Notre Dame Journal of Formal Logic
Volume XVIII, Number 3, July 1977
NDJFAM

THE "*CONDITIONATIM*"-CLAUSE: ONE OF THE PROBLEMS OF EXISTENTIAL IMPORT IN THE HISTORY OF LOGIC

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The discussion of the question whether one can legitimately infer 'Some man is white' from 'Every man is white' and of the related questions is almost as long as the history of the science of logic itself.[1] I intend, in this paper,* to sketch the already known stages of the discussion and then present in some detail a late-medieval phase which seems to have escaped the historians. This phase is the more interesting because it may be seen as a missing link between the so-called traditional and the modern views.

1 Looking at the Aristotelian syllogistic theory through the eyes of J. Łukasiewicz[2] we find the following as an assertable sentence:

(1) $$CAabIab \quad .$$

This sentence can be a logical thesis because it meets certain conditions of the metalanguage of elementary categorical syllogistic theory. Its terms, a and b, are replaceable only by terms which are general, non-negative, *and referential*.[3] Thus, the term 'Socrates' does not satisfy this requirement, nor does the term 'non-man', nor the term 'mermaid'—each for different reasons. In the object language, the requirement of referentiality appears in the form of the axiom[4]

(2) $$Iaa \quad .$$

As compared with this, modern logicians *à la* Boole or Russell do not allow the transition from 'every' to 'some' without some special, explicitly stated presuppositions added conjunctively to the universal clause. Thus neither the sentence

(3) $$\alpha \cap \bar{\beta} = 0 \;.\!\supset\!.\; \alpha \cap \beta \neq 0$$

nor

(4) $$(x)(Fx \supset Gx) \supset (\exists x)(Fx \,.\, Gx)$$

is assertible as a thesis, unless we add conjunctively an explicit existential clause or clauses to the antecedents.[5]

*An earlier version of this paper was presented at the Fourth International Congress for Logic, Methodology, and Philosophy of Science held in Bucharest, Rumania, August 29-September 4, 1971.

Received November 12, 1973

The two seemingly incompatible positions have given rise to various controversies between the "traditional" and the "modern" logicians, the former accusing the latter of not doing justice to our ordinary language, and the latter accusing the former of offering as theses sentential forms which may be true only contingently. Needless to say, the two positions are not incompatible, once we understand the metalanguage of the system within which the above sentential forms are considered.

The medieval version which seems to correspond to the Aristotelian system as interpreted by Łukasiewicz is found in those parts of those logical *summae* which go under the name of *logica vetus* or *logica antiqua*.[6] The presupposition of referentiality is implied by the examples given on the square of opposition. However, in their tracts on the *logica nova* and the *logica moderna* the same medieval authors often reflected on the "Aristotelian" materials which they had expounded in the former, primarily expository works. They asked such questions as whether 'Every chimaera is a chimaera'[7] is a tautology, whether negative propositions ever imply affirmative ones, etc. One of the results of such questioning seems to be a new, non-Aristotelian, kind of the square of opposition of categorical propositions which Professor E. A. Moody reconstructed in his *Truth and Consequence in Mediaeval Logic*,[8] viz.

(5)

$$(\exists x)Fx \,.\, \sim(\exists x)(Fx \,.\, \sim Gx) \qquad \sim(\exists x)(Fx \,.\, Gx)$$

$$(\exists x)(Fx \,.\, Gx) \qquad \sim(\exists x)Fx \vee (\exists x)(Fx \,.\, \sim Gx)$$

This interpretation of the four kinds of categorical propositions does meet the requirement of existential import, but it does not meet the requirements for conversion as applied in the reductive process of certain syllogistic figures where more presuppositions would have to be stated. It is worthwhile to observe that these medievals did not have any difficulty in recognizing both interpretations, i.e., the Aristotelian one and that of the *logica moderna,* as logically admissible; and furthermore, that neither F. Brentano's nor B. Russell's readings of categorical propositions coincides with either the Aristotelian or the medieval interpretation.

