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LES PROPRIÉTÉS DU FONCTEUR NICOD PAR RAPPORT À LA RÉCIPROCITÉ ET CONJONCTION. II

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Théorème 4.* *Les formes du groupe B admettent la forme normale*:

$$\mathbf{N}_4(D) = R^{\mathfrak{N}}\left(p_{n_v}^{v}\right)^{v}\left(p_{m_{v-1}}^{v-1}\right)^{v-1}\left(p_{m_{v-2}}^{v-2}\right)^{v-2}\ldots\left(p_{m_2}^{2}\right)^{2}p_{m_1}\left[\prod_{i=1}^{v-1}\left(Kp_{m_{v-1}}^{v-i}p_{m_{v-i}-1}^{v-i}K^{2}p_{m_{v-i}}^{v-i}\right.\right.$$
$$\left.\left.p_{m_{v-i}-1}^{v-i}p_{m_{v-i}-2}^{v-i}\ldots K^{m_{v-i}-3}p_{m_{v-i}-1}^{v-i}\ldots p_3^{v-i}K^{m_{v-i}-1}p_{m_{v-i}}^{v-i}\ldots p_1^{v-1}\right)^{v-i}\right]$$
$$\left[\left(\prod_{h=2}^{v}\prod_{i_k=0}^{m_{v-k+1}-3}K^{\mathfrak{M}}\prod_{u=1}^{h}\prod_{j=0}^{i_u}p_{m_{v-m+1}-j}^{v-u+1}\prod_{h=1}^{u-1}\prod_{j=1}^{h}K^{i_j+m_{u(v,v-1,\ldots,v-h)}}\prod_{t=0}^{i_j}p_{m_{v-h+1}-t}^{v-h+1}\right.\right.$$
$$\left.\prod_{j=0}^{m_{u(v,\ldots,v-h)}-1}p_{m_{u(v,\ldots,ph)}-t}^{u(v,\ldots,v-h)}\right)\left(K^{m_v+m_{v-1}-1}\prod_{t=0}^{m_v-1}p_{m_v-t}^{v}\prod_{t=0}^{m_{v-1}-1}p_{m_{v-1}-t}^{v-1}\right)\ldots$$
$$\left.K^{m_v+m_{v-1}+\ldots+m_1-1}\prod_{j=0}^{m_v-1}p_{m_v-j}^{v}\ldots\prod_{j=0}^{m_1-1}p_{m_1-j}^{1}\right]$$

où nous avons:

$$\mathfrak{M} = \sum_{u=1}^{h_i+h-1} u \qquad et \qquad \mathfrak{N} = \sum_{j=1}^{v}\sum_{i=1}^{j} m_i - v - 1$$

et où nous avons utilisé les notations du Théorème 3.

Nous démontrerons ce théorème. D'après du Théorème 2 nous avons:

$$\alpha = D^{v-1}D^{m_1-1}\prod_{i=1}^{m_1}p_iD^{m_2-1}\prod_{i=1}^{m_2}p_i\ldots D^{m_v-1}\prod_{i=1}^{v}p_i^{v}$$
$$\sim R^{v-1}1\left(D^{m_v-1}\prod_{i=1}^{m_v}p_i^{v}\right)\left(KD^{m_v-1}\prod_{i=1}^{m_v}p_i\,D^{m_{v-1}-1}\prod_{i=1}^{m_{v-1}}p_i^{v-1}\right)\left(K^{2}D^{m_v-1}\prod_{i=1}^{m_v}p_i^{v}D^{m_{v-1}-1}\right.$$
$$\left.\prod_{i=1}^{m_{v-1}}p_i^{v-1}D^{m_{v-2}-1}\prod_{i=1}^{m_{v-2}}p_i^{v-2}\right)\ldots\left(K^{v-3}D^{m_v-1}\prod_{i=1}^{m_v}p_i^{v}\ldots D^{m_3-1}\prod_{i=1}^{m_3}p_i^{3}\right)$$
$$\left(K^{v-1}D^{m_v-1}\prod_{i=1}^{m_v}p_i^{v}\ldots D^{m_1-1}\prod_{i=1}^{m_1}p_i^{1}\right)$$

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$$\sim R^{v-1}\, 1\Big(R^{m_v-1}\, 1p^v_{m_v}Kp^v_{m_v}p^v_{m_v-1}K^2p^v_{m_v}p^v_{m_v-1}p^v_{m_v-2}\ldots K^{m_v-3}p^v_{m_v}p^v_{m_v-1}p^v_3K^{m_v-1}$$
$$p^v_{m_v}\ldots p^v_3\Big)\Big[K\Big(R^{m_v-1}\, 1p^v_{m_v}Kp^v_{m_v}p^v_{m_v-1}K^2p^v_{m_v}p^v_{m_v-1}p^v_{m_v-2}\ldots K^{m_v-3}p^v_{m_v}\ldots$$
$$p^v_3K_3^{m_v-1}p^v_{m_v}\ldots p^v_1\Big)\Big(R^{m_{v-1}-1}\, 1p^{v-1}_{m_{v-1}}Kp^{v-1}_{m_{v-1}-2}K^2p^{v-1}_{m_{v-1}}p^{v-1}_{m_{v-1}-1}p^{v-1}_{m_{v-1}-2}\ldots$$
$$K^{m_{v-1}-3}p^{v-1}_{m_{v-1}}\ldots p^{v-1}_3K^{m_{v-1}-1}p^{v-1}_{m_{v-1}}\ldots p^{v-1}_1\Big)\Big]\Big[K^2\Big(R^{m_v-1}\, 1p^v_{m_v}Kp^v_{m_v}p^v_{m_v}\ldots$$
$$K^{m_v-3}p^v_{m_v}\ldots p^v_3K^{m_v-1}p^v_{m_v}\ldots p^v_1\Big)\Big(R^{m_{v-1}-1}\, 1p^{v-1}_{m_{v-1}}Kp^{v-1}_{m_{v-1}}p^{v-1}_{m_{v-1}-1}\ldots$$
$$K^{m_{v-1}-3}p^{v-1}_{m_{v-1}}\ldots p^v_3K^{m_{v-1}-1}p^{v-1}_{m_{v-1}}\ldots p^{v-1}_1\Big)\Big(R^{m_{v-2}-1}\, 1p^{v-2}_{m_{v-2}}Kp^{v-2}_{m_{v-2}}p^{v-2}_{m_{v-2}-1}\ldots$$
$$K^{m_{v-2}-3}p^{v-2}_{m_{v-2}}\ldots p^{v-2}_3K^{m_{v-2}-1}p^{v-2}_{m_{v-2}}\ldots p^{v-2}_1\Big)\Big]\ldots\Big[K^{v-3}\Big(R^{m_v-1}\, 1p^v_{m_v}Kp^v_{m_v}$$
$$p^v_{m_v-1}\ldots K^{m_v-3}p^v_{m_v}\ldots p^v_3K^{m_v-1}p^v_{m_v}\ldots p^v_1\Big)\ldots\Big(R^{m_3-1}\, 1p^3_{m_3}Kp^3_{m_3}p^3_{m_3-1}\ldots$$
$$K^{m_3-3}p^3_{m_3}\ldots p^3_3K^{m_3-1}p^3_{m_3}\ldots p^3_1\Big)\Big]\Big[K^{v-1}\Big(R^{m_v-1}\, 1p^v_{m_v}Kp^v_{m_v}p^v_{m_v-1}\ldots$$
$$K^{m_v-3}p^v_{m_v}\ldots p^v_3K^{m_v-1}p^v_{m_v}\ldots p^v_1\Big)\ldots\Big(R^{m_1-1}\, 1p^1_{m_1}Kp^1_{m_1}p^1_{m_1-1}\ldots$$
$$K^{m_1-3}p^1_{m_3}\ldots p^1_3K^{m_1-1}p^1_{m_1}\ldots p^1_3\Big)\Big]$$
$$=\omega.$$

Nous faisons les calculs des grandes parenthèses:

