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TEMPORAL MODALITIES AND THE FUTURE

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In [1][1] Robert McArthur defends the challenging thesis that the apparent semantic distinction between the factual future tense, e.g., 'There will be a sea fight tomorrow', and the modal future tenses, e.g., 'There may/must be a sea fight tomorrow', is without foundation. His strategy involves attempting to show that a semantical distinction between factual and modal future tenses cannot be sustained in either deterministic or indeterministic worlds.

> The argument for the deterministic case follows traditional lines by showing how, on a linear model of temporal succession, all three of the above statements have equivalent interpretations. In the indeterministic case the argument utilizes a branching model to demonstrate that only (future) possibility and (future) necessity admit of interpretations. ([2], p. 13.)

McArthur's claims strike us as puzzling, not least because we find it difficult to believe that metaphysical assumptions about determinism and indeterminism could have any direct bearing on the semantical interpretation of future-tensed statements. Of course it is not impossible to imagine metaphysical "scenarios" in which certain tensed expressions have no obvious application. However, we believe that the very conceivability of such cases testifies to the independence of the relevant semantical and metaphysical questions.

Our aim in this paper is thus to show, *contra* McArthur, that the distinction between factual and modal future tenses has a firm semantical foundation, and that this can be established independently of assumptions about the truth or falsity of determinism or indeterminism. In the process of demonstrating the semantical distinctness of factual and modal future

1. McArthur's paper along with an earlier version of the present article were both presented at the Western Division meeting of the American Philosophical Association in April, 1973.

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tenses we define a new class of model structures which we believe may be of some independent interest. Let us begin with the problem as it is alleged to arise in branching time contexts. McArthur has claimed that in a world in which time branches toward the future, i.e., in an indeterministic world, there is no semantical interpretation of 'Fp' which will render it distinct from both '$\langle F\rangle p$' and '$[F]p$' while making 'Fp' true. On McArthur's modal future tense explication of possibility, '$\langle F\rangle p$' (it may be the case that p) is to be glossed roughly as 'p will be true at some point in a possible future'. But given this interpretation of possibility, how should we interpret 'Fp' (it will be the case that p)? Parity of reasoning suggests something like the following: 'p will be true at some point in the actual future'. At just this point, however, McArthur makes the objection which he takes to be decisive. He writes,

> What prevents 'Fp' from finding an interpretation on the branching diagram is that in order to supply one we would have to have precisely what indeterminism denies—namely the ability to single out in advance the future state which becomes actual. If we could (per impossible) single out such a state of affairs, then 'Fp' would simply mean ''p' is the case in some future state which becomes the actual state at that time'. However, in the absence of this possibility there can be no interpretation of 'Fp' in an indeterministic context. ([1], p. 287)

Clearly everything hinges on how we are to understand the phrase, "the ability to single out in advance the future states which become actual."

Presumably what indeterminism denies is that it is possible in principle to identify or to determine in advance which state or sequence of states will be actualized. That is, if time really does branch, in the sense that all of the nodes on the branches which emanate from the present represent real alternative possibilities, then the ability to specify in advance which unique branch represents the actual future is ruled out. What is *not* ruled out is that there will be an actual future, i.e., a series of future possible states which will become actual. Indeed, without this assumption there would be nothing for indeterminism to deny us the possibility of identifying. But if this is so, the indeterminist has given us all we need to provide a semantical interpretation for 'Fp'.

In a branching time world, commitment to a real future is essentially commitment to the claim that there is some unique branch, i.e., some connected sequence of states, which will be actualized as time passes. Can we single out or identify this branch in advance? Not in any nontrivial way, because, *ex hypothesi*, we have no way of knowing which sequence of states will become actual. However this does not prevent us from "singling it out" trivially under the description, 'the branch which is the actual future'. Since we know there is at least one and, for the sake of simplicity, at most one, this minimal claim can be employed in the definition of a class of models capable of distinguishing 'Fp', '$\langle F\rangle p$' and '$[F]p$' in an indeterministic or branching time world.