2 Carl von Prantl, in his account of the accretions to and modifications of the *Summulae Logicales* of Peter of Spain in the late fifteenth and early sixteenth centuries in various Thomistic, Scotistic and other centers of study, mentions three versions of a compendium and gives a table of contents with some remarks about the indebtedness of the author to his predecessors or contemporaries. The three versions are almost word for word identical. I was able to examine two of them, the *Paruulus logice* and the *Textus paruuli logice una cum prebreui et perutili repetitione eiusdem*. Both are unpaginated and undated.[9] Of the many interesting features of this work, I would like to mention two: (a) the organization of materials, and (b) the unusual discussion of existential import of categorical propositions.

(a) The tract begins, not with a discussion of terms, in the manner of the *Port-Royal Logic* of the succeeding centuries,[10] but rather with a discussion of *propositio*. Next comes a brief treatment of terms, but only as they function in propositions. This is the doctrine of *suppositio*. The third chapter gives an interesting discussion of explication of those compound propositions which appear rather simple in ordinary languages, but which are logically complex; these are the *exponibilia*. Next comes a treatment of categorical propositions whose truth-conditions depend on temporal and modal functors. The fifth chapter corresponds to Porphyry's *Isagoge*, the sixth to the Aristotle's *Categories*, and the seventh to Aristotle's categorical syllogistic theory. This arrangement reminds one of Burleigh's *De puritate artis logicae*[11] in which, too, the syllogistic theory appears almost as an appendix to a logical theory which is considered to be more basic (even though Burleigh's other major logical work, *Super artem veterem*, follows the usual outline of *logica vetus*); however, it differs from it radically in that the doctrine of general consequential rules is placed at the very end of the treatise. It seems as if our anonymous author (henceforth referred to as 'AA' for short) thought of the object language first, and then provided the metalinguistic super-structure. This presumption is supported by the fact that within the seventh tract the valid syllogistic patterns are given first, and the most general syllogistic rules[12] last.

(b) The second respect, and the most important one for the topic of this paper, is the statement of what I shall call the *"conditionatim"*-clause-solution to the problem of existential import of categorical propositions. It is in the second chapter of *Paruulus logice* that this unusual view is presented. Having first pointed out why the descent to singulars is impossible in case of terms having discrete supposition, the author continues:

> Secunda regula est talis quod terminus communis super quem non cadit sinkathegoreuma confusiuum supponit determinate sub quo contingit descendere disiunctiue et conditionatim in affirmatiuis . . . vt quidam homo currit, vbi sic descenditur: ergo isto homo, *si sit*, currit, vel iste homo, *si sit*, currit, et sic de alijs; ergo quidam homo currit.[13]

The second rule of supposition thus states that even in a particular affirmative proposition the descent to singulars is allowed only if we add a conditional clause to each proposition, the disjunction of which would verify it. Previous logicians considered the following as a valid descent: Some man is running; therefore, this man is running or that man is running, etc., but the AA of *Paruulus logice* allows only: Some man is running; therefore this man, if he exists, is running, or that man, if he exists, is running, etc. It seems obvious that he considered even strictly singular terms to be capable of referential failure. It may, of course, be the case that in his example of descent to singular *'iste'* is not to be treated as a demonstrative pronoun but rather as a sort of metalinguistic device for grammatically proper names which might in fact be (disguised) definite descriptions. But even so, his *"conditionatim"* requirement for I-propositions seems unusual. If we should express it in any **PM**-like notation we would get:

(6) $(\exists x)(Fx \,.\, Gx) \therefore [Fa \supset (Fa \,.\, Ga)] \vee [Fb \supset (Fb \,.\, Gb)] \vee \ldots$

Now the disjunction expressing the descent to singulars is a very weak proposition, being conditional in each of its disjuncts. It could be true even on the simple condition that the antecedent in one of the disjuncts is not satisfied.