$$(a_1)\quad K\Big(R^{m_v-1}\, 1p^v_{m_v}Kp^v_{m_v}p^v_{m_v-1}p^v_{m_v-2}\ldots K^{m_v-3}p^v_{m_v}\ldots p^v_3K^{m_v-1}p^v_{m_v}\ldots p^v_1\Big)$$
$$\Big(R^{m_{v-1}-1}\, 1p^{v-1}_{m_{v-1}}Kp^{v-1}_{m_{v-1}}p^{v-1}_{m_{v-1}-1}K^2p^{v-1}_{m_{v-1}}p^{v-1}_{m_{v-1}-1}p^{v-1}_{m_{v-1}-2}\ldots K^{m_{v-1}-3}p^{v-1}_{m_{v-1}}\ldots$$
$$p^{v-1}_3K^{m_{v-1}-1}p^{v-1}_{m_{v-1}}\ldots p^{v-1}_1\Big)$$
$$\sim R^{m_vm_{v-1}-1}\, 1\Big(p^{v-1}_{m_{v-1}}Kp^{v-1}_{m_{v-1}-1}K^2p^{v-1}_{m_{v-1}}p^{v-1}_{m_{v-1}-1}p^{v-1}_{m_{v-1}-2}\ldots K^{m_{v-1}-3}p^{v-1}_{m_{v-1}}\ldots$$
$$p^{v-1}_3K^{m_{v-1}-1}p^{v-1}_{m_{v-1}}\ldots p^{v-1}_1\Big)\Big(p^v_{m_v}Kp^v_{m_v}p^{v-1}_{m_{v-1}}K^2p^v_{m_v}p^{v-1}_{m_{v-1}}p^{v-1}_{m_{v-1}-1}K^3p^v_{m_v}p^{v-1}_{m_{v-1}}$$
$$p^{v-1}_{m_{v-1}-1}p^{v-2}_{m_{v-1}-2}\ldots K^{m_{v-1}-2}p^v_{m_v}p^{v-1}_{m_{v-1}}\ldots p^{v-1}_3K^{m_v-1}p^v_{m_v}p^{v-1}_{m_{v-1}}\ldots p^{v-1}_1\Big).$$
$$\Big(Kp^v_{m_v}p^v_{m_v-1}K^2p^v_{m_v}p^v_{m_v-1}p^{v-1}_{m_{v-1}-1}K^3p^v_{m_v}p^v_{m_v-1}p^{v-1}_{m_{v-1}}p^{v-1}_{m_{v-1}}p^{v-1}_{m_{v-1}-1}K^4p^v_{m_v}p^v_{m_v-1}$$
$$p^{v-1}_{m_{v-1}}p^{v-1}_{m_{v-1}-1}p^{v-1}_{m_{v-1}-2}\ldots K^{m_{v-1}-1}p^v_{m_v}p^v_{m_v-1}p^{v-1}_{m_{v-1}}\ldots p^{v-1}_3K^{m_v+1}p^v_{m_v}p^v_{m_v-1}$$
$$p^{v-1}_{m_{v-1}}\ldots p^{v-1}_1\Big)\Big(K^2p^v_{m_v}p^{v-1}_{m_{v-1}}p^{v-2}_{m_v-2}K^3p^v_{m_v}p^v_{m_v-1}p^v_{m_v-2}p^{v-1}_{m_{v-1}}K^4p^v_{m_v}p^v_{m_v-1}p^v_{m_v-2}$$
$$p^{v-1}_{m_{v-1}}p^{v-1}_{m_{v-1}-1}\ldots K^{m_{v-1}-1}p^v_{m_v}p^v_{m_v-1}p^v_{m_v-2}p^{v-1}_{m_{v-1}}\ldots p^{v-1}_3K^{m_v+2}p^v_{m_v}p^v_{m_v-1}p^v_{m_v-2}$$
$$p^{v-1}_{m_v-2}p^{v-1}_{m_{v-1}}\ldots p^{v-1}_1\Big)\ldots\Big(K^{m_v-3}p^v_{m_v}p^v_{m_v-1}\ldots p^v_3K^{m_v-2}p^v_{m_v}\ldots p^v_3p^{v-1}_{m_{v-1}}$$
$$K^{m_v-1}p^v_{m_v}\ldots p^v_3p^{v-1}_{m_{v-1}}p^{v-1}_{m_{v-1}-1}\ldots K^{m_v+m_{v-1}-5}p^v_{m_v}\ldots p^v_3p^{v-1}_{m_{v-1}}\ldots$$
$$p^{v-1}_3K^{m_v+m_{v-1}-3}p^v_{m_v}\ldots p^v_3p^{v-1}_{m_{v-1}}\ldots p^v_1\Big)\Big(K^{m_v-1}p^v_{m_v}\ldots p^v_1K^{m_v}p^v_{m_v}p^v_{m_{v-1}}\ldots$$
$$p^v_1p^{v-1}_{m_{v-1}}K^{m_v+1}p^v_{m_v}\ldots p^v_1p^{v-1}_{m_{v-1}}p^{v-1}_{m_{v-1}-1}\ldots K^{m_v+m_{v-1}-3}p^v_{m_v}\ldots p^v_1p^{v-1}_{m_{v-1}}\ldots$$
$$p^{v-1}_3K^{m_v+m_{v-1}-1}p^v_m\ldots p^v_1p^{v-1}_{m_{v-1}}\ldots p^{v-1}_1\Big)$$
$$\sim R^{m_vm_{v-1}-1}\, 1p^v_{m_v}p^{v-1}_{m_{v-1}}\prod_{i=0}^{1}\Big(Kp^{v-i}_{m_{v-i}}p^{v-i}_{m_{v-1}-i}K^2p^{v-i}_{m_{v-i}}p^{v-i}_{m_{v-i}-2}\ldots K^{m_{v-i}-3}p^{v-i}_{m_{v-i}}\ldots$$
$$p^{v-i}_3K^{m_{v-i}-1}p^{v-i}_{m_{v-i}}\ldots p^{v-i}_1\Big)\left(\prod_{i_1=0}^{m_v-3}\prod_{i_2=0}^{m_{v-1}-3}K^{i_1+i_2+1}\prod_{j=0}^{i_1}p^v_{m_v-j}\prod_{j=0}^{i_2}p^{v-1}_{m_{v-1}-j}\right)$$

$$\left(\prod_{i_1=0}^{m_v-3} K^{i_1+m_{v-1}} \prod_{j=0}^{i_1} p^v_{m_v-j} \prod_{j=0}^{m_{v-1}-1} p^{v-1}_{m_{v-1}-j}\right)\left(\prod_{i_2=0}^{m_{v-1}-3} K^{i_2+m_v}\prod_{j=0}^{i_2} p^{v-1}_{m_{v-1}-j}\prod_{j=0}^{m_v-1} p^v_{m_v-j}\right)$$
$$\left(K^{m_v+m_{v-1}-1}\prod_{j=0}^{m_v-1}p^v_{m_v-j}\prod_{j=0}^{m_{v-1}-1}p^{v-1}_{m_{v-1}-j}\right)$$

(a_2) $K^2\Big(R^{m_v-1}Kp^v_{m_v}Kp^v_{m_v}p^v_{m_v-1}K^2p^v_{m_v}p^v_{m_v-1}p^v_{m_v-2}\dots K^{m_v-3}p^v_{m_v}\dots p^v_3K^{m_v-1}$
$p^v_{m_v}\dots p^v_1\Big)\Big(R^{m_{v-1}-1}1p^{v-1}_{m_{v-1}}Kp^{v-1}_{m_{v-1}}p^{v-1}_{m_{v-1}-1}K^2p^{v-1}_{m_{v-1}}-1p^{v-2}_{m_{v-1}-2}\dots$
$K^{m_{v-1}-3}p^{v-1}_{m_{v-1}}\dots p^v_3K^{m_{v-1}-1}p^{v-1}_{m_{v-1}}\dots p^{v-1}_1\Big)\Big(R^{m_{v-2}-1}1p^{v-2}_{m_{v-2}}Kp^{v-2}_{m_{v-2}}$
$p^{v-2}_{m_{v-2}-1}K^2p^{v-2}_{m_{v-2}}p^{v-2}_{m_{v-2}-1}p^{v-2}_{m_{v-2}-2}\dots K^{m_{v-2}-3}p^{v-2}_{m_{v-2}}\dots p^{v-2}_3K^{m_{v-2}-1}$
$p^{v-2}_{m_{v-2}}\dots p^{v-2}_1\Big)$

$\sim R^{m_vm_{v-1}m_{v-2}-1}1\Big(p^{v-2}_{m_{v-2}}Kp^{v-2}_{m_{v-2}}p^{v-2}_{m_{v-2}-1}K^2p^{v-2}_{m_{v-2}}p^{v-2}_{m_{v-2}-1}p^{v-2}_{m_{v-2}-2}\dots$
$K^{m_{v-2}-3}p^{v-2}_{m_{v-2}}\dots p^{v-2}_3K^{m_{v-2}-1}p^{v-2}_{m_{v-2}}\dots p^{v-2}_1\Big)\Big[p^v_{m_v}Kp^v_{m_v}p^{v-2}_{m_{v-2}}K^2p^v_{m_v}p^{v-2}_{m_{v-2}}$
$p^{v-2}_{m_{v-2}-1}K^3p^v_{m_v}p^{v-2}_{m_{v-2}}p^{v-2}_{m_{v-2}-1}p^{v-2}_{m_{v-2}-2}\dots K^{m_{v-2}-2}p^v_{m_v}p^{v-2}_{m_{v-2}}p^{v-2}_{m_{v-2}}\dots$
$p^{v-2}_3K^{m_{v-2}}p^v_{m_v}p^{v-2}_{m_{v-2}}\dots p^{v-2}_1\Big]\Big[Kp^v_{m_v}p^v_{m_v-1}K^2p^v_{m_v}p^v_{m_v-1}p^{v-2}_{m_{v-2}}K^3p^v_{m_v}p^v_{m_v-1}$
$p^{v-2}_{m_{v-2}-1}\dots K^{m_{v-2}-1}p^v_{m_v}p^v_{m_v-1}p^{v-1}_{m_{v-1}}\dots p^{v-1}_3K^{m_{v-2}+1}p^v_{m_v}p^{v-1}_{m_v}p^{v-2}_{m_{v-1}}\dots p^{v-1}_1\Big]$
$\dots\Big[K^{m_v-3}p^v_{m_v}\dots p^v_3K^{m_v-2}p^v_{m_v}\dots p^v_3p^{v-1}_{m_{v-1}}K^{m_v-1}p^v_{m_v}\dots p^v_3p^{v-1}_{m_{v-1}}p^{v-1}_{m_{v-1}-1}$
$\dots K^{m_v+m_{v-1}-5}p^v_3p^{v-1}_{m_{v-1}}\dots p^{v-1}_3K^{m_v+m_{v-1}-3}p^v_{m_v}\dots p^v_3p^{v-1}_{m_{v-1}}\dots$
$p^{v-1}_1K^{m_v-1}p^v_{m_v}\dots p^v_1K^{m_v}p^v_{m_v}\dots p^v_1p^{v-1}_{m_{v-1}}K^{m_v+1}p^v_{m_v}\dots p^v_1p^{v-1}_{m_{v-1}}p^{v-1}_{m_{v-1}-1}\dots$
$K^{m_v+m_{v-1}-3}p^v_{m_v}\dots p^v_1p^{v-1}_{m_{v-1}}\dots p^{v-1}_3K^{m_v+m_{v-1}-1}p^v_{m_v}\dots p^v_1p^{v-1}_{m_{v-1}}\dots p^{v-1}_1\Big]$
$\Big[K^{m_{v-1}-3}p^{v-1}_{v-1}Kp^{v-1}_{m_{v-1}}p^{v-2}_{m_{v-2}}K^2p^{v-1}_{m_{v-1}}p^{v-2}_{m_{v-2}}p^{v-2}_{m_{v-2}-1}K^3p^{v-1}_{m_{v-1}}p^{v-2}_{m_{v-2}}p^{v-2}_{m_{v-2}-1}p^{v-2}_{m_{v-2}-2}$
$\dots K^{m_{v-2}-2}p^{v-1}_{m_{v-2}}\dots p^{v-2}_3K^{m_{v-2}}p^{v-1}_{m_{v-1}}p^{v-2}_{m_{v-2}}\dots p^{v-2}_1\Big]\Big[Kp^{v-1}_{m_{v-1}}K^2p^{v-1}_{m_{v-1}-1}$
$p^{v-1}_{m_{v-2}}K^3p^{v-1}_{m_{v-1}}p^{v-1}_{m_{v-1}-1}p^{v-2}_{m_{v-2}}p^{v-2}_{m_{v-2}-1}\dots K^{m_{v-2}-1}p^{v-1}_{m_{v-1}}p^{v-1}_{m_{v-1}-1}p^{v-2}_{m_{v-2}}\dots p^{v-1}_3\Big]$
$\Big[K^{m_{v-2}+1}p^{v-1}_{m_{v-1}}p^{v-1}_{m_{v-1}-1}p^{v-2}_{m_{v-2}}\dots p^{v-2}_1K^2p^{v-1}_{m_{v-1}}p^{v-1}_{m_{v-1}-1}p^{v-1}_{m_{v-1}-2}K^3p^{v-1}_{m_{v-1}}p^{v-1}_{m_{v-1}-1}$
$p^{v-1}_{m_{v-2}}K^4p^{v-1}_{m_{v-1}}p^{v-1}_{m_{v-1}-1}p^{v-1}_{m_{v-1}-2}p^{v-2}_{m_{v-2}}p^{v-2}_{m_{v-2}-1}\dots K^{m_{v-2}}p^{v-1}_{m_{v-1}}p^{v-1}_{m_{v-1}-1}p^{v-1}_{m_{v-1}-2}$
$p^{v-2}_{m_{v-2}}\dots p^{v-2}_3K^{m_{v-2}+2}p^{v-1}_{m_{v-1}}p^{v-1}_{m_{v-1}-1}p^{v\ 1}_{m_{v-1}-2}p^{v-2}_{m_{v-2}}\dots p^{v-2}_1\Big]\dots$
$\Big[K^{m_{v-1}-3}p^{v-1}_{m_{v-1}}\dots p^{v-1}_3K^{m_{v-1}-2}p^{v-1}_{m_{v-1}}\dots p^{v-1}_3p^{v-2}_{m_{v-2}}K^{m_{v-1}-1}p^{v-1}_{m_{v-1}}\dots$
$p^{v-1}_3p^{v-2}_{m_{v-2}}p^{v-2}_{m_{v-2}-1}\dots K^{m_{v-1}+m_{v-2}-5}p^{v-1}_{m_{v-1}}\dots p^{v-1}_3p^{v-2}_{m_{v-2}}\dots p^{v-2}_3K^{m_{v-1}+m_{v-2}-3}$
$p^{v-1}_{m_{v-1}}\dots p^{v-1}_{m_{v-1}}\dots p^{v-1}_3p^{v-2}_{m_{v-2}}\dots p^{v-2}_1\Big]\Big[K^{m_{v-1}-1}p^{v-1}_{m_{v-1}}\dots p^{v-1}_1$
$K^{m_{v-1}}p^{v-1}_{m_{v-1}}\dots p^v_1p^{v-2}_{m_{v-2}}K^{m_{v-1}+1}p^{v-1}_{m_{v-1}}\dots p^{v-1}_1p^{v-2}_{m_{v-2}}p^{v-2}_{m_{v-2}-1}\dots$
$K^{m_{v-1}+m_{v-2}-3}p^{v-1}_{m_{v-1}}\dots p^{v-1}_1p^{v-2}_{m_{v-2}}\dots p^{v-2}_3K^{m_{v-1}+m_{v-2}-1}p^{v-1}_{m_{v-1}}\dots p^{v-1}_1p^{v-2}_{m_{v-2}}$
$\dots p^{v-2}_1\Big]\Big[p^v_{m_v}Kp^v_{m_v}p^{v-1}_{m_{v-1}}K^2p^v_{m_v}p^{v-1}_{m_{v-1}}K^2p^v_{m_v}p^{v-1}_{m_{v-1}}p^{v-1}_{m_{v-1}-1}\dots K^{m_{v-1}-2}$
$p^v_{m_v}p^{v-1}_{m_{v-1}}\dots p^{v-1}_3K^{m_{v-1}}p^v_{m_v}p^{v-1}_{m_{v-1}}\dots p^{v-1}_1\Big]\Big[Kp^v_{m_v}p^v_{m_v-1}K^2p^v_mp^v_{m_v-1}p^{v-1}_{m_v-1}$

