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NOTE ON AN INDEPENDENCE PROOF OF JOHANSSON

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In [1], p. 124, I. Johansson proves that the propositional formula $\neg\neg(\neg\neg a \supset a)$ is underivable in his minimal logic. He establishes this result by the well-known matrix-method: he gives certain matrices in which all the axioms of the minimal logic are valid, the rules of the system preserve validity, but $\neg\neg(\neg\neg a \supset a)$ is invalid. The matrices he uses are 5×5 matrices, i.e., of 5 rows and 5 columns for the binary connectives. The purpose of this short note is to point out that there are simpler 3×3 matrices which do the same job. The matrices for the connectives are given below. The only designated value is 1.

$\supset$	1	2	3
*1	1	2	3
2	1	1	3
3	1	1	1

$\wedge$	1	2	3
*1	1	2	3
2	2	2	3
3	3	3	3

$\vee$	1	2	3
*1	1	1	1
2	1	2	2
3	1	2	3

x	$\neg x$
*1	2
2	1
3	1

It is easy to check that all the axioms of the minimal logic are valid in these matrices, and the rules of the system preserve validity; yet $\neg\neg(\neg\neg a \supset a)$ is invalid, for if the value of 'a' is 3 then $\neg\neg(\neg\neg 3 \supset 3) = \neg\neg(2 \supset 3) = \neg\neg 3 = 2 \neq 1$.

REFERENCE

[1] Johansson, I., "Der Minimalkalkül, ein reduzierter intuitionistischer Formalismus," *Compositio mathematica*, vol. 4 (1936), pp. 119-136.

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