Let us see next what AA says about the corresponding A-proposition and the descent to its singulars. We find this information in the statement of the rule for descent from terms which have confused distributive supposition:

> Tertia regula est ista quod signum universale affirmatiuum confundit terminum communem inmediate sequentem cum eo constructum confuse distributive. Et quicquid aliud sequitur stat confuse tantum, vt omnis homo est animal. Contingit enim sub termino stante confuse et distributive descendere copulative et conditionatim in affirmatiuis . . . vt omnis homo currit, vbi sic descenditur: ergo isto homo, *si sit*, currit, et isto homo, *si sit*, currit, et sic de alijs: ergo omnis homo currit.[14]

Expressing, again, the descent from an A-proposition in a **PM**-like notation we get:

(7) $(x)(Fx \supset Gx) \therefore [Fa \supset (Fa\,.\,Ga)]\,.\,[Fb \supset (Fb\,.\,Gb)]\;.\;.\;.\;.$

The only difference, then, between an A- and an I-proposition consists in the kind of descent—either conjunctive or disjunctive respectively—and not in the elements constituting the "descended" proposition. This definitely secures the implication-relation between an A- and an I-proposition, since a conjunction always implies the corresponding disjunction. If the antecedent fails in one case, it will fail in the other case and thus both propositions will turn out true; and if the consequent will fail in one case, it will fail for the same reason in the other, and it will again depend on the truth-value of the antecedent whether the propositions are both true or both false, but the inferential relation between A- and I-propositions again cannot fail. So much for affirmative propositions.

3 Now what are the consequences of such an interpretation for the relation between the affirmative and negative categorical propositions. To answer this question I must first point out certain logical rules and laws to which AA had explicitly committed himself. In the first chapter of the *Paruulus logice* we find two basic versions of the so-called De Morgan Laws. Concerning the truth conditions for conjunctions AA says:

> Ad veritatem copulative requiritur vtramque partem esse veram . . . Ad falsitatem eius sufficit alteram partem esse falsam.[15]

And concerning the disjunction he says:

> Ad eius [i.e., disiunctive] veritatem sufficit alteram partem esse veram . . . Ad falsitatem eius requiritur vtramque esse falsam.[16]

If we now take the "descended" sentences as finite conjunctions and finite disjunctions in a universe of, say, two individuals *a* and *b*, and negate them, our O-proposition becomes:

(8) $$\sim\{[Fa \supset (Fa . Ga)] . [Fb \supset (Fb . Gb)]\} \quad ,$$
$$\text{i.e., } \sim[Fa \supset (Fa . Ga)] \vee \sim[Fb \supset (Fb . Gb)] \quad ,$$

and our E-proposition turns out to be:

(9) $$\sim\{[Fa \supset (Fa . Ga)] \vee [Fb \supset (Fb . Gb)]\} \quad ,$$
$$\text{i.e., } \sim[Fa \supset (Fa . Ga)] . \sim[Fb \supset (Fb . Gb)] \quad .$$

Considering next the fact that the falsity conditions of conditionals were also explicitly stated ("Ad falsitatem eius [i.e., conditionalis] requiritur quod antecedens potest esse verum sine consequente")[17] we get for O:

(10) $$[Fa . \sim(Fa . Ga)] \vee [Fb . \sim(Fb . Gb)]$$

and for E:

$$[Fa . \sim(Fa . Ga)] . [Fb . \sim(Fb . Gb)] \quad .$$

The result is curious. Instead of ending up with a square of *logica moderna* in which both affirmative propositions have existential import and neither of the negative ones has it, we have now just the reverse: both of the negative propositions have existential import and neither of the affirmative ones has it. And of the two, E and O, E is the stronger since it asserts the existence of *Fa* and *Fb* conjunctively, while O asserts their existence only disjunctively. The "*conditionatim*"-clause, built into the very statement of the affirmative propositions, is responsible for this result. AA most likely did not intend it. However, given his recognition of De Morgan's Laws, etc., the curious consequences are inevitable. His concern with existential import made him say things the implications of which are intolerable.