$$p^{v-1}_{m_{v-1}-1} \dots K^{m_{v-1}-1} p^v_{m_v} p^v_{m_v-1} p^{v-1}_{m_{v-1}} \dots p_3^{v-1} K^{m_{v-1}+1} p^v_{m_v} p^v_{m_v-1} p^{v-1}_{m_{v-1}} \dots p_1^{v-1}\Big]$$

$$\Big[K^2 p^v_{m_v} p^v_{m_v-1} p^v_{m_v-2} K^3 p^v_{m_v} p^v_{m_v-1} p^v_{m_v-2} p^{v-1}_{m_{v-1}} K^4 p^v_{m_v} p^v_{m_v-1} p^v_{m_v-2} p^{v-1}_{m_{v-1}} p^{v-1}_{m_{v-1}-1} \dots$$

$$K^{m_v-1} p^v_{m_v} p^v_{m_v-1} p^{v-2}_{m_v-2} p^{v-1}_{m_{v-1}} \dots p_3^{v-1} K^{m_{v-1}+2} K^{m_{v-1}+2} p^v_{m_v} p^v_{m_v-1} p^{v-2}_{m_{v-2}} p^{v-1}_{m_{v-1}} \dots$$

$$p_1^{v-1}\Big] \dots \Big[K^{m_v-3} p^v_{m_v} \dots p_3^v K^{m_v-2} p^v_{m_v} \dots p_3^v p^{v-1}_{m_{v-1}} K^{m_v-1} p^v_{m_v} \dots$$

$$p_3^v p^{v-1}_{m_{v-1}} p^{v-1}_{m_{v-1}-1} K^{m_v} p^v_{m_v} \dots p_3^v p^{v-1}_{m_{v-1}} p^{v-1}_{m_{v-1}-1} p^{v-1}_{m_{v-1}-2} \dots K^{m_v+m_{v-1}-5} p^v_{m_v} \dots$$

$$p_3^v p^{v-1}_{m_{v-1}} \dots p_3^{v-1} K^{m_v+m_{v-1}-3} p^v_{m_v} \dots p_3^v p^{v-1}_{m_{v-1}} \dots p_1^{v-1}\Big] \Big[K^{m_v-1} p^v_{m_v} \dots$$

$$p_1^v K^{m_v} p^v_{m_v} \dots p_1^v p^{v-1}_{m_{v-1}} K^{m_v+1} p^v_{m_v} \dots p_1^v p^{v-1}_{m_{v-1}} p^{v-1}_{m_{v-1}-1} \dots K^{m_v+m_{v-1}-3} p^v_{m_v} \dots$$

$$p_1^v p^{v-1}_{m_{v-1}} \dots p_3^{v-1} K^{m_v+m_{v-1}-1} p^v_{m_v} \dots p_1^v p^{v-1}_{m_{v-1}} \dots p_1^{v-1}\Big]$$

$$\sim R^{m_v m_{v-1} m_{v-2}-1} 1 p^v_{m_v} p^{v-1}_{m_{v-1}} p^{v-2}_{m_{v-2}} \Big(K p^{v-i}_{m_{v-i}} p^{v-i}_{m_{v-i}-1} K^2 p^{v-i}_{m_{v-i}} p^{v-i}_{m_{v-i}-1} p^{v-i}_{m_{v-i}-2} \dots$$

$$K^{m_{v-2}-3} p^{v-i}_{m_{v-i}} \dots p_3^{v-i} K^{m_{v-i}-1} p^{v-i}_{m_{v-i}} \dots p_1^{v-i}\Big) \Big(\prod_{i_1=0}^{m_v-3} \prod_{i_2=0}^{m_{v-1}-3} \prod_{i_3=0}^{m_{v-2}-1} K^{i_1+i_2+i_3+2}$$

$$\prod_{j=0}^{i_2} p^{v-1}_{m_{v-1}-j} \prod_{j=0}^{i_3} p^{v-2}_{m_{v-2}-j} \Big) \Big(\prod_{i_1=0}^{m_v-3} K^{i_1+m_v+m_{v-2}} \prod_{j=0}^{i_1} p^v_{m_{v-1}-j} \prod_{j=0}^{m_{v-1}-1} p^{v-1}_{m_{v-1}-j} \prod_{j=0}^{m_{v-2}-1}$$

$$p^{v-2}_{m_{v-2}-j} \Big) \Big(\prod_{i_2=0}^{m_{v-1}-3} K^{i_2+m_v+m_{v-2}} \prod_{j=0}^{i_2} p^{v-1}_{m_{v-1}-j} \prod_{j=0}^{m_v-1} p^v_{m_v-j} \prod_{j=0}^{m_{v-2}-1} p^{v-2}_{m_{v-2}-j} \Big) \Big(\prod_{i_3=0}^{m_{v-2}-3}$$

$$K^{i_3+m_v+m_{v-1}} \prod_{j=0}^{i_3} p^{v-2}_{m_{v-2}-j} \prod_{j=0}^{m_v-1} p^v_{m_v-j} \prod_{j=0}^{m_{v-1}-1} p^{v-1}_{m_{v-1}-j} \Big) \Big(K^{m_v+m_{v-1}+m_{v-2}-1} \prod_{j=0}^{m_v-1} p^j_{m_v-j}$$

$$\prod_{j=0}^{m_{v-1}-1} p^{v-1}_{m_{v-1}-j} \prod_{j=0}^{m_{v-2}-1} p^{v-2}_{m_{v-2}-j} \Big)$$

