and q are propositional variables. We can still regard $M(p . \sim(M(\sim q) . q))$ as providing the basis for defining $P(p)$. However, a definition of the unary $P(p)$ as $M(p . \sim(M(\sim q) . q))$ would have the undesirable result of not letting us assign a value to $P(p)$ simply upon an assignment of a value to p. So, to keep $P(p)$ unary, I shall require that the variable p be used where q is used in $M(p . \sim(M(\sim q) . q))$. Consequently, I give the following definition of $P(p)$. No constant Andersonian definition of $P(p)$:

$$P(p) =_{df} M(p . \sim(M(\sim p) . p)).$$

Before I simplify this definition of $P(p)$ to $ML(p)$, I should confess that my sole reason for calling the resulting Dawson modellings "Dawson modellings for Anderson's sense of 'ought'" is that they arise from the preceding alterations in Anderson's definition of $P(p)$. Even if such alterations are a serious distortion of Anderson's development of deontic logic, I think that the sequel will show that the resulting deontic logics are interesting for their own sake.

To carry out the argument that $P(p)$ as defined above is equivalent to $ML(p)$, I have to strengthen condition (b) of alethic normalcy so that it reads: "provably equivalent formulae can be substituted for one another." All systems that Anderson considers in [1], except the very first one he considers called X, have this stronger replacement rule. Abbreviate the name of this replacement rule as: Repl. In the following derivation, Repl. i, j says that the formula on the right of $\equiv$ in line i has replaced in line j some or all of the occurrences of the formula on the left of $\equiv$ in line i. I shall mark with N equivalences specifically required by conditions (c) of alethic normalcy. The following derivation is trivial, but I shall give it since it is crucial for my identification of $P(p)$ with $ML(p)$.

1)	$M(p . \sim(M(\sim p) . p)) \equiv M(p . \sim(M(\sim p) . p))$,	$A \equiv A$.
2)	$\sim(M(\sim p) . p) \equiv \sim M(\sim p) \vee \sim p$,	$\sim(A . B) \equiv \sim A \vee \sim B$.
3)	$M(p . \sim(M(\sim p) . p)) \equiv M(p . (\sim M(\sim p) \vee \sim p))$,	Repl. 2, 1.
4)	$\sim M(\sim p) \equiv L(p)$,	N.
5)	$M(p . \sim(M(\sim p) . p)) \equiv M(p . (L(p) \vee \sim p))$,	Repl. 4, 3.

6)	$p.(L(p) \vee \sim p) \equiv (p.L(p) \vee p.\sim p)$,	Distribution.
7)	$M(p.\sim(M(\sim p).p)) \equiv M(p.L(p) \vee p.\sim p)$,	Repl. 6, 5.
8)	$(p.L(p) \vee p.\sim p) \equiv p.L(p)$,	$(A \vee p.\sim p) \equiv A$.
9)	$M(p.\sim(M(\sim p).p)) \equiv M(p.L(p))$	Repl. 8, 7.
10)	$p.L(p) \equiv L(p)$	N.
11)	$M(p.\sim(M(\sim p).p)) \equiv M(L(p))$,	Repl. 10, 9.

Line 11 and the no constant definition of $P(p)$ enable us to identify $P(p)$ with $ML(p)$. Now since I am following Anderson's Pattern II, $O(p)$ will be identified with: $\sim ML(\sim p)$. In any normal alethic logic where provable equivalents are intersubstitutable we have: $\sim ML(\sim p) \equiv LM(p)$. So I shall identify $O(p)$ with $LM(p)$. With these identifications we have what I call a Dawson modelling for Anderson's sense of "ought" if we have an alethic system in which $ML(p)$ and $LM(p)$ are irreducible modalities, in which the formulae in List I are theorems, in which the formulae in List II are not theorems, and which is normal in the strong form of having Repl.