The following series of definitions provides a characterization of a class of models which accept at least some of the intuitively valid inferences involving these temporal modalities, and which nevertheless supply each with a distinct semantic interpretation. The class of models will be characterized relative to a language $\mathcal{L}$. We will assume that the well-formed formulae of $\mathcal{L}$ are built up from a denumerable set of sentential variables, the unary operators '$\sim$', 'F', '$\langle F \rangle$', and '$\boxed{F}$', and the binary operator '$\rightarrow$', in the usual way. The set of well-formed formulae of $\mathcal{L}$ will be called $\mathrm{E}(\mathcal{L})$. From an intuitive standpoint, M will represent the set of possible moments, R will be the relation of temporal accessibility, B will be the set of all possible futures with respect to all possible moments, and f will be a function that assigns each moment its actual future. First, some preliminary definitions:

D1 b is an *R-chain in M* if and only if

1. $R \subset M \times M$;
2. $b \subset M$;
3. R is transitive in M;
4. R is irreflexive in M;
5. R is asymmetric in M;
6. for every m and n in b, either $R(m,n)$ or $R(n,m)$, or $m = n$.

D2 V is a *standard valuation for $\mathcal{L}$ in M* if and only if

1. $V \subset (M \times \mathrm{E}(\mathcal{L})) \times \{0, 1\}$;
2. V is a function;
3. for every m in M and every α in $\mathrm{E}(\mathcal{L})$, $V(m,\alpha) = 0$ if and only if $V(m,\alpha) \neq 1$;
4. for every m in M and every α in $\mathrm{E}(\mathcal{L})$, $V(m, \sim\alpha) = 1$ if and only if $V(m,\alpha) = 0$;
5. for every m in M, and every α and β in $\mathrm{E}(\mathcal{L})$, $V(m, \alpha \rightarrow \beta) = 1$ if and only if either $V(m,\alpha) = 0$ or $V(m,\beta) = 1$.

D3 m is the *R-first element of b* if and only if

1. m is in b;
2. for every n, if n is in b and $n \neq m$, then $R(m,n)$.

We can now define the class of models that will supply the interpretations required:

D4 $\mathfrak{M}$ is a *τ-model for the language $\mathcal{L}$* if and only if

1. $\mathfrak{M} = \langle M, R, B, f, V \rangle$;
2. M is a non-empty set;
3. B is the non-empty set of all R-chains in M;
4. for every m in M, there is a b in B such that $f(m) = b$ and m is the R-first element of b;
5. for every m and n in M, if n is in $f(m)$, then $f(n) \subset f(m)$;
6. V is a standard valuation for $\mathcal{L}$ in M;
7. for every m in M and every α in $\mathrm{E}(\mathcal{L})$, $V(m, F\alpha) = 1$ if and only if for some n in $f(m)$, $V(n,\alpha) = 1$ and $n \neq m$;

8. for every m in M and every α in $\mathsf{E}(\mathcal{L})$, $V(m, \langle F\rangle\alpha) = 1$ if and only if for some n in M, $R(m, n)$ and $V(n, \alpha) = 1$;
9. for every m in M and every α in $\mathsf{E}(\mathcal{L})$, $V(m, \boxed{F}\alpha) = 1$ if and only if for every b in B, if m is in b, then there is an n in b, such that $R(m, n)$ and $V(n, \alpha) = 1$.

D5 α *is true in* $\mathfrak{M}$ if and only if

1. $\mathfrak{M}$ is a τ-model for the language $\mathcal{L}$;
2. α is in $\mathsf{E}(\mathcal{L})$;
3. $\mathfrak{M} = \langle M, R, B, f, V\rangle$;
4. for every m in M, $V(m, \alpha) = 1$.

D6 α is τ-*valid* if and only if, for every $\mathfrak{M}$, if $\mathfrak{M}$ is a τ-model for the language $\mathcal{L}$, then α is true in $\mathfrak{M}$.

In order to see that '$\langle F\rangle$' and 'F' are semantically distinguishable one need only examine clauses 7 and 8 of D4 where these two functors are in fact distinguished. According to clause 8, '$\langle F\rangle\alpha$' is true at moment m if and only if α is true at some moment temporally accessible from m (i.e., in some possible future with respect to m). On the other hand, '$F\alpha$' is true at moment m if and only if α is true at some moment in the branch assigned to m by the function f, i.e., at some moment in the actual future with respect to m. These conditions along with the characterization of implication are sufficient to guarantee that '$Fp \rightarrow \langle F\rangle p$' is τ-valid, while '$\langle F\rangle p \rightarrow Fp$' is not τ-valid. If 'Fp' is semantically distinct from '$\langle F\rangle p$' and '$\boxed{F}p$', it seems reasonable to suggest that it should be possible to represent this fact on a conventional branching time tree diagram.[2] But how might this be accomplished? In particular, how should 'Fp' be represented? One suggestion would be to pick out one of the branches arbitrarily to represent the distinguished actual future branch. The diagram for 'Fp' would then show 'p' occurring at one node of this branch.