Et généralement:

$$(a_{h-1}) K^{h-1} \Big(R^{m_v-1} 1 p^v_{m_v} K p^v_{m_v} p^v_{m_v-1} K^2 p^v_{m_v} p^v_{m_v-1} p^v_{m_v-2} \dots K^{m_v-3} p^v_{m_v} \dots p_3^v$$

$$K^{m_v-1} p^v_{m_v} \dots p_1^v\Big) \dots \Big(R^{m_{v-h}-1} 1 p^{v-h}_{m_{v-h}} K p^{v-h}_{m_{v-h}} p^{v-h}_{m_{v-h}-1} K^2 p^{v-h}_{m_{v-h}-1} p^{v-h}_{m_{v-h}-2} \dots$$

$$K^{m_{v-h}-3} p^{v-h}_{m_{v-h}} \dots p_3^{v-h} K^{m_{v-h}-1} p^{v-h}_{m_{v-h}} \dots p_1^{v-h}\Big)$$

$$\sim R^{m_v m_{v-1} m_{v-2} \dots m_{v-h}-1} 1 p^v_{m_v} p^{v-1}_{m_{v-1}} \dots p^{v-h}_{m_{v-h}} \prod_{i=1}^{h} \Big(K p^{v-i}_{m_{v-i}} p^{v-i}_{m_{v-i}-1} K^2 p^{v-i}_{m_{v-i}-1}$$

$$p^{v-i}_{m_{v-i}-1} p^{v-i}_{m_{v-i}-2} \dots K^{m_{v-i}-3} p^{v-i}_{m_{v-i}} \dots p_3^{v-i} K^{m_{v-i}-1} p^{v-i}_{m_{v-i}} \dots p_1^{v-i}\Big)$$

$$\Big(\prod_{i_1=0}^{m_v-1} \prod_{i_2=0}^{m_{v-1}-1} \dots \prod_{i_h=0}^{m_{v-h}-1} K^{i_1+i_2+\dots+i_h+h-1} \prod_{j=0}^{i_1} p^v_{m_v-j} \prod_{j=0}^{i_2} p^{v-1}_{m_{v-1}-j} \dots \prod_{j=0}^{i_h} p^{v-h}_{m_{v-h}-j}$$

$$\prod_{i_1=0}^{m_v-3} K^{i_1+m_{v-1}+m_{v-2}+\dots+m_{v-h}} \prod_{j=0}^{i_1} p^v_{m_v-j} \prod_{j=0}^{m_{v-1}-1} p^{v-1}_{m_v-1-j} \dots \prod_{j=0}^{m_{v-h}-1} p^{v-h}_{m_{v-h}-j} \Big)$$

$$\Big(\prod_{i_2=0}^{m_{v-1}-3} K^{i_2+m_v+m_{v-2}+\dots+m_{v-h}} \prod_{j=0}^{i_2} p^{v-1}_{m_{v-1}-j} \prod_{j=0}^{m_v-1} p^v_{m_v-j} \prod_{j=0}^{m_{v-2}-1} p^{v-2}_{m_{v-2}-j} \dots$$

$$\prod_{j=0}^{m_{v-h}-1} p^{v-h}_{m_{v-h}-j} \Big) \Big(\prod_{j=0}^{i_3} p^{v-2}_{m_{v-2}-j} K^{i_2+m_v+m_{v-1}+m_{v-3}+\dots+m_{v-h}} \prod_{j=0}^{i_3} p^{v-2}_{m_{v-2}-j} \prod_{j=0}^{m_v-1} p^v_{m_v-j}$$

$$\prod_{j=0}^{m_{v-1}-1} p_{m_{v-1}-j}^{v-1} \prod_{j=0}^{m_{v-3}-1} p_{m_{v-3}-j}^{v-3} \cdots \prod_{j=0}^{m_{v-h}-1} p_{m_{v-h}-j}^{v-h}\Big)\Big(\prod_{j=0}^{ih} K^{ih+m_v+\ldots+m_{v-h}-1} \prod_{j=0}^{ih}$$

$$p_{m_{v-h}-j}^{v-h} \prod_{j=0}^{m_v-1} p_{m_v-j}^{v} \cdots \prod_{j=0}^{m_{v-h-1}-1} p_{m_{v-h-1}-j}^{v-h-1}\Big)\Big(K^{m_v+m_{v-1}+\ldots+m_{v-h}-1} \prod_{j=0}^{m_v-1} p_{m_v-j}^{j} \cdots$$

$$\prod_{j=0}^{m_{v-h}-1} p_{m_{v-h}-j}^{v-h}\Big)$$

D'après ces relations, nous déduisons la forme normale $\mathsf{N}_4(D)$.

Théorème 5. *Si α est une forme du groupe, c'est-à-dire:*

$$\alpha = D^{v-1}\alpha_1\alpha_2 \ldots \alpha_v$$

où

$$\alpha_h = Dp_1^h K p_2^h \ldots Dp_{m_h-2}^h D p_{m_h-1}^h p_{m_h}^h \qquad (h = 1, 2, \ldots, v)$$

alors α admet la forme normale:

$$\mathsf{N}_5(D) = R^{\mathfrak{N}}(p_1^v)^v(p_1^{v-1})^{v-1} \ldots (p_1^2)^2(p_1^1)\left[\prod_{i=1}^{v}\Big(Kp_1^i p_2^i K^2 p_1^i p_2^i p_3^i \ldots K^{m_i-1} p_1^i p_2^i \ldots\right.$$

$$\left.p_{m_i}^i\Big)^i\right]\left[\prod_{k=v-1}^{v}\Big(\prod_{i_k=0}^{m_{v-k+1}-3} K^{\mathfrak{P}} \prod_{i=v-1}^{v}\prod_{j=0}^{m_i-1} p_{j+1}^i\Big) \ldots \Big(\prod_{k=v-h}^{v}\prod_{i_k=0}^{m_{v-k+1}-3} K^{\mathfrak{R}} \prod_{i=v-h}^{v}\right.$$

$$\left.\prod_{j=0}^{m_i-1} p_{j+1}^i\Big) \ldots \Big(\prod_{k=1}^{v}\prod_{i_k=0}^{m_{v-k+1}-3} K^{\mathfrak{S}} \prod_{i=1}^{v}\prod_{j=0}^{m_i-1} p_{j+1}^i\Big)\right] \ldots \left[\prod_{j=1}^{2}\prod_{i_j=1}^{m_{v-j+1}-3}\right.$$

$$K^{i_j+m_{u(v,v-1)}} \prod_{k=0}^{i_j} p_{k+1}^{v-j+1} \prod_{k=1}^{m_{t(v,v-1)}} p_k^{t(v,v-1)} \ldots \prod_{j=1}^{h}\prod_{i_j=0}^{m_{v-j+1}-3}$$

$$\left.K^{i_j+m_{u(v,v-1,\ldots,v-h)}} \prod_{k=0}^{i_j} p_{k+1}^{v-j+1} \prod_{k=1}^{m_{t(v,v-1,\ldots,v-h)}} p_k^{t(v,v-1,\ldots,v-j)}\right] \ldots$$

$$\left[\prod_{j=1}^{v}\prod_{i_j=0}^{m_{v-j+1}} K^{i_j+m_{(v,\ldots,1)}} \prod_{k=1}^{m_{t(v,\ldots,1)}} p_k^{m_{t(v,\ldots,1)}}\Big(K^{m_v+m_{v-1}+\ldots+m_1-1}\right.$$

$$\left.\prod_{j=1}^{m_v} p_j^v \ldots \prod_{j=1}^{m_1} p_j^1\Big)\right]$$

avec les notations du Théorème 3 *et*

$$\mathfrak{N} = \sum_{j=1}^{v}\sum_{i=1}^{j} m_i + v - 1, \qquad \mathfrak{P} = \sum_{u=1}^{2} i_u + 1,$$

$$\mathfrak{R} = \sum_{u=1}^{h} i_u + h - 1, \qquad \mathfrak{S} = \sum_{u=1}^{v} i_u + v - 1$$

D'après les Théorèmes 1 et 2 et le lemme du Théorème 3, nous avons:

$$K^h R^{v_1-1} \prod_{i_1=1}^{v_1} q_1^{i_1} R^{v_2-1} \prod_{i_2=1}^{v_2} q_2^{i_2} \ldots R^{v_{h+1}-1} \prod_{i_{h+1}=1}^{v_{h+1}} q_{h+1}^{i_{h+1}}$$

$$\sim R^{v_1 v_2 \cdots v_{h+1}-1} \prod_{i_1=1}^{v_1} \prod_{i_2=1}^{v_2} \cdots \prod_{i_{h+1}=1}^{v_{h+1}} K^h q_1^{i_1} q_2^{i_2} \cdots q_{h+1}^{i_{h+1}}$$

Et c'est ainsi, nous avons la formule suivante:

$$(1)\ \alpha \sim R^{v-1}\, 1\alpha_v K\alpha_v\alpha_{v-1}K^2\alpha_v\alpha_{v-1}\alpha_{v-2}\cdots K^{v-3}\alpha_v\alpha_{v-1}\cdots\alpha_3K^{v-1}\alpha_v\alpha_{v-1}\cdots\alpha_1$$

Utilisant le Théorème 2 et le lemme ci-dessous, nous faisons le calcul pour la forme:

$$\alpha = D^{h-1}\alpha_v\alpha_{v-1}\cdots\alpha_{v-h}$$

Nous avons:

$$(2)\ \alpha_v = R^{v-1}\, 1p_1^vKp_1^vp_2^vK^2p_1^vp_2^vp_3^v\cdots K^{m_v-3}p_1^vp_2^v\cdots p_{m_v-2}^vK^{m_v-1}p_1^vp_2^v\cdots p_{m_v}^v$$

$$(3)\ K\alpha_v\alpha_{v-1} = K\left(Kp_1^vKp_2^v\cdots Kp_{m_v-2}^vKp_{m_v-1}^vp_{m_v}^v\right)\left(Kp_2^{v-1}Kp_2^{v-1}\cdots Kp_{m_{v-1}-2}^{v-1}Kp_{m_{v-1}-1}^{v-1}p_{m_{v-1}}^{v-1}\right)$$

$$\sim K\left(R^{m_v-1}\,1p_1^vKp_1^vp_2^vK^2p_1^vp_2^vp_3^v\cdots K^{m_v-3}p_1^vp_2^v\cdots p_{m_v-2}^vK^{m_v-1}p_1^vp_2^v\cdots p_{m_v}^v\right)\left(R^{m_{v-1}-1}\,1p_1^{v-1}Kp_1^{v-1}p_2^{v-1}K^2p_1^{v-1}p_2^{v-1}p_3^{v-1}\cdots K^{m_{v-1}-3}p_1^{v-1}p_2^{v-1}\cdots p_{m_{v-1}-2}^{v-1}\right)K^{m_{v-1}-1}p_1^vp_2^v\cdots p_{m_{v-1}}^{v-1}$$

