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A NEW AXIOMATIZATION OF MODAL SYSTEM K1.2

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In [5], p. 316, system K1.2 is defined as an extension of S4 obtained by the addition of the axiom

H1 $\mathfrak{C}pLCMpp$

to that system, and in [4], pp. 352-355, section **2.5**, it has been proved that in the field of S4 the axiom **H1** is inferentially equivalent to its consequence

H2 $\mathfrak{C}LMpCpLp$.

In [2], p. 396, Goldblatt has shown that the same equivalence holds in the field of system S2. In this note it will be proved that the addition either of **H1** or of **H2**, as a new axiom, to S2 generates system K1.2.

Proof: Since it is self-evident that in the field of S2 **H1** implies **H2**, let us assume system S2 (classically formalized) and formula **H2**. Then:

Z1	$\mathfrak{C}Lpp$	[S1]
Z2	$\mathfrak{C}LCpqCLpLq$	[S1°]
Z3	$\mathfrak{C}\mathfrak{C}Cpqr\mathfrak{C}qr$	[S2°, **BR** (Becker Rule), *cf.* [1], p. 73, **46.1** and **46.2**]
Z4	$\mathfrak{C}\mathfrak{C}pCpq\mathfrak{C}pq$	[S2°, **BR**]
Z5	$\mathfrak{C}CMpLqLCpq$	[S1, *cf.* [3], p. 156]
Z6	$\mathfrak{C}CLpMqMCpq$	[S2°, *cf.* [6], pp. 71-72, Lemma]
Z7	$\mathfrak{C}MCMpLqMLCpq$	[*Z5*; S2°, **BR**]
Z8	$\mathfrak{C}LMCMpLqLMLCpq$	[*Z7*; S2°, **BR**]
Z9	$\mathfrak{C}LCLpMqLMCpq$	[*Z6*; S2°, **BR**]
Z10	$\mathfrak{C}MLCLpMqMLMCpq$	[*Z9*; S2°, **BR**]
Z11	$\mathfrak{C}LMLCLpMqLMLMCpq$	[*Z10*; S2°, **BR**]
Z12	$LMCMpp$	[*Z9*, p/Mp, q/p; *Z1*, p/Mp]
Z13	$LMLCLpp$	[*Z8*, p/Lp, q/p; *Z12*, p/Lp]
Z14	$LMLMCMpp$	[*Z11*, p/Mp, q/p; *Z13*, p/Mp]
Z15	$CLMCMppLLMCMpp$	[**H2**, $p/LMCMpp$; *Z14*]
Z16	$LLMCMpp$	[*Z15*; *Z12*; S1°, *cf.* [1], p. 53, **32.221**]
Z17	$LLMCpLp$	[*Z16*, p/Np; S1°]
Z18	$\mathfrak{C}LLMp\mathfrak{C}pLp$	[**H2**; S2°, **BR**]

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Z19	$\mathfrak{C}CpLpLCpLp$	[*Z18*, $p/CpLp$; *Z17*]
Z20	$\mathfrak{C}LpLCpLp$	[*Z3*, q/Lp, $r/LCpLp$; *Z19*]
Z21	$\mathfrak{C}LpCLpLLp$	[*Z20*; *Z2*, q/Lp; S1°]
C10	$\mathfrak{C}LpLLp$	[*Z4*, p/Lp, q/LLp, *Z21*]

Since C10 is the proper axiom of S4 and we have S1, the proof is complete. Thus, *cf.* [4], p. 355, it is established:

$\{K1.2\} \rightleftarrows \{S4;\ H1\} \rightleftarrows \{S4;\ H2\} \rightleftarrows \{K1;\ L1\} \rightleftarrows \{K1.1;\ L1\} \rightleftarrows \{S2;\ H1\} \rightleftarrows \{S2;\ H2\}$

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