Notre Dame Journal of Formal Logic
Volume XVII, Number 2, April 1976
NDJFAM

NOR LOGIC: A SYSTEM OF NATURAL DEDUCTION

LAURENCE S. GAGNON

It has long been known that classical sentential logic can be based on either **NAND** or **NOR** operations. Only recently, however, has a natural deduction system for **NAND** been developed by Price [1]. In a similar vein, the aim of this paper is to present a consistent and complete set of inference rules for the **NOR** operator. The metalinguistic notation used is basically that of Goodstein [2].

The present **NOR** system contains an introduction rule and two elimination rules, each of which has two forms.

$$\text{Xi:}\quad \frac{\ldots A \vdash B,\ \ldots A \vdash B \downarrow B}{A \downarrow A}$$

$$\text{Xe:}\quad \frac{A \downarrow B, A}{C} \qquad \frac{A \downarrow B, B}{C}$$

$$\text{XXe:}\quad \frac{(A \downarrow B) \downarrow (A \downarrow B), A \downarrow A}{B} \qquad \frac{(A \downarrow B) \downarrow (A \downarrow B), B \downarrow B}{A}$$

The introduction rule, Xi, is the only one of the set which allows one to discharge an assumption (hypothesis) from a proof. Since the standard matrix for **NOR** validates all of the inference rules, the system is consistent. Moreover, the rules are independent of one another as can be seen by the following reinterpretations of the **NOR** operator.

(1) If $A \downarrow B$ is reinterpreted in terms of the classical matrix for $\neg(B \rightarrow A)$, then Xe and XXe are valid but Xi is not.

(2) If $A \downarrow B$ is reinterpreted in terms of the classical matrix for $\neg(A \& B)$, then Xi and XXe are valid but Xe is not.

(3) If $A \downarrow B$ is reinterpreted as follows, where 1 is the designated value, then Xi and Xe are valid but XXe is not.

Received July 14, 1975

↓		B 1	2	3
	1	3	3	3
A	2	3	1	2
	3	3	2	1

Finally, given the standard definitions of the remaining connectives in terms of **NOR**, the system is truth-functionally complete, since it yields the following complete set of sentential rules.

Ni: $\dfrac{\ldots A \vdash B, \ldots A \vdash \neg B}{\neg A}$ Ne: $\dfrac{\neg\neg A}{A}$

Ki: $\dfrac{A, B}{A \& B}$ Ke: $\dfrac{A \& B}{A}$ $\dfrac{A \& B}{B}$

Like the **NAND** system presented in Price [1], the present **NOR** system can be extended to a full predicate logic with identity simply by adding a standard set of introduction and elimination rules for $\forall$ and $=$.

REFERENCES

[1] Price, R., "The stroke function in natural deduction," *Zeitschrift für mathematische Logik und Grundlagen der Mathematik*, vol. 7 (1961), pp. 117-123.

[2] Goodstein, R., *Development of Mathematical Logic*, Springer-Verlag, New York (1971).

Colgate University
Hamilton, New York