$$\sim R^{m_vm_{v-1}-1}\,1\left(p_1^vKp_1^vp_2^vK^2p_1^vp_2^vp_3^v\cdots K^{m_v-3}p_1^vp_2^v\cdots p_{m_v-2}^vK^{m_v-1}p_1^vp_2^v\cdots p_{m_v}^v\right)\left(p_1^{v-1}\ Kp_1^{v-1}p_2^{v-1}K^2p_1^{v-1}p_2^{v-1}p_3^{v-1}K^3p_1^{v-1}p_2^{v-1}p_3^{v-1}\cdots K^{m_{v-1}-3}p_1^{v-1}p_2^{v-1}\cdots p_{m_{v-1}-2}^{v-1}K^{m_{v-1}-1}p_1^{v-1}p_2^{v-1}\cdots p_{m_{v-1}}^{1}\right)\Big[\left(Kp_1^vp_1^{v-1}K^2p_1^vp_1^{v-1}p_2^{v-1}K^2p_1^vp_1^{v-1}p_2^{v-1}K^3p_1^vp_1^{v-1}p_2^{v-1}p_3^{v-1}\cdots K^{m_{v-1}-2}p_1^vp_1^{v-1}p_2^{v-1}\cdots p_{m_{v-1}-2}^{v-1}K^{m_{v-1}}p_1^vp_1^{v-1}p_2^{v-1}\cdots p_{m_{v-1}}^{v-1}\right)\left(K^2p_1^vp_2^vp_1^{v-1}K^3p_1^vp_2^vp_1^{v-1}p_2^{v-1}p_3^{v-1}K^4p_1^vp_2^vp_1^{v-1}p_2^{v-1}p_3^{v-1}\cdots K^{m_{v-1}-1}p_1^vp_2^vp_1^{v-1}p_2^{v-1}\cdots p_{m_{v-1}-2}^{v-1}K^{m_{v-1}+1}p_1^vp_2^vp_1^{v-1}p_2^{v-1}\cdots p_{m_{v-1}}^{v-1}\right)$$

$$\left(K^3p_1^vp_2^vp_3^vp_1^{v-1}K^4p^vp^vp^vp_1^{v-1}p_2^{v-1}\cdots K^{m_{v-1}}p_1^vp_2^vp_3^vp_1^{v-1}p_2^{v-1}p_3^{v-1}\cdots p_{m_{v-1}-2}^{v-1}K^{m_{v-1}+2}p_1^vp_2^vp_3^vp_1^{v-1}p_2^{v-1}\cdots p_{m_{v-1}}^{v-1}\right)\cdots\left(K^{m_v-2}p_1^vp_2^v\cdots p_{m_v-2}^vp^{v-1}K^{m_v-1}p_1^vp_2^v\cdots p_{m_v-2}^vp_1^{v-1}p_2^{v-1}K^{m_v}p_1^vp_2^v\cdots p_{m_v-2}^vp_1^{v-1}p_2^{v-1}\cdots K^{m_v+m_{v-1}-5}p_1^vp_2^v\cdots p_{m_v-2}^vp_1^{v-1}p_2^{v-1}\cdots p_{m_{v-1}-2}^{v-1}K^{m_v+m_{v-1}-3}p_1^vp_2^v\cdots p_{m_v-2}^vp_1^{v-1}p_2^{v-1}\cdots p_{m_{v-1}}^{v-1}\right)$$

$$\left(K^{m_v}p_1^vp_2^v\cdots p_{m_v}^vp_1^{v-1}K^{m_v+2}p_1^vp_2^v\cdots p_{m_v}^vp_1^{v-1}p_2^{v-1}K^{m_v+2}p_1^vp_2^v\cdots p_{m_v}^vp_1^{v-1}p_2^{v-1}p_3^{v-1}\cdots K^{m_v+m_{v-1}-3}p_1^vp_2^v\cdots p_{m_v}^vp_1^{v-1}p_2^{v-1}\cdots p_{m_{v-1}-2}^{v-1}K^{m_v+m_{v-1}-1}p_1^vp_2^v\cdots p_{m_v}^vp_1^{v-1}p_2^{v-1}\cdots p_{m_{v-1}}^{v-1}\right)\Big]$$

$$\sim R^{m_vm_{v-1}-1}\,1\left(p_1^{v-1}Kp_1^{v-1}p_2^{v-1}p_3^{v-1}\cdots K^{m_{v-1}-3}p_1^{v-1}p_2^{v-1}\cdots p_{m_{v-1}-2}^{v-1}K^{m_{v-1}-2}K^{m_{v-1}-1}p_1^{v-1}p_2^{v-1}\cdots p_{m_{v-1}}^{v-1}\right)\left(p_1^vKp_1^vp_2^vK^3p_1^vp_2^vp_3^v\cdots K^{m_v-3}p_1^vp_2^v\cdots p_{m_v-2}^vK^{m_v-1}p_1^vp_2^v\cdots p_{m_v}^v\right)\left(\prod_{i_1=0}^{m_v-3}\prod_{i_2=0}^{m_{v-1}-3}K^{i_1+i_2+1}\prod_{j=0}^{i_1}p_{j+1}^v\prod_{j=0}^{i_2}p_{j+1}^{v-1}\right)\left[\left(\prod_{i_1=0}^{m_v-3}K^{i_1+m_{v-1}}\right.\right.$$

$$\prod_{j=0}^{i_1} p_{j+1}^{v} \prod_{j=0}^{m_{v-1}-1} p_{j+1}^{v-1}\Big)\Big(\prod_{i_2=0}^{m_{v-1}-3} K^{i_2+m_v} \prod_{j=0}^{i_2} p_{j+1}^{v-1} \prod_{j=0}^{m_v-1} p_{j+1}^{v}\Big)\Big]\Big(K^{m_v+m_{v-1}-1} \prod_{j=1}^{m_v} p_j^{v} \prod_{j=1}^{m_v-1} p_j^{v-1}\Big)$$

$$(4)\ K^2\alpha_v\alpha_{v-1}\alpha_{v-2} = K^2\Big(R^{m_v-1}\, 1p_1^vKp_1^vp_2^vK^2p_1^vp_2^vp_3^v \ldots K^{m_v-3}p_1^vp_2^v \ldots p_{m_v-2}^v K^{m_v-1}p_1^vp_2^v \ldots p_{m_v}^v\Big)\Big(R^{m_{v-1}-1}\, 1p_1^{v-1}Kp_1^{v-1}p_2^{v-1}K^2p_1^{v-1}p_2^{v-1}p_3^{v-1} \ldots K^{m_{v-1}-3}p_1^vp_2^v \ldots p_{m_{v-1}-2}^{v-1}K^{m_{v-1}-1}p_1^vp_2^v \ldots p_{m_v}^v\Big)\Big(R^{m_{v-2}-1}\, 1p_1^{v-2}Kp_1^{v-2}p_2^{v-2} K^2p_1^{v-2}p_2^{v-2}p_3^{v-2} \ldots K^{m_{v-2}-3}p_1^{v-2}p_2^{v-2} \ldots p_{m_{v-2}-2}^{v-2}K^{m_{v-2}-1}p_1^{v-2}p_2^{v-2} \ldots p_{m_{v-2}}^{v-2}\Big)$$

$$\sim R^{m_vm_{v-1}m_{v-2}-1}\, 1p_1^vp_2^{v-1}p_3^{v-2}\Big[\prod_{i=v-2}^{v}\Big(Kp_1^ip_2^iK^2p_1^ip_2^ip_3^iK^3p_1^ip_2^ip_3^ip_4^i \ldots K^{m_i-3}p_1^ip_2^i \ldots p_{m_i-2}^iK^{m_i-1}p_1^ip_2^i \ldots p_{m_i}^i\Big)\Big]\Big[\Big(\prod_{i_1=0}^{m_v-3}\prod_{i_2=0}^{m_{v-1}-3}\prod_{i_3=0}^{m_{v-2}-1} K^{i_1+i_2+i_3+2} \prod_{j=0}^{i_1} p_{j+1}^v \prod_{j=0}^{i_2} p_{j+1}^{v-1} \prod_{j=0}^{i_3} p_{j+1}^{v-2}\Big)\Big(\prod_{i_1=0}^{m_v-3} K^{i_1+m_{v-1}+m_{v-2}} \prod_{j=0}^{i_1} p_{j+1}^v \prod_{j=0}^{m_{v-1}-1} p_{j+1}^{v-1} \prod_{j=0}^{m_{v-2}-1} p_{j+1}^{v-2}\Big)\Big]$$

$$\Big[\Big(\prod_{i_2=0}^{m_{v-1}-3} K^{i_2+m_v+m_{v-2}} \prod_{j=0}^{i_2} p_{j+1}^{v-1} \prod_{j=0}^{m_v-1} p_{j+1}^{v} \prod_{j=1}^{m_{v-2}-1} p_{j+1}^{v-2}\Big)\Big]\Big[\Big(\prod_{i_3=0}^{m_{v-2}-3} K^{i_3+m_v+m_{v-1}} \prod_{j=0}^{i_3} p_{j+1}^{v-2} \prod_{j=0}^{m_v-1} p_{j+1}^{v} \prod_{j=0}^{m_{v-1}-1} p_{j+1}^{v-1}\Big)\Big]\Big(K^{m_v+_{v-1}+m_{v-2}-1} \prod_{j=0}^{m_v-1} p_{j+1}^v \prod_{j=0}^{m_{v-2}-1} p_{j+1}^{v-2}\Big)$$

$$= R^{m_vm_{v-1}m_{v-2}-1}\, 1p_1^vp_1^{v-1}p_1^{v-2}\Big[\prod_{i=v-2}^{v}\Big(Kp_1^ip_2^iK^2p_1^ip_2^ip_3^i \ldots K^{m_i-3}p_1^ip_2^i \ldots p_{m_i-2}^i K^{m_i-1}p_1^ip_2^i \ldots p_{m_i}^i\Big)\prod_{k=1}^{3}\prod_{i_k=0}^{m_{v-k+1}-3}\Big] K\mathfrak{P} \prod_{i=v-2}^{v}\prod_{j=0}^{m_i-1} p_{j+1}^i \Big(\prod_{j=1}^{3}\prod_{i_j=0}^{m_{v-j+1}-3} K^{i_j+m_{(v,v-1,v-2)}} \prod_{k=0}^{i_j} p_{k+1}^{v-j+1} \prod_{k=1}^{m_{t(v,v-1,v-2)}} p_k^{t(v,v-1,v-2)}\Big)\Big(K^{m_v+m_{v-1}+m_{v-2}-1} \prod_{j=0}^{m_t-1} p_{j+1}^{v} \prod_{j=0}^{m_{v-1}-1} p_{j+1}^{v-1} \prod_{j=0}^{m_{v-2}-1} p_{j+1}^{v-2}\Big)$$

où nous avons

$$\mathfrak{P} = \sum_{u=1}^{3} i_u + 2$$

et

si $j = 1,\ m_{(v,v-1,v-2)} = m_{v-1} + m_{v-2}$
si $j = 2,\ m_{(v,v-1,v-2)} = m_v + m_{v-2}$
si $j = 3,\ m_{(v,v-1,v-2)} = m_v + m_{v-1}$

et

si $j = 1,\ \prod_{k=1}^{t(v,v-1,v-2)} = \prod_{k=1}^{m_v-1} p_k^{v-1} \prod_{k=1}^{m_v-2} p_k^{v-2}$

si $j = 2,\ \prod_{k=1}^{t(v,v-1,v-2)} = \prod_{k=1}^{m_v} p_k^{v} \prod_{k=1}^{m_v-2} p_k^{v-2}$

si $j = 3$, $\prod_{k=1}^{t(v,v-1,v-2)} = \prod_{k=1}^{m_v} p_k^v \prod_{k=1}^{m_v-1} p_k^{v-1}$

