Notre Dame Journal of Formal Logic
Volume XVII, Number 3, July 1976
NDJFAM

VENN DIAGRAMS EXTENDED: MAP LOGIC

JOHN RYBAK AND JANET RYBAK

1 *From Venn to Karnaugh* The Venn System of logic diagrams is a beautiful and effective system, it has, however, hitherto been of such restricted range that it has been relatively neglected.* To the best of our belief the only established writer to see its real possibilities is W. V. O. Quine. In his *Methods of Logic*[1] he touches on the use of diagrams for complex terms as in E a $(S \vee P)$, or $(W \cdot S)$ a E and he suggests one might go beyond three or four variables by splitting a longer argument into several three-variable parts.

Some six years ago the authors encountered the Karnaugh Map (a version of the Veitch-diagram adapted by Dr. M. Karnaugh, who presented it as an improved method of simplifying the design of computer circuitry[2]) and saw that this provided a mechanical method for handling a large range of logical problems.

In using Karnaugh Maps for logic, maps for two, three, and four variables are drawn as follows; Figures 1-3:

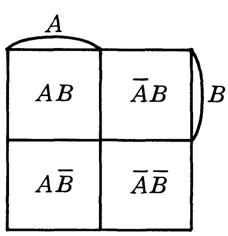


Figure 1

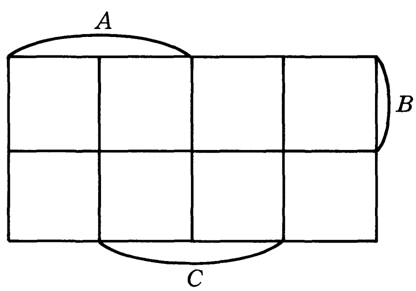


Figure 2

*The substance of this paper was originally presented as part of an informal staff-seminar to some senior members of Sydney University Philosophy Department, November 28, 1973.

1. W. V. O. Quine, *Methods of Logic*, Second Edition, Routledge & Kegan Paul, London (1966), pp. 79-81.

2. M. Karnaugh, "The map method for synthesis of combinational logic circuits," *Transactions, American Institute of Electrical Engineers*, pt. 1, Communications & Electronics, vol. 72 (1953), pp. 593-599.

Received July 7, 1974

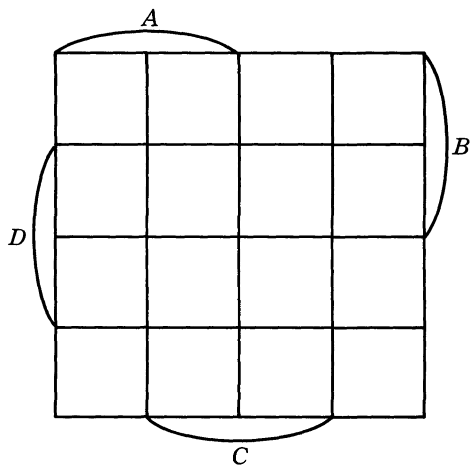


Figure 3

2 *Demonstrating Given Conclusions* Let us at once proceed to work a 5-variable argument and fill in the details as we go along. Take the sorites:

$$A \text{ a } B,\ \bar{C} \text{ e } B,\ C \text{ a } D,\ D \text{ e } E\ /\ \therefore A \text{ e } E.$$

Draw a 4-variable map and then halve the vertical columns, thus doubling the number of cells. We now have a 5-variable map, Figure 4. Labelling may be carried out in *any* order.

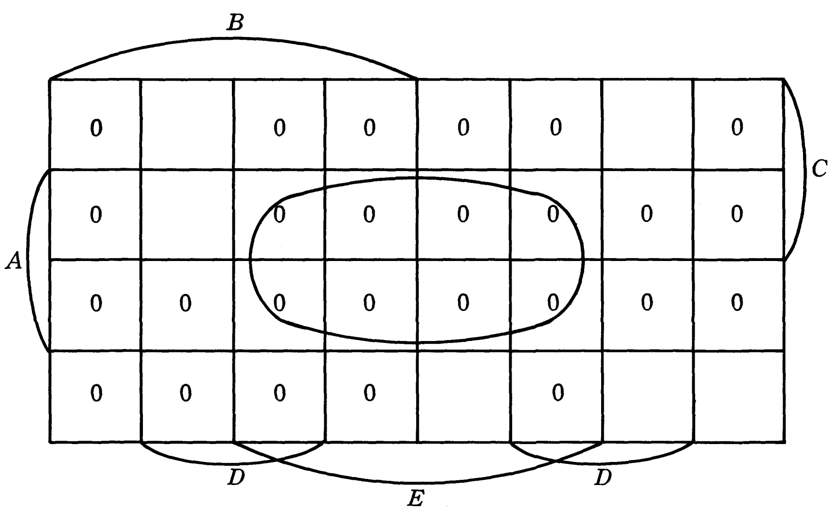


Figure 4

Write the argument in the usual Boole-Venn form:

$$A\bar{B} = 0,\ \ \bar{C}B = 0,\ \ C\bar{D} = 0,\ \ DE = 0\ \ /\ \therefore\ AE = 0.$$