List I

1)	$LM(p) \supset ML(p)$	1′)	$O(p) \supset P(p)$
2)	$ML(p \vee q) \equiv (ML(p) \vee ML(q))$	2′)	$P(p \vee q) \equiv (P(p) \vee P(q))$
3)	$LM(p) \equiv \sim ML(\sim p)$	3′)	$O(p) \equiv \sim P(\sim p)$

List II

1)	$p \supset ML(p)$	1′)	$p \supset P(p)$
2)	$ML(p) \supset p$	2′)	$P(p) \supset p$
3)	$M(p) \supset ML(p)$	3′)	$M(p) \supset P(p)$

Are there any such Dawson modellings? System K1 of Sobociński/McKinsey provides such a Dawson modelling. Informally K1 is S4 plus $LM(p) \supset ML(p)$ as and additional axiom. The rest of the essay will presuppose some familiarity with K1. (See McKinsey [13], Sobociński [17], [18], Prior [15], Bull [7], and pp. 265-267 of [11] for some background on K1.) When we allow $O(\ )$ to abbreviate $LM(\ )$ and $P(\ )$ to abbreviate $ML(\ )$, I shall call K1 "DK1." DK1 is the only Dawson modelling for Anderson's sense of "ought" to which I shall give any attention in this essay. I want to consider what DK1 shows about the iteration of deontic operators, about the juxtaposition of alethic and deontic operators, deontic operators lying within the scope of deontic operators, and to close with some philosophical speculation on the significance of DK1 for ethics.

The sequences of equivalences below show that in DK1 all iterations of deontic operators reduce to $O(p)$ or $P(p)$ and that juxtaposition of an alethic operator with a deontic one gives a deontic operator. The K1 version of each sequence will enable the reader, familiar with K1, quickly to verify the equivalences.

The ought-sequence:

DK1 version) $O(p) \equiv OO(p) \equiv OP(p) \equiv LO(p) \equiv LP(p) \equiv OL(p) \equiv OM(p)$
K1 version) $LM(p) \equiv LMLM(p) \equiv LMML(p) \equiv LLM(p) \equiv LML(p) \equiv LML(p) \equiv LMM(p)$

The permitted-sequence:

DK1 version) $P(p) \equiv PO(p) \equiv PP(p) \equiv MO(p) \equiv MP(p) \equiv PL(p) \equiv PM(p)$
K1 version) $ML(p) \equiv MLLM(p) \equiv MLML(p) \equiv MLM(p) \equiv MML(p) \equiv MLL(p) \equiv MLM(p)$

Of course, of more importance than the mere drawing of the logical consequences listed above is a detailed ethical discussion of whether or not we can accept these equivalences as expressing moral truths. Such an ethical discussion is beyond the scope of this essay. I think that a major value of this Dawson modelling is that it quickly leads us to consider our moral beliefs in order to see whether or not we can accept these equivalences. Even though it is beyond the scope of this essay, let me say a word about the fact that we have $O(p) \supset LO(p)$ in DK1 because it seems to reflect a profound fact about morality. If I think that something is obligatory, I cannot think of what it would be like for it not to be obligatory. For instance, if I really think that artificial birth control is wrong, I cannot think that it can be made permissible simply by the Pope saying that it is permitted. Ask yourself whether or not you can think of what it would be like to have torturing an infant for amusement permissible.

Ethical reflection may lead one to think that we cannot accept reduction to $O(p)$ and $P(p)$ of all iterations of deontic operators and juxtapositions of alethic operators with deontic ones. Still one may want to accept identification of $P(p)$ with $ML(p)$ and $O(p)$ with $LM(p)$. In this case, one could investigate systems such as the following to see whether they gave results closer to our moral intuitions. Consider S2 or S3 with $L(LM(p) \supset ML(p))$ and $L(ML(p \vee q) \supset (ML(p) \vee ML(q)))$ as new axioms or T with these axioms as material implications.

