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Syllogisms Using "Few", "Many", and "Most"

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In this paper I show that it is possible to expand syllogistic logic to include the intermediate quantifiers "few", "many", and "most".

1 A five-quantifier square of opposition The relations which hold between statements on a square of opposition will depend upon whether the square expresses the Aristotelian Framework or the Boolean Framework. Copi [3] and others explain the difference between the Boolean and the Aristotelian Frameworks by saying that in the Aristotelian Frameworks both universal and particular statements are understood to be making, assuming, presupposing, or implying an existential claim, while in Boolean Framework only particular statements are so understood. That is, in the Aristotelian Framework, any statement of the form "All *S* are *P*" may be taken as asserting the existence of members in both the *S* class and the *P* class, while in the Boolean Framework a universal statement makes neither claim.

My recommendation is that universal and particular statements do *not* make existential claims, nor do they assume, presuppose, or imply such a claim. It seems to me that it makes perfect sense, when talking of unicorns, to assert "Some are male and some are female," since unicorns are fictionally conceived to be capable of sexual reproduction. Yet, the speaker clearly would not wish to be understood as asserting the existence of unicorns. This understanding of particular statements, in my opinion, rescues the Aristotelian Framework from many of the paradoxes with which it might otherwise be saddled.[1] I shall also

*I wish to thank Philip L. Peterson, not only for a variety of terminological suggestions which I have adopted, but also for important criticisms which helped me avoid serious blunders. In addition, Professor Peterson suggested a number of substantial changes in the text of the paper, most of which I have adopted verbatum.

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understand the quantifiers "few", "many", and "most" to be making the same kind of existential claim as the quantifiers "all", "some", and "none".

In keeping with tradition I call statements quantified by "all" and "some" *universal* statements and *particular* statements, respectively. In addition, I adopt the following new terminology. Statements quantified by "most" are referred to as *majority* statements, and statements quantified by "many" are referred to as *common* statements. For reasons which will be explained later, statements quantified by "few" are referred to as *predominant* statements.[2] The terms "negative" and "affirmative" are used with their familiar meanings. Intermediate statements are represented by:

P	predominant affirmative
B	predominant negative
T	majority affirmative
D	majority negative
K	common affirmative
G	common negative.[3]

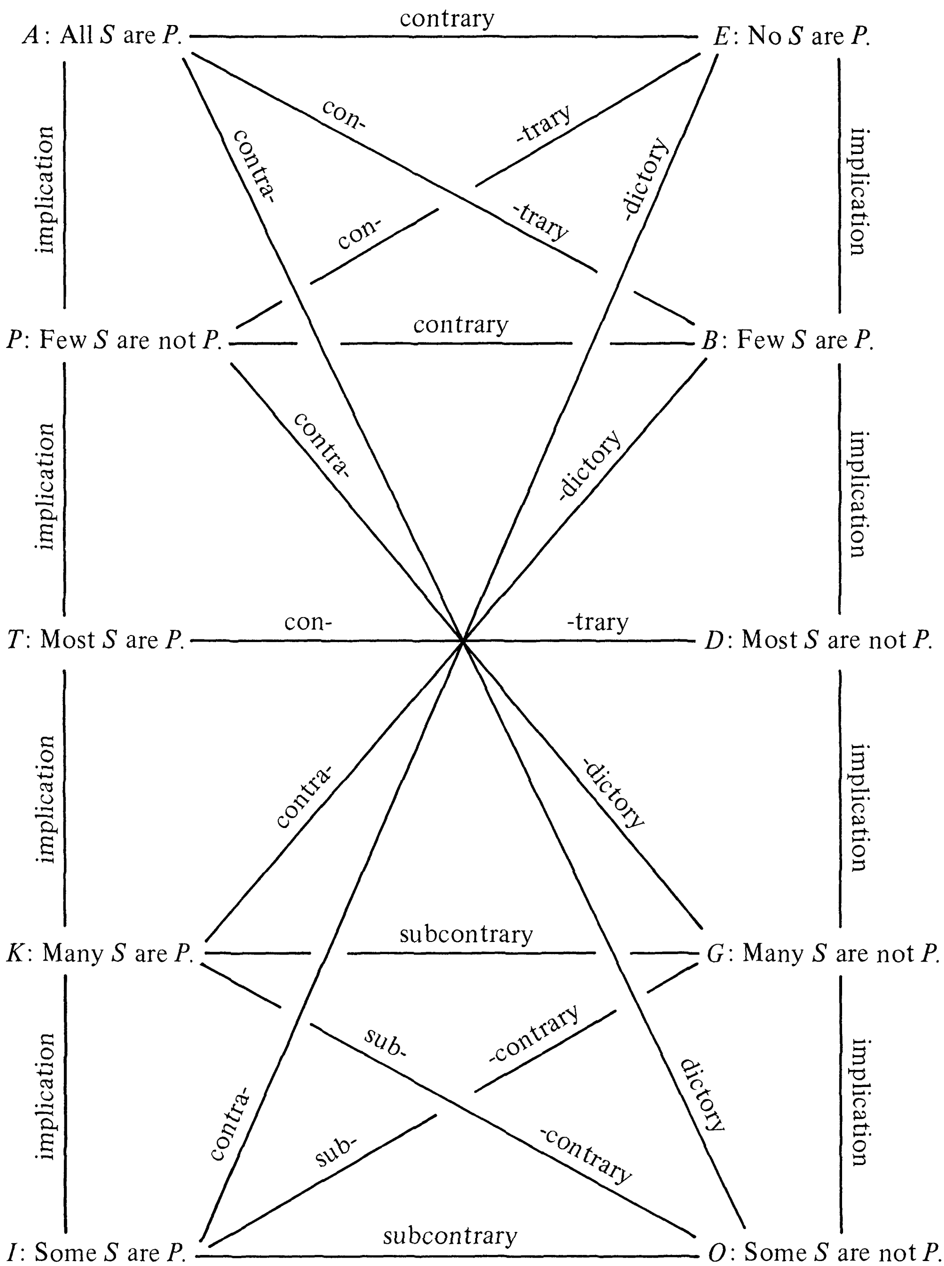
As in classical syllogistic logic, we use:

A	universal affirmative
E	universal negative
I	particular affirmative
O	particular negative.

Quantifiers may be understood in any of three senses, which may be called the *minimal* sense, the *maximal* sense, and the *exact* sense. A quantifier understood in the minimal sense is understood to be saying "at least" or "no less than" the quantity named. A quantifier understood in the maximal sense is understood to be saying "only" or "no more than". A quantifier understood in the exact sense combines the minimal and maximal senses, so that it says "no more nor less than" (cf. [4]). The quantifier "some" is traditionally understood in the minimal sense to mean "at least some (and possibly all)". I propose to use "many" and "most" in the same sense.[4] Following Peterson [5], I propose to ignore the minimal use of "a few", since it is too nearly synonymous with "some" to be of logical significance, and restrict "few", for purposes of syllogistic logic, to its maximal sense, "no more than few (if any)". The result of adopting this convention is that "few", like "none", must be understood to be making a denial. Thus statements of the form "Few *S* are *P*" must be classified as negative statements, while statements of the form "Few *S* are not *P*" are logically equivalent to affirmative statements. That is, "Few *S* are not-*P*" is a kind of double negation directly parallel to the double negative in "No *S* are not-*P*".

The resulting extended square of opposition is illustrated on the following page.

Not all of the relations which occur on the square are shown. However, it is important to understand that every statement on the square is related to every other statement on the square in some manner or other. The relations not stated may be derived by simple extrapolation from the relations that *are*



stated. In this manner it would be possible to derive six additional contrary relations *A-D, A-G, E-T, E-K, P-D, B-T,* and six subcontrary relations *I-B, I-D, P-O, T-O, K-D, T-G.*

The contrariety of *A* and *E* is part of classical syllogistic logic, as is the subcontrariety of *I* and *O*. That *A* and *O*, and *E* and *I*, are contradictory is also part of classical syllogistic logic. All of the remaining relations that occur on the square of opposition should also conform to our normal intuitions. Let us examine some of them to see that this is true.

Given the previously stated assumptions, that "few" is to be understood maximally, and that it is to be understood to be making the same claim with respect to existence as "all", it should be obvious that *P* statements follow from *A* statements by implication. If "No Songs are Pretty" is true, it must be true that "Few Songs are Pretty". Similarly, if "All Songs are Pretty" is true, then "Few Songs are not Pretty" is true also.

Provided that we understand "most" to mean "at least most (and possibly all)," it should be equally clear that "Most Songs are Pretty" must follow from "Few Songs are not Pretty." Peterson [5] believes there is a strict sense of "most" which permits these two statements to be logically equivalent. I will not use "most" in this special sense. In my opinion "Few *S* are not *P*" makes a strong enough claim that it would be invalid to infer it from the weaker claim "Most *S* are *P*" (where the latter is just the ordinary use of "most", i.e., what Peterson calls the *generic* sense).