4 The first and the second rule of supposition cited in section 2 above were given in isolation from the corresponding rules for affirmative propositions. We must now state AA's rules of descent for negative propositions and explore the resulting consequences. AA claims that as far as negative propositions are concerned, the descent is to either simple disjunctive or simple conjunctive propositions; the "*conditionatim*"-clause is not required as was the case with affirmative propositions. Thus, from 'A man is not running' we can infer 'This man is not running or that man is not running, etc.';[18] and from 'No man is running' we can infer 'This man is not running and that man is not running, etc.'[19] These descents can be represented by

(12) $$(\exists x)(Fx . \sim Gx) \therefore (Fa . \sim Ga) \vee (Fb . \sim Gb) \vee \ldots$$

and

(13) $$(x)(Fx \supset \sim Gx) \therefore (Fa \supset \sim Ga) . (Fb \supset \sim Gb) \ldots .$$

Limiting ourselves again to the set of two individuals, *a* and *b*, our O and E propositions, i.e.,

(14) $$(Fa . \sim Ga) \vee (Fb . \sim Gb)$$

and

(15) $$(Fa \supset \sim Ga) \,.\, (Fb \supset \sim Gb)$$

would no longer be contradictories of (10) and (11), and thus either the laws of contradictory opposition or the laws of duality for the denials of conjunction and disjunction, or both, must be given up. The results are again intolerable.

5 Assume now that AA required the "*conditionatim*"-clause only for A-propositions. Here, the clause seems reasonable, since we sometimes do want to make general statements about things which do not exist or which never existed or even never could exist. "Every dinosaur is a vertebrate" may be taken as an example of the first case, "Every mermaid is pretty" of the second, and "Every body moving in a certain direction and not impeded by external forces moves interminably in the same direction" of the third.[20] The square would then turn out to be a Russellian one, or an equivalent of it, in which both A and E are essentially negative and both I and O essentially affirmative with respect to the existence of the subject:

(16)

$[Fa \supset (Fa \,.\, Ga)] \,.\, [Fb \supset (Fb \,.\, Gb)]$	$\sim (Fa \,.\, Ga) \,.\, \sim (Fb \,.\, Gb)$
$(Fa \,.\, Ga) \vee (Fb \,.\, Gb)$	$[Fa \,.\, \sim (Fa \,.\, Ga)] \vee [Fb \,.\, \sim (Fb \,.\, Gb)]$

Given the rule that if a proposition implies another, it implies itself along with it,[21] we could simplify our initial statement of A-proposition into

(17) $$(x)(Fx \supset Gx)$$

and our O-proposition would turn out as

(18) $$\sim (x)(Fx \supset Gx) \;,$$

i.e., as $(\exists x)(Fx \,.\, \sim Gx)$, which would give us the so-called "modern" square. Our AA, it seems to me, made important steps in the direction of such an interpretation of the standard categorical propositions. The recent scholastic Franz Brentano needed merely to pull the strings together to give us the existentially-neutral A- and E-propositions and existentially-committing I- and O-propositions on the "modern" square of opposition.[22] However, the precise struggles of the AA of *Paruulus* seem to have escaped all historians of logic up to this day.

NOTES

1. The historical works of Łukasiewicz, Bocheński, Kneales, Moody, Boehner, etc., as well as elementary textbooks on logic testify to this. Alonzo Church made an interesting survey of the history of this problem in his contribution to the volume *Logic, Methodology and Philosophy of Science*, Y. Bar-Hillel, ed., North Holland Publishing Co., Amsterdam 1965, pp. 417-425. The closest hint at the "*conditionatim*"-clause comes from P. Boehner, *Medieval Logic*, Manchester, Manchester University Press, 1952, pp. 50f. He mentions a manuscript, MS. 153

of the Dominican Library at Vienna, written in the fifteenth century, containing an anonymous tract beginning with "Ad clariorem circa terminorum suppositiones . . ." which might be used by historians as a clue to the (perhaps more embracing) prototype of our *Paruulus.* Its placement of the condition for making a descent under terms with determinate supposition, the confused and distributive supposition, and also the pure confused supposition, is exactly the same as in our treatise. However, Boehner does not explore the logical consequences of such an interpretation for the rest of logical theory.

