Notre Dame Journal of Formal Logic
Volume XV, Number 2, April 1974
NDJFAM

THE INADEQUACY OF HUGHES AND CRESSWELL'S SEMANTICS FOR THE CI SYSTEMS

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The purpose of this note is to show that the semantics developed by Hughes and Cresswell in [1], pp. 198-199, for the contingent identity systems T + CI, S4 + CI, and S5 + CI is inadequate in that none of these systems is sound with respect to the corresponding notion of validity. Since theorems of LPC are theorems of each of the CI systems,

(i) $$\varphi x_0 x_1 \supset (\exists x_1)(\exists x_0)\varphi x_1 x_0$$

is a theorem in each of the CI systems. (We suppose variables to be indexed by the non-negative integers.) However, (i) is not S5 + CI-valid and so, is not T + CI- or S4 + CI-valid. We construct an S5 + CI countermodel to (i) as follows. Let $W = \{w\}$, $R = \{\langle w, w\rangle\}$, D = the set of non-negative integers, and for each variable x_i, let $V_1(x_i, w) = i$ and let $V_1(\varphi) = \{\langle\langle 0, 1\rangle, w\rangle\}$. Finally, let θ be the smallest set of value-assignments A such that $V_1 \epsilon A$ and if $V \epsilon A$, a and b are variables, and V' is a value-assignment which is the same as V except that $V(\mathsf{a}, w) = V'(\mathsf{b}, w)$, then $V' \epsilon A$. Evidently, $\langle W, R, D, V_1, \theta\rangle$ is an S5 + CI-model. Moreover, $V_1(\varphi x_0 x_1, w) = 1$ since

$$\langle\langle V_1(x_0, w), V_1(x_1, w)\rangle, w\rangle = \langle\langle 0, 1\rangle, w\rangle \epsilon V_1(\varphi).$$

A bit of computation reveals that $V_1((\exists x_1)(\exists x_0)\varphi x_1 x_0, w) = 1$ only if there is a $V \epsilon \theta$ differing from V_1 only in assignment to x_0 and x_1 such that $V(\varphi x_1 x_0, w) = 1$, i.e., $\langle\langle V(x_1, w), V(x_0, w)\rangle, w\rangle \epsilon V(\varphi) = V_1(\varphi)$. Only that value-assignment V which makes $V(x_0, w) = 1$, $V(x_1, w) = 0$, and which is otherwise the same as V_1 satisfies the second part of the condition, but this $V \not\epsilon \theta$. To see this last, we note that a simple induction on θ shows that for any $V' \epsilon \theta$, either $V' = V_1$ or $\{V'(\mathsf{a}, w)$: a is a variable$\}$ is a *proper* subset of D. Since V fails to satisfy this condition, $V \not\epsilon \theta$. So, $V_1((\exists x_1)(\exists x_0)\varphi x_1 x_0, w) = 0$. So, $V_1((i), w) = 0$. So, (i) is not S5 + CI-valid.

We conjecture that the following modification in the condition on θ in a model $\langle W, R, D, V_1, \theta\rangle$ will yield an adequate semantics: for every $V \epsilon \theta$ and for any individual variables $\mathsf{a}_1, \ldots, \mathsf{a}_n, \mathsf{b}_1, \ldots, \mathsf{b}_n$, there is a $V' \epsilon \theta$ which is the same as V except that $V(\mathsf{a}_1, w) = V'(\mathsf{b}_1, w), \ldots,$ and $V(\mathsf{a}_n, w) = V'(\mathsf{b}_n, w)$ for every $w \epsilon W$. We shall not pursue that question here.

Received June 28, 1973

REFERENCE

[1] Hughes, G. E., and M. J. Cresswell, *An Introduction to Modal Logic*, London (1968).

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