In DK1 do we have to have any deontic operators lying in the scope of other deontic operators? For instance, can we take a formula such as $O(p \supset O(p))$ and find an equivalent version in which no deontic operator occurs in the scope of another? Of course, one can trivialize the problem by noting that we never need to have an $O(\ )$ or a $P(\ )$ in the scope of another $O(\ )$ or $P(\ )$ because we can simply un-abbreviate any $O(\ )$ or $P(\ )$ back into $LM(\ )$ or $ML(\ )$. So let us ask if every formula in K1 is equivalent to a formula in which no sequence LM or ML is in the scope of another LM or ML. Possibly, the result of Makinson and Schumn in [14] and [16] that in K1 there are infinitely many non-equivalent formulas in a single variable suggests that the answer is: No. However, we can observe that in K1 every formula is equivalent to one in which no $L(\ )$ or $M(\ )$ lies within the scope of any LM or ML. The observation is readily established by considering that the formulae are written with only: L, M, $\vee$, $.$, $\sim$, with no $\sim$ in front of any L or M, and reduced so that there are no sequences of Ls and Ms longer than LM and ML. Now the K1 equivalences listed below plus those on the K1 versions of the ought and permitted equivalence sequences show how any formula with an L or M in the scope of an LM or an ML can be reduced to one in which the scope of LM or ML is over a shorter formula if there are any LM or ML at all.

1)	$ML(q \vee ML(p)) \equiv ML(q) \vee ML(p)$,	1′)	$P(q \vee P(p)) \equiv P(q) \vee P(p)$
2)	$ML(q \vee LM(p)) \equiv ML(q) \vee ML(p)$,	2′)	$P(q \vee O(p)) \equiv P(q) \vee P(p)$
3)	$LM(q\,.\,LM(p)) \equiv LM(q)\,.\,LM(p)$,	3′)	$O(q\,.\,O(p)) \equiv O(q)\,.\,O(p)$
4)	$LM(q\,.\,ML(p)) \equiv LM(q)\,.\,LM(p)$,	4′)	$O(q\,.\,P(p)) \equiv P(q)\,.\,O(p)$
5)	$ML(q\,.\,ML(p)) \equiv M(L(q)\,.\,LM(p))$,	5′)	$P(q\,.\,P(p)) \equiv M(L(q)\,.\,O(p))$
6)	$ML(q\,.\,LM(p)) \equiv M(L(q)\,.\,LM(p))$,	6′)	$P(q\,.\,O(p)) \equiv M(L(q)\,.\,O(p))$
7)	$LM(q \vee LM(p)) \equiv L(M(q) \vee ML(p))$,	7′)	$O(q \vee O(p)) \equiv L(M(q) \vee P(p))$
8)	$LM(q \vee ML(p)) \equiv L(M(q) \vee ML(p))$,	8′)	$O(q \vee P(p)) \equiv L(M(q) \vee P(p))$
9)	$ML(q \vee M(p)) \equiv ML(q) \vee ML(p)$,	9′)	$P(q \vee M(p)) \equiv P(q) \vee P(p)$
10)	$ML(q \vee L(p)) \equiv ML(q) \vee ML(p)$,	10′)	$P(q \vee L(p)) \equiv P(q) \vee P(p)$
11)	$LM(q\,.\,M(p)) \equiv LM(q)\,.\,LM(p)$,	11′)	$O(q\,.\,M(p)) \equiv O(q)\,.\,O(p)$
12)	$LM(q\,.\,L(p)) \equiv LM(q)\,.\,LM(p)$,	12′)	$O(q\,.\,L(p)) \equiv O(q)\,.\,O(p)$
13)	$ML(q\,.\,M(p)) \equiv M(L(q)\,.\,LM(p))$,	13′)	$P(q\,.\,M(p)) \equiv M(L(q)\,.\,O(p))$
14)	$ML(q\,.\,L(p)) \equiv M(L(q)\,.\,L(p))$,	14′)	$P(q\,.\,L(p)) \equiv M(L(q)\,.\,L(p))$
15)	$LM(q \vee M(p)) \equiv L(M(q) \vee M(p))$,	15′)	$O(q \vee M(p)) \equiv L(M(q) \vee M(p))$
16)	$LM(q \vee L(p)) \equiv L(M(q) \vee ML(p))$,	16′)	$O(q \vee L(p)) \equiv L(M(q) \vee P(p))$