Et généralement:

$K^{h-1}\alpha_v\alpha_{v-1}\ldots\alpha_{v-h}$

$$\sim R^{m_v m_{v-1}\ldots m_{v-h}+1}\, 1p_1^v p_1^{v-1} p_1^{v-2}\ldots p_1^{v-h}\left[\prod_{i=v-h}^{v}\left(Kp_1^i p_2^i K^2 p_1^i p_2^i p_3^i\ldots K^{m_i-3} p_1^i p_2^i \ldots p_{m_i-2}^i K^{m_i-1} p_1^i p_2^i \ldots p_{m_1}^i\right)\right]\left[\prod_{k=1}^{h}\left(\prod_{i_k=0}^{m_{v-k+1}-3} K^{\mathfrak{P}}\prod_{i=v-h}^{v}\prod_{j=0}^{m_i-1} p_{j+1}^i\right)\right]$$

$$\left[\left(\prod_{j=1}^{h}\prod_{i_j=0}^{m_{v-j+1}-3} K^{i_j+m_{(v,\ldots,v-h)}}\prod_{k=0}^{i_j} p_{k+1}^{v-j+1}\prod_{k=1}^{m_t(v,\ldots,v-h)} p_k^{m_t(v,\ldots,v-h)}\right)\right]$$

$$\left(K^{m_v+\ldots+m_{v-h}}\prod_{j=1}^{m_v} p_j^v \prod_{j=1}^{m_v-1} p_j^{v-1} p_j^{v-2}\ldots\prod_{j=1}^{m_n-1} p_j^1\right)$$

où nous avons

$$\mathfrak{P} = \sum_{u=1}^{k} i_u + h - 1, \quad \mathfrak{S} = \sum_{u=1}^{v} + v - 1$$

et

si $j = 1$, $m_{u(v,\ldots,v-h)} = m_{v-1} + \ldots + m_{v-h}$
si $j = 2$, $m_{u(v,\ldots,v-h)} = m_v + m_{v-3} + \ldots + m_{v-h}$
si $j = h$, $m_{u(v,\ldots,v-h)} = m_{v-1} + \ldots + m_{v-h-1}$

et

si $j = 1$, $\prod_{k=1}^{m_t(v,\ldots,v-h)} p_k^{t(v,v-1,\ldots v-h)} = \prod_{k=1}^{m_v-1} p_k^{v-1}\prod_{k=1}^{m_v-2} p_k^{v-2}\ldots\prod_{k=1}^{m_v-h} p_k^{v-h}$

si $j = 2$, $\prod_{k=1}^{m_t(v,\ldots,v-h)} p_k^{t(v,v-1,\ldots,v-h)} = \prod_{k=1}^{m_v} p_k^v\prod_{k=1}^{m_v-3} p_k^{v-3}\ldots\prod_{k=1}^{m_v-h} p_k^{v-h}$

si $j = h$, $\prod_{k=1}^{m_t(v,\ldots,v-h)} p_k^{t(v,v-1,\ldots,v-h)} = \prod_{k=1}^{m_v} p_k^v\prod_{k=1}^{m_v-1} p_k^{v-1}\ldots\prod_{k=1}^{m_v-h-1} p_k^{v-h-1}$

D'après ces relations, nous avons avec les notations derivées précédemment:

$$\alpha\sim R^{\mathfrak{R}}\, 1^v(p_1^v)^v(p_1^{v-1})^{v-1}\ldots(p_1^2)^2(p_1^1)\left[\prod_{i=1}^{v}\left(Kp_1^{v-i} p_2^i K^2 p_1^i p_2^i p_3^i\ldots K^{m_i-3} p_1^i p_2^i\ldots p_{m_i-2}^i K^{m_i-1} p_1^i p_2^i\ldots p_{m_i}^i\right)^i\right]\left[\left(\prod_{k=v-1}^{v}\prod_{i_k=0}^{m_v-k+1-3} K^{\mathfrak{P}}\prod_{i=v-1}^{v}\prod_{j=0}^{m_i-1} p_{j+1}^i\right)\ldots\left(\prod_{k=v-h}^{v}\prod_{i_k=0}^{m_v-k+1}\right.\right.$$

$$\left.\left. K^{\mathfrak{R}}\prod_{i=v-h}^{v}\prod_{j=0}^{m_i-1} p_{j+1}^i\right)\right]\left[\ldots\left(\prod_{k=1}^{v}\prod_{i_k=0}^{m_{v-k+1}-3} K^{\mathfrak{S}}\prod_{i=1}^{v}\prod_{j=0}^{m_i-1} p_{j+1}^i\right)\right]\left[\left(\prod_{j=1}^{2}\prod_{i_j=0}^{m_{v-j+1}-3}\right.\right.$$

$$K^{i_j+m_{u(v,v-1)}}\prod_{k=0}^{i_j} p_{k+1}^{v-j+1}\prod_{k=1}^{m_t(v,v-1)} p_k^{t(v,v-1)}\Big)\ldots\Big(\prod_{j=1}^{h}\prod_{i_j=0}^{m_{v-j-1}-3} K^{i_j+m_{u(v,\ldots,v-h)}}$$

$$\prod_{k=0}^{i_j} p_{k+1}^{u-j+1}\prod_{k=1}^{m_t(v,\ldots,v-h)} p_k^{t(v,\ldots,v-h)}\Big)\ldots\Big(\prod_{j=1}^{v}\prod_{i_j=0}^{m_{v-j+1}} K^{i_j+m_{u(v,\ldots,v-1)}}\prod_{k=0}^{i_j} p_{k-1}^{v-j+1}$$

$$\prod_{k=1}^{m_{t(v,\ldots,1)}} p_k^{m_{t(v,\ldots,1)}}\Big)\Big]\Big(K^{m_v+m_{v-1}}K^{m_v+m_{v-1}\ldots+m_1-1}\prod_{j=1}^{m_v}p_j^v\ldots\prod_{j=1}^{m_1}p_j^1\Big)$$

Théorème 6. *Chaque forme α du group* D, *c'est-a-dire:*

$$\alpha = DD^{m_1-1}\prod_{i=1}^{m_1}p_i^1 DD^{m_2-1}\prod_{i=1}^{m_2}p_i^2 DD^{m_3-1}\prod_{i=1}^{m_3}p_i^3\ldots DD^{m_{v-2}-1}\prod_{i=1}^{m_{v-2}}p_i^{v-2}$$

$$DD^{m_{v-1}-1}\prod_{i=1}^{m_{v-1}}p_i^{v-1}K^{m_v-1}\prod_{i=1}^{m_v}p_i^v$$

admet la forme normale:

$$\mathbf{N}_6(D) = R^{\mathfrak{N}}\left(p_{m_1}^1\right)^v\left(p_{m_2}^2\right)^{v-1}\ldots\left(p_{m_{v-1}}^{v-1}\right)^2\left(p_{m_{v-1}}^{v-1}\right)^2\left(p_{m_v}^v\right)\Bigg[\prod_{i=1}^{v}\Big(Kp_{m_i}^i p_{m_i-1}^i$$

$$K^2p_{m_i}^i p_{m_i-1}^i p_{m_i-2}^i\ldots K^{m_i-3}p_{m_i}^i p_{m_i-1}^i\ldots p_3^i K^{m_i-1}p_{m_i}^i p_{m_i-1}^i\ldots p_1^i\Big)^i\Bigg]$$

$$\Bigg[\Bigg(\prod_{i_1=1}^{m_1-1}\prod_{i_2=0}^{m_2-1}\ldots\prod_{i_v=0}^{m_v-1}K^{\mathfrak{M}}\prod_{j=0}^{i_1}p_{m_1-j}^1\prod_{j=0}^{i_2}p_{m_2-j}^2\ldots\prod_{j=0}^{m_v-1}p_{m_v-j}^v\Bigg)$$

$$\Bigg(\prod_{h=1}^{v}\prod_{j=1}^{m_h-3}K^{i_j+m_{u(1,2,\ldots,h)}}\prod_{i=1}^{i_j}p_{m_h-i}^h\prod_{i=1}^{m_{t(1,2,\ldots,h)}-1}p_{m_{t(1,2,\ldots,h)}}^{t(1,2,\ldots,h)}\Bigg)$$

$$K^{\mathfrak{L}}\prod_{i=0}^{m_1-1}p_{m_1-1}^i\prod_{i=0}^{m_2-1}p_{m_2-1}^2\ldots\prod_{i=1}^{m_v-1}p_{m_v-i}^v\Bigg]$$

où nous avons:

$$\mathfrak{N} = \sum_{j=1}^{v}\sum_{i=1}^{j}m_i + v - 1,\quad \mathfrak{M} = \sum_{j=1}^{v}i_j + v - 1,\quad \mathfrak{L} = \sum_{i=1}^{h}m_i + h - 1$$

et

si $j = 1$, $m_{u(1,2,\ldots,h)} = m_2 + m_3 + \ldots + m_h$

si $j = 2$, $m_{u(1,2,\ldots,h)} = m_1 + m_3 + \ldots + m_h$

et

$$\text{si } j = 1,\quad \prod_{i=0}^{m_{t(1,2,\ldots,h)}}p_{m_{t(1,2,\ldots,h)}}^{t(1,2,\ldots,h)} = \prod_{i=0}^{m_2-1}p_{m_2-i}^2\prod_{i=0}^{m_3-1}p_{m_3-1}^3\ldots\prod_{i=0}^{m_h-1}p_{m_h-1}^i$$

$$\text{si } j = 2,\quad \prod_{i=0}^{m_{t(1,2,\ldots,h)}}p_{m_{t(1,2,\ldots,h)}}^{t(1,2,\ldots,h)} = \prod_{i=0}^{m_1-1}p_{m_1-i}^1\prod_{i=0}^{m_3-1}p_{m_1-i}^3\prod_{i=0}^{m_h}p_{m_h-1}^h$$

$$\text{si } j = h,\quad \prod_{i=0}^{m_{t(1,2,\ldots,h)}}p_{m_{t(1,2,\ldots,h)}-i}^{t(1,2,\ldots,h)} = \prod_{i=0}^{m_1-1}p_{m_i-1}^i\prod_{i=0}^{m_2-1}p_{m_2-i}^2\ldots\prod_{i=0}^{m_{h-1}-1}p_{m_{h-1}-i}^{h-1}$$

