A NOTE ON "TRANSITIVITY, SUPERTRANSITIVITY AND INDUCTION"

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In the review of our paper 'Transitivity, Supertransitivity and Induction,' [1] that occurred in [2], the reviewer pointed out two apparent errors. We will here clarify the points in mention.

The reviewer first stated that Lemma 9 "seems to be in error." The difficulty, as we see it, is that the transition from step (1) to step (2) was unclear, so we will present a somewhat more complete proof. We will assume

\[ \begin{align*}
(1) & \quad \forall y (\forall x \in \text{Fld}_\epsilon \land (x \in y \to \psi(x)) \to \psi(y)) \to (\forall y (\forall x \in \text{Fld}_\epsilon \land \psi(x))) \\
(2) & \quad \forall u (\forall v \in \text{Fld}_R \land (v \in u \to \psi(v)) \to (\forall v \in \text{Fld}_R \land \psi(v))) \\
(3) & \quad \forall u (\forall v \in \text{Fld}_R \land \psi(v) \to (v \in u)) \\
(4) & \quad \forall u (\forall v \in \text{Fld}_R \land \psi(v)) \\
\end{align*} \]

for formulas \( \psi(x) \) not containing \( y \) or \( u \) and show that

\[ \psi(x) \equiv \exists x \in \text{Fld}_\epsilon \land \exists (f'(x)) \]

We will first show that \( \psi \) satisfies the hypothesis of (1). Suppose that

\[ \psi(y) \]

We must show that \( \psi(y) \). It is clear from (6) that the first part of the definition of \( \psi \) is satisfied. It remains to show that \( \psi(f'y) \). Since \( f \) is an isomorphism, there exists \( u \) such that

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Thus, we must show \( \varphi(u) \). To do this, we need only show the hypothesis of (3). The first part of the hypothesis is clear from (8). Now suppose that

\[ vRu. \]

Therefore, \( v \in \text{Fld}R \), hence there exists \( x \) such that \( x \in \text{Fld}_\epsilon \) and \( f'x = v \). Since \( f \) is an isomorphism and by (10), we have that \( x \in y \). Therefore, by (7), \( \psi(x) \); in particular, \( c(f'x) \) and hence \( c(v) \). Therefore, we have

\[ (v)(vRu \rightarrow \varphi(v)). \]

By (8), (9), (11), and (3), we have that \( \varphi(u) \). This shows that \( \psi \) satisfies the hypothesis for (1). Since (1) is assumed true, the conclusion must follow. Therefore, we have

\[ (y)(y \in \text{Fld}_\epsilon \rightarrow \psi(y)). \]

We now return to the proof of (4). Suppose \( u \in \text{Fld}R \). Therefore, there is a \( y \in \text{Fld}_\epsilon \) such that \( f'y = u \). By (12), we have that \( \psi(y) \). By (5), we have that \( c(f'y) \); therefore, we have \( c(u) \), which completes the proof of (4) and hence of (2), and so Lemma 9 is proved.

The second remark that the reviewer makes in [2] is that Theorem 19, part (ii) seems to be false. Actually it is vacuously true. Sets \( A^*_n \) are defined such that \( A = \bigcup_{n=0}^{\infty} A^*_n \subseteq B^\epsilon \). However, \( A^*_n = \emptyset \) for \( n \geq 1 \), easily seen from Lemma 17, making parts (ii) and (iii) of Theorem 19 redundant.

REFERENCES


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