It is clear that *K* statements follow from *T* statements by implication. Consider a class containing only three members, such as a set of three paintings by the same artist. Two of the paintings have been damaged but the third is in perfect condition. Since it is true that more than half of the paintings are damaged we are justified in asserting "Most of the paintings are damaged". But two is not the kind of number that we usually characterize as "many" so it sounds peculiar at best to assert "Many of the paintings are damaged". As long as we understand the quantifier "many" to mean "many *of* the paintings which we are now talking about"–and not to mean simply "many paintings"–I maintain that the assertion "Many of the paintings are damaged" may sound peculiar, but it is not logically incorrect. Given this understanding, *K* statements do indeed follow from *T* statements by implication.[5]

Finally, *I* statements follow from *K* statements by implication: if "Many Songs are Pretty" is true, then clearly "Some Songs are Pretty" is also true, but not vice versa. Also, of course, the relations of implication which have been established on the affirmative side of the square of opposition should work in exactly the same way on the negative side of the square.

Given the relations of implication that have been established, and given the contradiction which holds between *A* and *O* statements, and between *E* and *I* statements, many of the remaining contrary and subcontrary relations can be derived by simple extrapolation: for example, *O* and *P* statements are subcontraries since the falsity of *O* entails the truth of the corresponding *A* statement by contradiction. The truth of *A* entails the truth of the corresponding *P* statement by implication. Thus the falsity of *O* entails the truth of the corresponding *P*, though the truth of *O* tells us nothing about the corresponding *P* statement. Other contrary and subcontrary relations may be verified in the same manner.

The contrariety of *T* and *D* statements is one of the relations which cannot be derived. Nevertheless, both cannot be true: If "Most Songs are Pretty" is true, it cannot possibly be that "Most Songs are not Pretty" is true. However, both *can* be false in those rare cases in which precisely half of a class has a particular attribute which the other half lacks. For example, of a dozen eggs, if precisely six are broken, then "Most of the eggs are broken" is false, but so is "Most of the eggs are not broken."

Peterson [5] argues—and I agree—that "Few *S* are *P*" and "Many *S* are *P*" directly contradict each other. "Few" understood maximally, and "many" understood minimally, are in effect antonyms like "hot" and "cold". Thus replacing one for the other in the context of a proposition should reverse the truth-value of the proposition. Notice that for "Many *S* are *P*" to be true, no assumption is made as to whether the *S* that are *P* are more, less, or the same in number or amount as the *S* that are not-*P*.[6]

2 Immediate inferences In addition to the immediate inferences stated on the extended square of opposition (and the inferences extrapolated from it), obversion is valid for *P, T, K, B, D,* and *G* statements, just as it is for *A, E, I,* and *O* statements. Conversion and contraposition are not valid for any intermediate statement, although, as in classical syllogistic logic, conversion is valid for *E* and *I* statements and contraposition is valid for *A* and *O* statements.

3 Rules for a five-quantifier syllogistic logic The following rules seem to capture our intuitions regarding arguments in syllogistic form.

Distribution:

1. Any universal statement distributes its subject.
2. Any negative statement distributes its predicate.
3. Any predominant, majority, or common statement distributes its subject if and only if its subject is the minor term.

The third distribution rule is, of course, the backbone of the whole system. It captures, in a rather *ad hoc* fashion, the fact that intermediate propositions sometimes behave as if they were universal and sometimes as if they were particular, depending (as it turns out) on whether their subject is the minor term or not. Besides sounding *ad hoc*, this rule does serious violence to the old notion of distribution. Therefore, the concept of distribution is here intended to be understood merely functionally, as "that which a universal statement does to its subject, . . . etc." In other words, I offer no justification for the rules of distribution other than that they allow the Rules of Syllogism successfully to sort out valid syllogisms from invalid ones.

Rules of Syllogism:

1. Rules of Distribution
   a. The middle term must be distributed in at least one premiss.
   b. Any term which is distributed in the conclusion must also be distributed in the premisses.
2. Rules of Quality
   a. There must be at least one affirmative premiss.
   b. If the conclusion is negative, there must be at least one negative premiss.
   c. If there is a negative premiss, then the conclusion must be negative.
3. Rules of Quantity
   a. If there is a predominant premiss, the conclusion may not be universal.

b. If there is a majority premiss, the conclusion may not be universal or predominant.
c. If there is a common premiss, the conclusion may not be universal, predominant, or majority.