The problem with this solution is that it seems to commit us to more than we wanted. While our intention was to distinguish an arbitrary branch, so long as we are confined to an actual diagram our choice will always involve picking out some particular branch. This maneuver thus gives the appearance of violating the assumption that we do not know which sequence of future states will become actual. In this case, though, appearances are misleading. To see why, we need only recall that τ-validity was defined in terms of truth in every τ-model. Since, for every branch b, there is some τ-model in which b is designated as the actual future with respect to b's first element, the mere designation of particular branches does not violate the strictures of indeterminism.

We turn now to the other half of McArthur's thesis, to the claim that the semantic distinction between factual and modal future tenses cannot be

2. It is the apparent insolubility of the problem posed in these terms which seems to have led McArthur astray.

upheld in a deterministic or linear time world. Here McArthur is obviously on much stronger ground for the simple reason that in linear time models we are considering only the series of actual states of the world. If we have no means for representing possibilities that are not actualized then it follows immediately that we have been deprived of the semantical resources required to explicate the concept of a modal future tense. McArthur summarizes this situation by saying that in *deterministic contexts*, factual and modal future tenses are semantically equivalent. However, it seems to us that it would be more perspicuous to say that modal future tenses are not in general well-defined in *linear time models*. The question of how we should interpret the single future tense which can be semantically represented in such models is not itself a semantical question. It is rather a matter to be settled by one's metaphysical predilections, on the basis of extra-semantical assumptions. For example, if we construe linear time models deterministically, then the concepts of contingency and future possibility will lose independent significance.[3] However, such an interpretation in no way affects the semantical properties of the model. Finally, it should perhaps go without saying that the particular "modal collapse" we have been discussing does not affect the interpretation of other modal concepts in linear time models. The logical, physical and Diodorian temporal modalities, for example, all receive straightforward interpretations even in deterministic contexts.

By way of conclusion we would like at least to raise the possibility that McArthur's explicit formulation of the theses we have been criticizing misrepresents his real concerns. Our reasons for so thinking can be made clear by quoting briefly from the last page of his article.

> An essential feature of the logic of indeterminism (is) that all future tense statements should be viewed as being either overtly or covertly (when in a factual guise) modal. When faced with an apparently factual future tense statement in an indeterministic context it would seem more useful to investigate the beliefs, intentions, etc. of the speaker to decide whether he is prepared to go all the way to '$\boxed{F}p$'. . . or is making a guess, an unfounded prediction, is only reasonably sure, and so forth, in which case we can take him as asserting '$\langle F \rangle p$'. ([1], p. 288)

We have already argued against the view that there is any distinctive logic of determinism or indeterminism *per se*, but the real interest of this passage is that it points in a quite different direction. What McArthur seems most concerned with here is the range of epistemic grounds available for claims about the future, rather than the tense-structure of the sentences in which such claims might be expressed.

3. Of course linear time models need not be construed deterministically for we are free to think of the different "worlds" of such models as representing instantaneous states of *the* world, temporally individuated. The relation R of accessibility may then be stipulated to hold between two such "worlds" just in case they are identical or one is a temporal successor of the other without prejudicing the question of whether one is, in any sense, determined by the other.

Two different considerations appear to speak in favor of this interpretation. First, McArthur's modal functors are susceptible to a fairly straightforward epistemic interpretation, and one which preserves their lack of interdefinability. Read: 'it is credible that p will occur' for 'it may be the case that p', and for 'it must be the case that p' read 'it is known that p will occur' (in the sense of being, say, nomologically rather than logically certain). Second, on this epistemic construal of the functors, 'KFp' (it is known that p will occur) and 'CFp' (it is credible that p will occur) differ from 'Fp' in a crucial respect. The truth-values of statements of the first two types will be decidable in principle at the time of their utterance, whereas the truth-value of future contingent statements is not decidable in advance. This consequence fits in neatly with McArthur's own suggestion that "all future tense statements should be viewed as overtly or covertly modal," if we now read 'epistemically qualified' for 'modal'.

REFERENCES

[1] McArthur, R. P., "Factuality and modality in the future tense," *Noûs*, vol. 8 (1974), pp. 283-288.

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