Enter the premises on the map, as in Fig. 4. In each of the eight cells which are $A \cdot \bar{B}$ (A but not B) enter a '0'. For the fourth premise enter a '0' in each of the cells which are both D and E and so on for the rest, excepting the conclusion. When we inspect the conclusion-cells, the A which are E, ("circled"), we find they are *already* filled with '0's. The conclusion has been demonstrated. If the procedure had been correctly followed and yet we found one or more cells within the "circle" failed to carry a '0' we would have demonstrated the invalidity of the argument; thus the system is *effective*.

3 *Particular Premises* Arguments containing particular premises require only that a diagonal line, as in Figure 5, or some other convenient mark, be placed in the relevant cell or cells. This is adequately illustrated with a brief argument.

1.	A a B	$A \cdot \bar{B} = 0$
2.	C o B	$C \cdot \bar{B} \neq 0$
∴	C o A	∴ $C \cdot \bar{A} \neq 0$

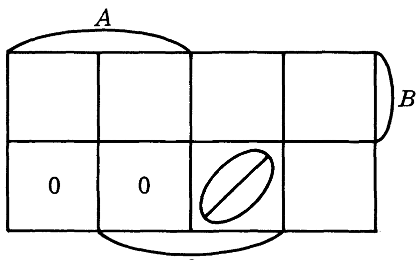


Figure 5

Record universals before particular premises; here this removes the need to decide whether the cell $AC\bar{B}$ should also be marked $\neq 0$, for premise 1 shows that it is $= 0$. The premises being entered, the conclusion (circled) is found to be thereby shown; $C\bar{A}$ is seen to be not entirely empty; the argument is valid.

Cases arise where the '/' is required in more than one cell; e.g., the premise $Y\bar{Z} \neq 0$ may need to be marked in some such manner as is shown in Figure 6—indicating that one of these effectively adjacent (see section *Larger Maps*) cells is non-empty—we do not know which.

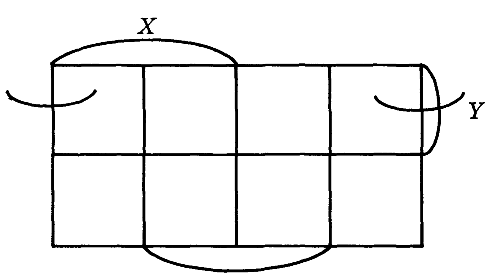


Figure 6

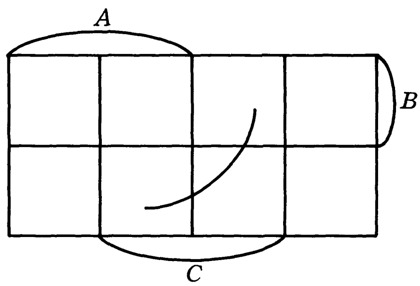
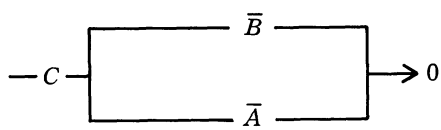


Figure 7

Figure 7 shows a useful way of recording such a premise as C o $(B \cdot A)$; the boolean form of which is $C\bar{B} \vee C\bar{A} \neq 0$. For a particular premise with an 'ored' boolean form, like $C\bar{B} \vee C\bar{A} \neq 0$, only one disjunct need be non-empty on the map, but for a universal (like $C\bar{B} \vee C\bar{A} = 0$) it is necessary that *both* disjuncts are = 0, as is shown by the following simplified circuit diagram:



Elementary Boolean Laws are needed in handling propositions with complex terms like A o $(B \vee C)$.

	A o $(B \vee C)$	(Extended Trad. Form)
	$A \cdot \overline{B \vee C} \neq 0$	(Boolean Form)
	$A \cdot (\bar{B} \cdot \bar{C}) \neq 0$	(DeMorgan)
Mapping Form:	$A \cdot \bar{B} \cdot \bar{C} \neq 0$	(Association)

Of course, one quickly learns to carry out such moves "in one's head". Fairly frequent use is also made of the laws of Complements, Identity, Idempotency, Distribution, Commutation and Double Negation.

4 *Larger Maps* Some diagrams for larger arguments follow: The cells marked 'X' on the 6-variable map below, Figure 8, are "effectively adjacent" to each other; similarly the pair marked 'Y'. (Imagine the map rolled into a cylinder in either direction.)