The attempt here has been to preserve the classical Aristotelian rules in as close to their traditional form as possible. The Rules of Distribution, for example, have not been altered, but they range over a much more complex set of problems. The way in which the Rules of Distribution work to prevent invalid reasoning can be shown by considering some invalid syllogisms. First the *TAT* syllogism in the third figure, which is intuitively invalid:

Most *M* are *P*.
All *M* are *S*.
Most *S* are *P*.

This syllogism meets all the rules of Quality and Quantity, and the minor premiss distributes the middle term. Indeed, if this were an IAI syllogism it would be valid on all counts. But because the conclusion is a majority statement, it follows that the minor term is distributed in the conclusion, since a *T* statement distributes the minor term when the minor term appears as its subject. But the minor term is not distributed in the premisses. The syllogism is therefore invalid due to the fallacy of an illicit minor term.

The following syllogism—*TAK* syllogism in the third figure—is invalid for the same reason. Here, however, it is less clear that the syllogism is intuitively invalid:

Most *M* are *P*.
All *M* are *S*.
Many *S* are *P*.

Suppose that the class *S* has 100 members, three of which belong to class *M*, while the rest do not. Of the three members of class *S* which belong to class *M*, two also belong to class *P*. This situation is consistent with the truth of the premisses in the above syllogism. It follows from what has so far been said that at least two members of class *S* also belong to class *P*, but we have no reason to believe that this is true of any more than two members. It is perhaps not clear exactly what ratio would be required to justify the assertion "Many *S* are *P*", but, assuming the least favorable case, I doubt that anyone would claim that a ratio of 100:2 was sufficient. So the above syllogism must be invalid—as indeed it is, due to an illicit minor term.

Finally, the *TED* syllogism in the second figure:

Most *P* are *M*.
No *S* are *M*.
Most *S* are not *P*.

This syllogism also meets all of the rules of Quality and Quantity. Again, the middle term is distributed by the minor premiss. However, both the major and the minor terms are distributed in the conclusion, but only the minor term is distributed in the premisses. Because a majority statement distributed its subject only if its subject is the minor term, the major term remains undistributed in the premisses, and the syllogism is invalid.

By contrast, the *ATT* syllogism in the first figure is valid:

All *M* are *P*.
Most *S* are *M*.
Most *S* are *P*.

Again, all of the rules of Quality and Quantity are met. The major premiss, being an *A* statement, distributes the middle term. The minor term is distributed in the conclusion, just as in the first examples. However, the minor premiss is also a *T* statement with the minor term as its subject. Therefore, the minor term is also distributed in the premisses, and all the conditions for a valid syllogism have been met.

The *EKG* syllogism in the second figure is also valid:

No *P* are *M*.
Many *S* are *M*.
Many *S* are not *P*.

As in the earlier example, it may not be clear exactly what ratio would be required to justify the assertion that "Many *S* are not *P*", but for the minor premiss to be true, we must suppose that that ratio has been achieved, no matter what numbers we assign to the various classes. Therefore, this syllogism is valid.

The Rules of Quality which occur in classical syllogistic logic have not been changed in any way by the addition of predominant, majority, and common statements. But it is important to remember that "few" understood maximally is a denial word, so that "Few *S* are *P*" is a negative statement, while "Few *S* are not *P*" is equivalent to the affirmative, "Almost all *S* are *P*." This gives rise to some curiosities. For example, the *EPO* syllogism in the fourth figure is valid. It can be expressed:

No *P* are *M*.
Few *M* are not *S*.
Some *S* are not *P*.

This syllogism appears to violate rule 2a, that there must be at least one affirmative premiss. But, of course, the minor premiss *is* affirmative, despite its initial appearance.

Given the relatively strong generality of both the premisses, it is tempting to make a somewhat stronger claim in the conclusion. This, however, would not be permissible since any claim stronger than the particular would require that the subject of the conclusion be distributed (since it is the minor term). But the minor term is *not* distributed in the premisses, since "Few *S* are not *P*" is to be treated as an affirmative, not a negative, statement. Therefore, only a particular conclusion may be valid.

In classical syllogistic logic, Rules of Quantity are not, strictly speaking, necessary; though the principle that the conclusion cannot be more general than its least general premiss is implicitly contained in the fact that a syllogism cannot violate this principle without breaking one of the other Rules of

Syllogism. The addition of predominant, majority, and common statements to syllogistic logic requires that at least some parts of this principle be stated explicitly, as the following syllogism demonstrates:

All *M* are *P*.
Most *S* are *M*.
All *S* are *P*.