Two examples may clarify the reduction process. Consider the idealistic: $O(p \supset O(p))$. Write it as: $LM(\sim p \vee LM(p))$. By equivalence (7) above this reduces to: $L(M(\sim p) \vee ML(p))$. And this last formula can be rewritten as: $L(L(p) \supset P(p))$ in DK1. This reduction tells us that if someone utters the idealistic "It ought to be that whatever happens is something that ought to be," he is saying that the necessity of something strictly implies its permissibility. $O(O(p) \supset p)$ is also idealistic. It says 'It ought to be that if something ought to be it happens." Write it as $LM(ML(\sim p) \vee p)$ and use equivalence (8) to get: $L(ML(\sim p) \vee M(p))$. This last formula can be rewritten in DK1 as: $L(L(\sim p) \supset P(\sim p))$. I grant that the deontic version of many of the equivalences above paralyzes our moral intuitions. However, trying to see what could lead us to assert such equivalences belongs to the ethical evaluation of DK1.

I shall now close with some philosophical speculations about DK1. In [7] Castañeda admitted that it is not perfectly clear what the naturalistic fallacy is. However, Castañeda points out that we are clear enough about the naturalistic fallacy to know that if we reduce moral statements to claims of logical necessity we have reduced morality to something which is not morality. He suggests that this is just what Anderson did by equating ought-statements with claims of a strict implication via definition Pattern I. But if this Dawson modelling is not a total distortion of Anderson's sense of "ought," it provides a threefold defense against a charge that he reduced moral statements to logical ones. First the fact that $O(p)$, viz. $LM(p)$, is not equivalent to $L(p)$ shows that ought-statements are not assertions of logical necessity if that is what $L(\ )$ is to be used for. The ought-operator contains $L(\ )$ but it is not $L(\ )$. The fact that LM is an irreducible modality in K1 can be construed as showing that $O(p)$ is what it is and nothing else as G. E. Moore required at the beginning of his *Principia Ethica*. Secondly, the fact that $O(p)$, viz. $LM(p)$, is not a K1 theorem shows that not all ought-statements are logical truths in the sense of being provable formulae.

Thirdly, and of most significance for showing that there is no reduction of moral claims to logical claims, is the fact that any Dawson modelling for Anderson's sense of "ought" will have: $LM(p) . LM(q) \supset LM(p . q)$, as a theorem. To be sure, this is not the elementary modal fallacy: $M(p) . M(q) \supset M(p . q)$. But it is close to it! It is not at all clear that any natural, i.e., used, sense of "necessity" and "possibility" could fit into it. I am not saying that what is alethically bad is deontically good. I am saying only that if a reduction of deontic to alethic logic requires an alethic logic with no natural interpretation for $L(\ )$ and $M(\ )$ as necessity and possibility, the claim that such a reduction is a reduction of the moral to the non-moral is weakened.

This suggestion that K1 involves a sense of "necessity" with a moral tinge together with the identification of $P(p)$ with $ML(p)$ leads me to close with a speculation about Kant's deontic logic. This speculation suggests that looking at the K-family of modal systems may be helpful towards understanding how Kant used "necessity." Consider Kant's "Act only on that maxim whereby thou canst at the same time will that it should become a universal law." This suggests that an action A is permissible if and only if its maxim p_a meets the above condition. It is not too rash to read p_a's meeting the above condition as: $ML(p_a)$, where $ML(p_a)$ would say "it could be that maxim p_a is a necessary truth in the sense of being a natural law." So, maybe we could start to develop a Kantian deontic logic by identifying his "permitted" with $ML(p)$ where p is somehow restricted to what can be called maxims.

I shall not pursue these Kantian speculations any further here. I shall simply note the curiosity that we began by investigating the totally heteronomous sense of "ought" of A. R. Anderson but ended up by speculating that we may have a deontic logic for the autonomous sense of Kant.[2]

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2. Some of the basic ideas of this essay were sketched at the Association of Symbolic Logic meetings held at Atlantic City, New Jersey on January 21-22, 1971. I should add that the stimulus for this essay came when Charles Eaker of the SUNY college at Oswego asked me what would happen if Anderson tried to develop deontic logic without adding anything to alethic logic.

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