Figure 9 shows the halving of each vertical column to transform a 6-variable map into a 7-variable map. An 8-variable map would halve each horizontal row of a 7-variable map. Readers might find it a convenience to roneo or mimeograph a supply of 7-, 8-, 9-, 10-variable maps just as mathematicians have graph paper prepared for them to work on. The curved lines to be labelled with the variables could be included on the map. (Alternatively, all maps could be 10-variable maps; by halving one of these with a coloured line we have a 9-variable map, and so on).

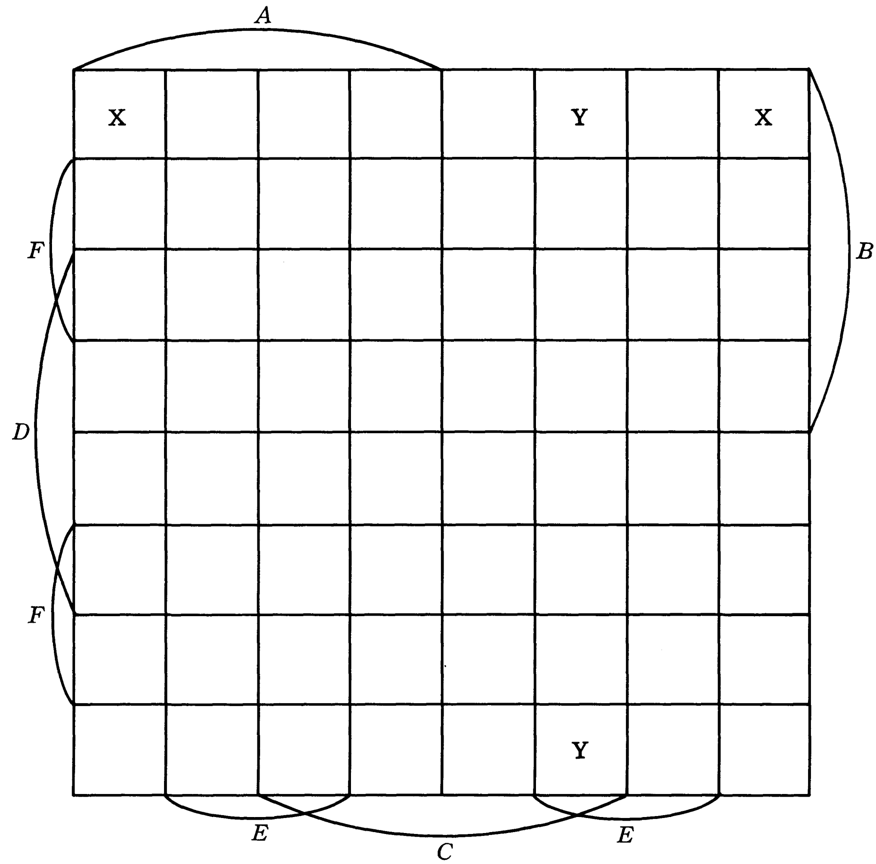


Figure 8

It will be seen that a 3-variable map has 2^3 cells, a 4-variable map 2^4 cells, an n-variable 2^n cells—maps grow exponentially and to work arguments with 9 or 10 variables is often too tedious. After 8 variables, perhaps one should consider such devices as 'splitting arguments' as Quine suggested. We have comfortably split a 10-variable argument into two 5-variable maps followed by one 6-variable map.[3] To do this one groups premises to eliminate common terms and terms appearing only once in the premises and not in the conclusion, and to retain conclusion terms.

For even larger arguments the authors have a rough, working computer program which can handle up to fourteen variables. Finance permitting, we think this might be extended to some twenty-two variables or even a few more with some kinds of computer, but even here the exponential growth of the number of cells required sets limits.

3. For further details of this and the computer program, etc., see our *Map logic and other extensions of traditional logic* (Published by the authors, c/o Department of Economic Statistics, University of Sydney, Australia, December 1973).