This clearly invalid syllogism meets every requirement for validity except rule 3b, that a syllogism with a majority premiss may not have a universal conclusion. It should be easy to discover other syllogisms which show that rules 3a and 3c are also not superfluous. However the rule "If there is a particular premiss, the conclusion must be particular" was not stated in the Rules of Syllogisms since, as in classical syllogistic logic, no syllogism would violate this rule without also violating one of the other rules.

4 Valid syllogisms Under the Aristotelian Framework there are twenty-four valid syllogisms using familiar *A, E, I,* and *O* statements.[7] The addition of *P, T, K, B, D,* and *G* statements increases this number to 93: thirty in each of the first and second figures, eighteen in the third figure, and fifteen in the fourth figure. They are as follows:

Figure I	Figure II	Figure III	Figure IV
Syllogisms valid in the Aristotelian tradition:			
AAA	*EAE*	*AAI*	*AAI*
EAE	*AEE*	*IAI*	*AEE*
AII	*EIO*	*AII*	*IAI*
EIO	*AOO*	*EAO*	*EAO*
AAI	*EAO*	*OAO*	*EIO*
EAO	*AEO*	*EIO*	*AEO*
With the addition of majority statements:			
AAT	*AED*	*ATI*	*AED*
ATT	*ADD*	*ETO*	*ETO*
ATI	*ADO*	*TAI*	*TAI*
EAD	*EAD*	*DAO*	
ETD	*ETD*		
ETO	*ETO*		
With the addition of common statements:			
AAK	*AEG*	*AKI*	*AEG*
ATK	*ADG*	*EKO*	*EKO*
AKI	*AGO*	*KAI*	*KAI*
AKK	*AGG*	*GAO*	
EAG	*EAG*		
ETG	*ETG*		
EKO	*EKO*		
EKG	*EKG*		

Figure I	Figure II	Figure III	Figure IV
With the addition of predominant statements:			
AAP	*AEB*	*PAI*	*AEB*
APP	*ABB*	*EPO*	*PAI*
APT	*ABD*	*BAO*	*EPO*
APK	*ABG*	*API*	
API	*ABO*		
EAB	*EAB*		
EPB	*EPB*		
EPD	*EPD*		
EPG	*EPG*		
EPO	*EPO*		

NOTES

1. A more complete argument is given by Angell [1]. Peterson [5] adopts the opposite understanding (that all affirmatives and negatives presuppose the existence of instances of their subject term). Note that it doesn't appear to matter which view—mine or Peterson's—is adopted for what follows. In either case it is possible to infer the truth of "Some *S* are *P*" given the truth of "All *S* are *P*"; or the falsity of "All *S* are *P*", given the falsity of "Some *S* are *P*". It is only the mixed view, i.e. the Boolean Framework, that would cause difficulties.

2. Peterson [5] called the common statements "more than particular" and the predominant statements "less than universal". He formulated predominant statements with the term "most", but said it was the nongeneric sense of "most" that was used in predominant statements, in contrast to the generic sense of "most" used in majority statements. My terminology is, I believe, much less awkward.

3. The use of these six consonants has been adopted following a suggestion from Peterson, who gives this rationale:

> There are not enough additional vowel names (or letters for them) to easily attach mnemonic labels to the intermediate propositions (six are needed). So, why not move to consonants? The most obvious consonants to my mind (the place to *begin* in teaching phonetics) are the stops. There just happen to be *very obvious* stops (nonproblematic consonantal sounds). These three stops are P, T, and K—i.e., the consonantal sounds that begin the words "pat," "tat," and "cat" (the initial sound of the latter should be represented by a "K"). These are *unvoiced* stops. Matching these three are three voiced stops (the very same consonantal sounds but with the vocal cords operating)—namely, B, D, and G (the initial sounds of "bad," "dad," and "god"). What's more, this order is front-to-back with respect to points of articulation (phonetically). P and B are bi-labials (formed mainly with the two lips). T and D are alveolars (formed by tongue tip touching the ridge just behind the top teeth). K and G are velars (formed by the middle of the tongue touching the roof of the mouth). So, we have two varieties (voiced and unvoiced) of three things (three stops, front to back), exactly the right pattern to label the intermediate propositions.

This arrangement also lends itself to the following mnenomic arrangement:

P = Predominant
T = MajoriTy
K = Kommon

4. Peterson made the same presumption, though he called the minimal sense the 'broad' or 'liberal' interpretation ([5], p. 157).

5. Cf. Peterson [5], pp. 166-167 and footnote 13, for additional discussion.

6. Peterson makes the same presumption, though this is obscured by an important typographical error in his footnote 7. In the second line after (iii) "Q(SP)" should read "$Q(S\bar{P})$".

7. Cf. Bird [2], pp. 22-23.

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