D'après le Théorème 2 nous avons:

$$\alpha \sim R^{v-1}1K^{m_1-1}\prod_{i=1}^{m_1}p_i^1KK^{m_1-1}\prod_{i=1}^{m_1}p_i^1K^{m_2-1}\prod_{i=1}^{m_2}p_i^2K^2K^{m_1-1}\prod_{i=1}^{m_1}p_i^1$$

$$K^{m_2-1}\prod_{i=1}^{m_2} p_i^2 K^{m_3-1}\prod_{i=1}^{m_3} p_i^3 \dots K^{\nu-3}K^{m_1-1}\prod_{i=1}^{m_1} p_i^1 K^{m_2-1}\prod_{i=1}^{m_2} p_i^2 \dots$$
$$K^{m_{\nu-3}-1}\prod_{i=1}^{m_{\nu-3}} p_i^{\nu-2} K^{\nu-1}K^{m_1-1}\prod_{i=1}^{m_1} p_i^1 K^{m_2-1}\prod_{i=1}^{m_2} p_i^2 \dots K^{m_\nu-1}\prod_{i=1}^{m_\nu} p_i^\nu .$$

D'après le Théorème 2 et le lemme du Théorème 3 nous avons les relations suivantes:

$$(1)\ D^{m_1-1}\prod_{i=1}^{m_1} p_i^1 \sim R^{m_1-1} 1 p_{m_1}^1 K p_{m_1}^1 p_{m_1-1}^1 K^2 p_{m_1}^1 p_{m_1-1}^1 p_{m_1-2} \dots K^{m_1-3} p_{m_1}^1 p_{m_1-1}^1 \dots$$
$$p_3^1 K^{m_1-1} p_{m_1}^1 p_{m_1-1}^1 \dots p_1^1 p_2^1 p_3^1 .$$

$$(2)\ KD^{m_1-1}\prod_{i=1}^{m_1} p_i^1 D^{m_2-1}\prod_{i=1}^{m_2} p_i^2$$

$$\sim K\Big(R^{m_1-1} 1 p_{m_1}^1 K p_{m_1}^1 p_{m_1-1}^1 K^2 p_{m_1}^1 p_{m_1-1}^1 p_{m_1-2}^1 \dots K^{m_1-3} p_{m_1}^1 \dots p_3^1 K^{m_1-1} p_{m_1}^1 \dots$$
$$p_1^1\Big)\Big(R^{m_1-1} 1 p_{m_2}^2 K p_{m_2}^2 p_{m_2-1}^2 K^2 p_{m_2}^2 p_{m_2-1}^2 \dots K^{m_2-3} p_{m_2}^2 \dots p_3^2 K^{m_2-1} p_{m_2}^2 \dots p_1^2\Big)$$
$$\sim R^{m_1 m_2-1}\Big(1 p_{m_1}^1 K p_{m_1}^1 p_{m_1-1}^1 K^2 p_{m_1}^1 p_{m_1-1}^1 p_{m_1-2}^1 \dots K^{m_1-3} p_{m_1}^1 \dots p_3^1 K^{m_1-1} p_{m_1}^1 \dots$$
$$p_1^1 p_{m_1}^2 K p_{m_2}^2 p_{m_2-1}^2 K^2 p_{m_2}^2 p_{m_2-1}^2 p_{m_2-2}^2 \dots K^{m_2-3} p_{m_2}^2 \dots p_3^2 K^{m_2-1} p_{m_2}^2 \dots p_1^2\Big)$$
$$\Big[\Big(K p_1^1 p_{m_2}^2 K^2 p_1^1 p_{m_2}^2 p_{m_2-1}^2 K^3 p_{m_1}^1 p_{m_2}^2 p_{m_2-1}^2 p_{m_2-2}^2 \dots K^{m_2-2} p_{m_1}^1 p_{m_2}^2 p_{m_2-1}^2 \dots$$
$$p_3^2 K^{m_2} p_{m_1}^1 p_{m_2}^2 p_{m_2-1}^2 \dots p_1^2\Big)\Big(K^2 p_{m_1}^1 p_{m_1-1}^1 p_{m_2}^2 K^3 p_{m_1}^1 p_{m_1-1}^1 p_{m_2}^2 p_{m_2-1}^2 \dots$$
$$p_3^2 K^{m_2+1} p_{m_1}^1 p_{m_1-1}^1 p_{m_2}^2 p_{m_2-1}^2 \dots p_1^2\Big)\Big(K^3 p_{m_1}^1 p_{m_1-1}^1 p_{m_1-2}^1 p_{m_2}^2 K^4 p_{m_1}^1 p_{m_1-1}^1 p_{m_1-2}^1$$
$$p_{m_2}^2 p_{m_2-1}^2 \dots K^{m_2} p_{m_1}^1 p_{m_1-1}^1 p_{m_1-2}^1 p_{m_2}^2 p_{m_2-1}^2 p_{m_2-2}^2 \dots p_2^2 K^{m_2+2} p_{m_1}^1 p_{m_1-1}^1 p_{m_1-2}^1$$
$$p_{m_2}^2 p_{m_2-1}^2 \dots p_1^2\Big)\Big] \dots \Big[\Big(K^{m_1-2} p_{m_1}^1 p_{m_1-1}^1 \dots p_3^1 p_{m_2}^2 K^{m_1-1} p_{m_1}^1 p_{m_1-1}^1 \dots$$
$$p_3^1 p_{m_2}^2 p_{m_2-1}^2 \dots K^{m_1 m+_2-5} p_{m_1}^1 p_{m_1-1}^1 \dots p_{m_1}^1 p_{m_1-1}^1 \dots p_3^1 p_2^2 p_{m_2-1}^2 \dots p_3^2\Big)$$
$$\Big(K^{m_1} p_{m_1}^1 p_{m_1-1}^1 \dots p_1^1 p_{m_2}^2 K^{m_1+1} p_{m_1}^1 \dots p_{m_2}^2 p_{m_2-1}^2 \dots K^{m_1+m_2-3} p_{m_1}^1 \dots$$
$$p_m^1 p_{m_2}^2 \dots p_3^2 K^{m_1+m_3-1} p_{m_1}^1 \dots p_1^1 p_{m_2}^2 \dots p_1^1\Big)\Big]$$

$$\sim R^{m_1 m_2-1}\prod_{i=1}^{2}\Big(p_{m_i}^i K p_{m_i}^i p_{m_i-1}^i K^2 p_{m_i}^i p_{m_i-1}^i p_{m_i-2}^i \dots K^{m_i-3} p_{m_i}^i p_{m_i-1}^i \dots p_1^i$$
$$K^{m_i-1} p_{m_i}^i p_{m_i-1}^i \dots p_1^i\Big)\Bigg[\prod_{i_1=0}^{m_1-3}\prod_{i_2=0}^{m_2-3}\Big(K^{i_1+i_2+1}\prod_{j=0}^{i_1} p_{m_1-j}^1 \prod_{j=0}^{i_2} p_{m_2-j}^2 \prod_{i_1=0}^{m_1-3} K^{i_1+m_2}$$
$$\prod_{j=0}^{i_1} p_{m_1-1}^1 \prod_{j=0}^{m_2-1} p_{m_2-j}^2 \prod_{i_2=0}^{m_2-3} K^{i_2+m_1}\prod_{j=0}^{i_2} p_{m_2-j}^2 \prod_{j=0}^{m_1-1} p_{m_2-j}^1\Big)\Bigg]$$
$$\Big(K^{m_1-m_2-1} p_{m_1}^1 \dots p_1^1 p_{m_2}^2 \dots p_1^2\Big).$$

$$(3)\ K^2 D^{m_1-1}\prod_{i=1}^{m_1} p_1^1 D^{m_1-1}\prod_{i=1}^{m_2} p_i D^{m_1-1}\prod_{i=1}^{m_3} p_i^1$$

$$\sim K^2\Big(R^{m_1-1} 1 p_{m_1}^1 K p_{m_1}^1 p_{m_1-1}^1 K^2 p_{m_1}^1 p_{m_1-1}^1 p_{m_1-2}^2 \dots K^{m_1-1} p_{m_1}^1 \dots p_3^1 K^{m_1-1} p_{m_1}^1$$

$$\ldots p_1^1\big)\Big(R^{m_2-1}\mathit{1}p_{m_2}^2Kp_{m_2}^2p_{m_2-1}^2K^2p_{m_2}^2p_{m_2-1}^2p_{m_2-2}^2\ldots K^{m_2-3}p_{m_2}^2p_{m_2-1}^2\ldots p_3^2K^{m_2-1}p_{m_2}^2p_{m_2-1}^2\ldots p_1^2\Big)\Big(R^{m_3-1}\mathit{1}p_{m_3}^3Kp_{m_3}^3p_{m_3-1}^3K^2p_{m_3}^3p_{m_3-1}^3p_{m_3-2}^3\ldots K^{m_3-3}p_{m_3}^3p_{m_3-1}^3\ldots p_3^3K^{m_3-1}p_{m_3}^3p_{m_3-1}^3\ldots p_1^3\Big)R^{m_1m_2m_3-1}\mathit{1}$$