2. *Cf. Aristotle's Syllogistic* (second enlarged edition), Oxford, Clarenden Press, (1957).

3. *Ibid.*, p. 130.

4. *Ibid.*, p. 88.

5. E.g., $\alpha \neq 0 . \alpha \cap \bar{\beta} = 0 .\supset. \alpha \cap \beta \neq 0$; or $(\exists x) Fx . (x)(Fx \supset Gx) \supset (\exists x)(Fx . Gx)$. To secure all the laws of conversion and all the syllogistic moods we should add in some cases existential assumptions concerning terms in the predicate position.

6. *Logica vetus* is that part of logic which was available in the Latin West before the twelfth century, i.e., the *Categories* and *On Interpretation* of Aristotle with Ciceronian and Boethian accretions, Porphyry's *Isagoge*, and the *Liber de Sex Principiis* of Gilbertus Porretanus. *Logica nova* is that body of Aristotelian Organon which was made readily available by the middle of the twelfth century, i.e., *Prior Analytics*, *Posterior Analytics*, *Topics* and *Sophistical Refutations*. Collectively the "old" and the "new" logics became known as *logica antiqua*, in contradistinction to the emerging *logica moderna*. The latter consisted of such tracts as those on properties of terms, on syncategorematics, on consequences, on insolubles, on sophisms, etc.

7. Abelard, for example, would say that this proposition is simply false on the ground that there are no chimaeras. *Cf.* his *Dialectica*, L. M. de Rijk, ed., Assen, Van Gorcum, (1956), p. 176, 11. 21f.

8. *Cf.* his *Truth and Consequence in Mediaeval Logic*, Amsterdam (1953), p. 52.

9. The third version mentioned by Prantl is entitled: *Textus parvuli modernorum* and was printed by Melchior Lotter in Leipzig. My *Textus paruuli logice . . .* was printed by Frederick Kreusner. All text references given below are to Kreusner's edition.

   Prantl distinguishes this work from similar compedia which went by the name of *"parvulus antiquorum"* and which were limited to the matter of the first seven chapters of Peter's *Summulae*, while "the new" compilations also took into account the works of Paulus Venetur. *Cf. Geschichte der Logik im Abendlande*, Graz, Akademische Druck u. Verlagsanstalt, (1955), Band IV, pp. 219f.

10. R. M. Eaton's *General Logic*, New York, C. Scribner's Sons, (1931), Part II, is a typical example of the great influence which this post-scholastic work had on the subsequent text-book tradition down to our own century. Compared with the high formalities and rigor of medieval logical treatises, Arnauld's work is definitely a retrogression which only the development of mathematical logic was able to stop.

11. I have in mind the *Tractatus brevior*, St. Bonaventure (1951), p. 22, where only fifteen lines are devoted to the statement of categorical-syllogistic consequential rules.

12. I.e., the rule governing unanalyzed propositions and the rules governing inferential relations among those general propositions whose formal properties are determined by the supposition of their terms.

13. *Paruulus logice*, Kreusner's edition, p. 7. Italics are mine.

14. *Ibid.*, p. 8.

15. *Ibid.*, p. 5.

16. *Ibid.*, p. 5.

17. *Ibid.*, p. 5.

18. *Ibid.*, pp. 7f.

19. *Ibid.*, p. 8.

20. Assuming that there is in the universe more than one body.

21. The statement of this generally accepted rule is given, for example, in W. Burleigh, *De puritate artis logicae, tractatus longior*, with a revised edition of the *Tractatus Brevior*, St. Bonaventure, N.Y., The Fransiscan Institute, (1955), p. 203: "Quaelibet propositio infert seipsam cum suo consequente", which rule is used there to establish the general rule: "Quidquid sequitur ex consequente et ex antecedente, sequitur ex antecendente per se".

22. *Cf.* his *Psychologie vom empirischen Standpunkt*, Leipzig F. Meiner (1874), II, *Ch.* 7.

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