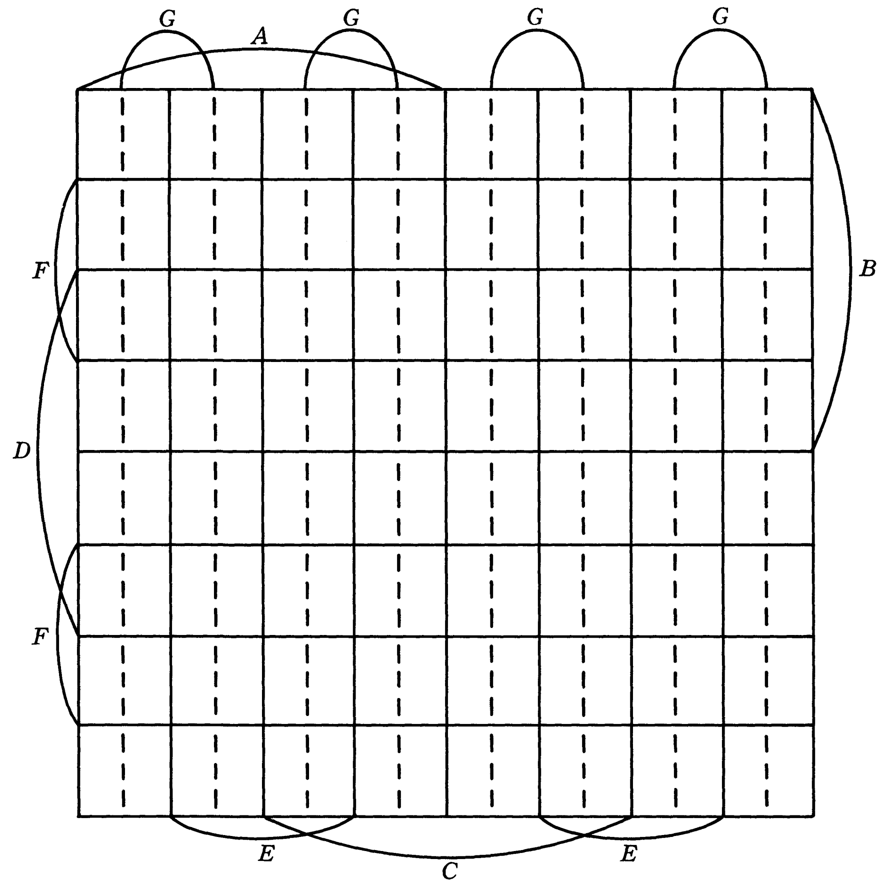


Figure 9

5 *Discovering Conclusions* An interesting development is the use of the map-method to find what conclusions a given set of premises implies. To illustrate, we shall limit the conclusions sought to those which are universal, though particular conclusions present no special difficulty. Given the premises:

1. A a $\overline{B}$	$AB = 0$
2. B a $(C \cdot D)$	$B\overline{C} \vee B\overline{D} = 0$
3. D a $(B \vee C)$	$D\overline{B}\overline{C} = 0$
4. A e $(\overline{D} \cdot \overline{C})$	$A\overline{D}\overline{C} = 0$

What can we conclude? A 4-variable map is drawn and labelled, Figure 10. Instead of '0's, enter the premise-numbers in the appropriate cells to avoid later reading premises from the map as possible conclusions, and as an aid to seeing which premises co-operate to produce selected conclusions. To find conclusions first look for patterned combinations of adjacent cells, especially those which bring in all or most of the premises.

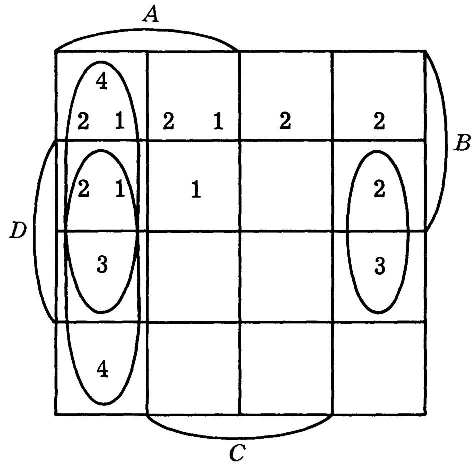


Figure 10

Inspection shows that one such grouping is the left-hand vertical row of cells. The simplest reading of this column is $A\overline{C} = 0$; i.e., A e $\overline{C}$ or $\overline{C}$ a $\overline{A}$ or A a C. Another conclusion embracing only three of the premises comprises the two middle left-hand and right-hand cells. This may be read as $D\overline{C} = 0$ or D a C. But we could also consider a single cell; e.g., the top left cell is $AB\overline{C}\overline{D} = 0$ which has several possible traditional readings: $(A \cdot B)$ a $(D \vee C)$ or $(A \cdot B \cdot \overline{D})$ a C or B a $(\overline{A} \vee D \vee C)$ and many more[4].

The map-method also detects incompatible premises: a '/' appearing in a cell which already contains a '0', or in a group of cells all of which already contains '0's. For arguments with many variables we suggest that the work be turned over to a computer .which could print out a great many possible conclusions (together with a map) from amongst which a researcher may select for closer investigation those that interest him. Like the Venn diagram, the map may also be employed for certain set-arithmetical problems and probability-calculations, with the improvement that a useful increase in the number of variables is afforded by the map method.[5] It would not seem unlikely that still further extensions are possible.

University of Sydney
Sydney, New South Wales, Australia

4. For further details see the work *Map Logic* already cited.

5. See Janet Rybak, "Diagrams for set theory and probability problems of four or more variables," *The American Statistician*, vol. 29 (1975), pp. 91-93.