$$\left[\prod_{i=1}^{3}\Big(p_{m_i}^iKp_{m_i}^ip_{m_i-1}^iK^2p_{m_i}^ip_{m_i-1}^ip_{m_i-2}^i\ldots K^{m_i-1}p_{m_i}^ip_{m_i}^i\ldots p_3^iK^{m_i-1}p_{m_i}^ip_{m_i-1}^i\ldots p_1^i\Big)\right]\left[\left(\prod_{i_1=0}^{m_1-3}\prod_{i_2=0}^{m_2-3}\prod_{i_3=0}^{m_3-3}K^{i_1+i_2+i_3+2}\prod_{j=0}^{i_1}p_{m_1-j}^1\prod_{j=0}^{i_2}p_{m_2-j}^2\prod_{j=0}^{i_3}p_{m_3-j}^3\right)\right.$$
$$\left(\prod_{i_1=0}^{m_1-3}K^{i_1+m_2+m_3}\prod_{j=0}^{i_1}p_{m_1-j}^1\prod_{j=0}^{m_2-1}p_{m_2-j}^2\prod_{j=0}^{m_3-1}p_{m_3-j}^3\right)\left(\prod_{i_2=0}^{m_2-3}K^{i_2+m_1+m_3}\prod_{j=0}^{i_2}p_{m_2-j}^2\prod_{j=0}^{m_1-1}p_{m_1-j}^1\prod_{j=0}^{m_3-1}p_{m_3-j}^3\right)\left(\prod_{i_3=0}^{m_3-3}K^{i_3+m_1+m_2}\prod_{j=0}^{i_3}p_{m_3-j}^3\prod_{j=0}^{m_1-1}p_{m_1-j}^1\prod_{j=0}^{m_2-1}p_{m_2-j}^2\right)$$
$$\left.\left(K^{m_1+m_2+m_3-1}\prod_{j=0}^{m_1-1}p_{m_1-j}^1\prod_{j=0}^{m_2-1}p_{m_2-j}^2\prod_{j=0}^{m_3-1}p_{m_3-j}^3\right)\right]$$

$$=R^{m_1m_2m_3-1}\mathit{1}\prod_{i=1}^{3}\Big(p_{m_i}^iKp_{m_i}^ip_{m_i-1}^iK^2p_{m_i}^ip_{m_i-1}^ip_{m_i-2}^i\ldots K^{m_i-3}p_{m_i}^ip_{m_i-1}^i\ldots p_3^iK^{m_i-1}p_{m_i}^ip_{m_i-1}^i\ldots p_1^i\Big)\left(\prod_{i_1=0}^{m_1-3}\prod_{i_2=0}^{m_2-3}\prod_{i_3=0}^{m_3-3}K^{i_1+i_2+i_3+2}\prod_{j=0}^{i_2}p_{m_1-j}^1\prod_{j=0}^{i_2}p_{m_2-j}^2\prod_{j=0}^{i_3}p_{m_3-j}^3\right)\left[\prod_{h=1}^{3}\prod_{j=1}^{m_h-2}K^{i_j+m_{u(1,2,3)}}\prod_{t=0}^{i_j}p_{m_h-t}^h\prod_{j=0}^{m_{t(1,2,3)}-1}\left(K^{m_1+m_2+m_3-1}\prod_{j=0}^{m_1-1}p_{m_1-j}^m\prod_{j=0}^{m_2-1}p_{m_2-j}^2\prod_{j=0}^{m_3-1}p_{m_3-j}^3\right)\right]$$

où nous avons:

si $j = 1$, $m_{u(1,2,3)} = m_2 + m_3$
si $j = 2$, $m_{u(1,2,3)} = m_1 + m_3$
si $j = 3$, $m_{u(1,2,3)} = m_1 + m_2$

et

$$\text{si } j=1,\quad \prod_{i=0}^{m_{t(1,2,3)}-1}p_{m_{t(1,2,3)}-i}^{t(1,2,3)}=\prod_{i=0}^{m_2-1}p_{m_2-1}^2\prod_{i=0}^{m_3-1}p_{m_3-1}^3$$
$$\text{si } j=2,\quad \prod_{j=0}^{m_{t(1,2,3)}-1}p_{m_{t(1,2,3)}-i}^{t(1,2,3)}=\prod_{i=0}^{m_1-1}p_{m_1-i}^1\prod_{i=0}^{m_3-1}p_{m_3-i}^3$$
$$\text{si } j=3,\quad \prod_{i=0}^{m_{t(1,2,3)}-1}p_{m_{t(1,2,3)}-i}^{t(1,2,3)}=\prod_{i=0}^{m_1-1}p_{m_1-i}^1\prod_{i=0}^{m_2-1}p_{m_2-i}^2$$

En général, si $j = h$, alors

$$(4)\quad K^{h-1}K^{m_1-1}\prod_{i=1}^{m_1}p_i^1K^{m_2-1}\prod_{i=1}^{m_2}p_i^2\ldots K^{m_h}\prod_{i=1}^{m_h-1}p_i\sim R^{m_1m_2\ldots m_h+h-1}$$

$$1p_{m_1}\left[\prod_{i=1}^{h}\left(p_{m_i}^i K p_{m_i}^i p_{m_i-1}^i K^2 p_{m_i}^i p_{m_i-1}^i p_{m_i-2}^i \ldots K^{m_i-3} p_{m_i}^i p_{m_i-1}^i \ldots p_3^i\right.\right.$$

$$\left.\left. K^{m_i-1} p_{m_i}^i p_{m_i-1}^i \ldots p_1^i\right)\right]\left[\left(\prod_{i_1=0}^{m_1-3}\prod_{i_2=0}^{m_2-3}\ldots\prod_{i_h=0}^{m_h-1} K^{\mathfrak{M}} \prod_{j=0}^{i_1} p_{m_1-j}^1 \prod_{j=0}^{i_2} p_{m_2-j}^2 \ldots\right.\right.$$

$$\left.\left.\prod_{j=0}^{i_h} p_{m_h-j}^h\right)\left(\prod_{h=1}^{h}\prod_{j=1}^{m_h-3} K^{i_j+m_{u(1,2,\ldots,h)}} \prod_{i=0}^{i_j} p_{m_1-i} \prod_{i=0}^{m_{t(1,2,\ldots,h)}-1} p_{m_{t(1,2,\ldots,h)}}^{t(1,2,\ldots,h)}\right)\right]$$

$$\left(K^{\mathfrak{L}} \prod_{j=0}^{m_1-1} p_{m_1-j}^1 \prod_{j=0}^{m_2-1} p_{m_2-j}^2 \ldots \prod_{j=1}^{m_h-1} p_{m_h-j}^h\right)$$

où nous avons noté:

$$\mathfrak{M} = \sum_{j=1}^{h} i_j + h - 1 \quad \text{et} \quad \mathfrak{L} = \sum_{i=1}^{u} m_i - 1$$

et

si $j = 1,\ m_{u(1,2,\ldots,h)} = m_2 + m_3 + \ldots + m_h$
si $j = 2,\ m_{u(1,2,\ldots,h)} = m_1 + m_3 + \ldots + m_h$
si $j = h,\ m_{u(1,2,\ldots,h)} = m_1 + m_2 + \ldots + m_{h-1}$

et

$$\text{si } j = 1,\quad \prod_{i=0}^{m_{t(1,2,\ldots,h)}-1} p_{m_{t(1,2,\ldots,h)}-i}^{t(1,2,\ldots,h)} = \prod_{i=0}^{m_2-1} p_{m_2-i}^2 \prod_{i=0}^{m_3-1} p_{m_2-i}^3$$

$$\text{si } j = 2,\quad \prod_{i=0}^{m_{t(1,2,\ldots,h)}-1} p_{m_{t(1,2,\ldots,h)}-i}^{t(1,2,\ldots,h)} = \prod_{i=0}^{m_1-1} p_{m_1-i}^1 \prod_{i=0}^{m_3-1} p_{m_2-i}^3 \ldots \prod_{i=0}^{m_h-1} p_{m_h-i}^h$$

$$\text{si } j = h,\quad \prod_{i=0}^{m_{t(1,2,\ldots,h)}-1} p_{m_{t(1,2,\ldots,h)}-i}^{t(1,2,\ldots,h)} = \prod_{i=0}^{m_1-1} p_{m_1-i}^1 \prod_{i=0}^{m_2-1} p_{m_2-i}^2 \ldots \prod_{i=0}^{m_{h-1}-1} p_{m_{h-1}-i}^{h-1}$$

D'après ces relations, la forme α admet la forme normale:

$$\alpha \sim R^{\mathfrak{N}} 1^v \left(p_{m_1}^1\right)^v \left(p_{m_2}^2\right)^{v-1} \ldots \left(p_{m_{v-1}}^{v-1}\right)^2 \left(p_{m_v}^v\right)\left[\prod_{i=1}^{v}\left(K p_{m_i}^i p_{m_i-1}^i K^2 p_{m_i}^i p_{m_i-1}^i p_{m_i-2}^i \ldots\right.\right.$$

$$\left.\left. K^{m_i-3} p_{m_i}^i p_{m_i-1}^i \ldots p_3^i K^{m_i-1} p_{m_i}^i p_{m_i-1}^i \ldots p_1^i\right)^i\right]\left(\prod_{i=1}^{m_1-1}\prod_{i_2=0}^{m_2-1}\ldots\prod_{i=1}^{m_v-1} K^{\mathfrak{M}} \prod_{j=0}^{i_1}\right.$$

$$\left. p_{m_1-j}^1 \prod_{j=0}^{i_2} p_{m_2-j}^2 \ldots \prod_{j=0}^{i_v} p_{m_v-j}^v\right)\left(\prod_{h=1}^{v}\prod_{j=1}^{m_h-3} K^{i_j+m_{u(1,2,\ldots,v)}} \prod_{i=0}^{i_j} p_{m_{h-1}}^h \prod_{i=0}^{m_{t(1,2,\ldots,v)}}\right.$$

$$\left. p_{m_{t(1,2,\ldots,h)}-i}^{t(1,2,\ldots,h)}\right)\left(K^{\mathfrak{L}} \prod_{i=0}^{m_1-1} p_{m_1-1}^1 \prod_{i=1}^{m_2-1} p_{m_2-i}^2 \ldots \prod_{i=1}^{m_v-1} p_{m_v-i}^v\right)$$

où

$$\mathfrak{N} = \sum_{j=1}^{v}\sum_{i=1}^{j} m_j + v - 1, \quad \mathfrak{M} = \sum_{j=1}^{v} i_j + v - 1, \quad \text{et} \quad \mathfrak{L} = \sum_{i=1}^{v} m_i - 1$$

c'est-à-dire la forme normale $\mathbf{N}_6(D)$.

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