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A Note Concerning the Notion of Mereological Class. Postscript

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Since the publication of my note concerning the notion of mereological class [1] I have noticed that a system of mereology—I shall refer to it as System $\mathfrak{B}_1$—can be based on the following single axiom:

B₁A1 $[AB] :: A\ \varepsilon\ el(B) .\equiv \because [\exists a] \because B\ \varepsilon\ a \because [CD] \because [E] \therefore E\ \varepsilon\ C .\equiv: [F] : [\exists G] . G\ \varepsilon\ el(E) . G\ \varepsilon\ el(F) .\equiv. [\exists HI] . H\ \varepsilon\ a . I\ \varepsilon\ el(F) . I\ \varepsilon\ el(H) :: B\ \varepsilon\ el(C) . B\ \varepsilon\ el(D) ::\supset. A\ \varepsilon\ el(D).$

In B_1A1, just as in BA1, which is the axiom of System $\mathfrak{B}$, E2 is embedded as the definition of the notion of mereological class, but B_1A1 is shorter than BA1 by one ontological unit, and for this reason is of interest. It happens to be the shortest known single axiom for the notion of mereological elementhood.

The idea behind B_1A1 becomes apparent as soon as one realises that the set of presuppositions $\{B_1A1, E2\}$ is inferentially equivalent to the set of presuppositions consisting of E2 and

$B_1A1.1$ $[AB] \therefore A\ \varepsilon\ el(B) .\equiv: [\exists a] : B\ \varepsilon\ a : [C] : B\ \varepsilon\ el(Kl(a)) . B\ \varepsilon\ el(C) .\supset. A\ \varepsilon\ el(C),$

which is shorter than BA1.1.

In order to prove that System $\mathfrak{B}_1$ and System $\mathfrak{B}$ are inferentially equivalent we first continue the deductions within the framework of System $\mathfrak{B}$ as follows:

BT18	$[Aa] : A\ \varepsilon\ a .\supset. A\ \varepsilon\ el(Kl(a))$	
Proof:	$[Aa] : \mathrm{Hp}(1) .\supset.$	
(2)	$Kl(a)\ \varepsilon\ Kl(a)$.	[BT5; 1]
	$A\ \varepsilon\ el(Kl(a))$	[BT8; 2; 1]

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BT19 $[AB] :: A\ \varepsilon\ el(B) .\supset\because [\exists a] \because B\ \varepsilon\ a \because [CD] \because [E] \therefore E\ \varepsilon\ C .\equiv: [F] : [\exists G] . G\ \varepsilon\ el(E) . G\ \varepsilon\ el(F) .\equiv. [\exists HI] . H\ \varepsilon\ a . I\ \varepsilon\ el(F) . I\ \varepsilon\ el(H) :: B\ \varepsilon\ el(C) . B\ \varepsilon\ el(D) ::\supset. A\ \varepsilon\ el(D)$

Proof: $[AB] :: \mathrm{Hp}(1) .\supset\because$

(2) $B\ \varepsilon\ B \because$ [BT1; 1]

(3) $[CD] \because [E] \therefore E\ \varepsilon\ C .\equiv: [F] : [\exists G] . G\ \varepsilon\ el(E) . G\ \varepsilon\ el(F) .\equiv. [\exists HI] . H\ \varepsilon\ B . I\ \varepsilon\ el(F) . I\ \varepsilon\ el(H) :: B\ \varepsilon\ el(C) . B\ \varepsilon\ el(D) ::\supset. A\ \varepsilon\ el(D) ::$ [BT7; 1]

$[\exists a] \because B\ \varepsilon\ a \because [CD] \because [E] \therefore E\ \varepsilon\ C .\equiv: [F] : [\exists G] . G\ \varepsilon\ el(E) . G\ \varepsilon\ el(F) .\equiv. [\exists HI] . H\ \varepsilon\ a . I\ \varepsilon\ el(F) . I\ \varepsilon\ el(H) :: B\ \varepsilon\ el(C) . B\ \varepsilon\ el(D) ::\supset. A\ \varepsilon\ el(D)$ [2; 3]

BT20 $[ABa] :: B\ \varepsilon\ a :: [CD] \because [E] \therefore E\ \varepsilon\ C .\equiv: [F] : [\exists G] . G\ \varepsilon\ el(E) . G\ \varepsilon\ el(F) .\equiv. [\exists HI] . H\ \varepsilon\ a . I\ \varepsilon\ el(F) . I\ \varepsilon\ el(H) :: B\ \varepsilon\ el(C) . B\ \varepsilon\ el(D) ::\supset. A\ \varepsilon\ el(D) \because\supset. A\ \varepsilon\ el(B)$

Proof: $[ABa] :: \mathrm{Hp}(2) \because\supset::$

(3) $[E] \therefore E\ \varepsilon\ Kl(a) .\equiv: [F] : [\exists G] . G\ \varepsilon\ el(E) . G\ \varepsilon\ el(F) .\equiv. [\exists HI] . H\ \varepsilon\ a . I\ \varepsilon\ el(F) . I\ \varepsilon\ el(H) ::$ [BT3; 1]

(4) $B\ \varepsilon\ el(Kl(a)) .$ [BT18; 1]

(5) $B\ \varepsilon\ el(B) .$ [BT2; 1]

$A\ \varepsilon\ el(B)$ [2; 3; 4; 5]

BT21 $(= B_1A1)$ [BT19; BT20]

It is evident from BT21 that any thesis derivable within the framework of System $\mathfrak{B}_1$ can be derived within the framework of System $\mathfrak{B}$.

With a view to proving that, conversely, any thesis derivable within the framework of System $\mathfrak{B}$ can be derived within the framework of System $\mathfrak{B}_1$. We now deduce, within the latter system, the following theses:

B_1A1 $[AB] :: A\ \varepsilon\ el(B) .\equiv\because [\exists a] \because B\ \varepsilon\ a \because [CD] \because [E] \therefore E\ \varepsilon\ C .\equiv: [F] : [\exists G] . G\ \varepsilon\ el(E) . G\ \varepsilon\ el(F) .\equiv. [\exists HI] . H\ \varepsilon\ a . I\ \varepsilon\ el(F) . I\ \varepsilon\ el(H) :: B\ \varepsilon\ el(C) . B\ \varepsilon\ el(D) ::\supset. A\ \varepsilon\ el(D)$ [Axiom]

B_1D1(=E2) $[Aa] \therefore A\ \varepsilon\ A : [B] : [\exists C] . C\ \varepsilon\ el(A) . C\ \varepsilon\ el(B) .\equiv. [\exists DE] . D\ \varepsilon\ a . E\ \varepsilon\ el(B) . E\ \varepsilon\ el(D) :\equiv. A\ \varepsilon\ Kl(a)$ [Definition]

B_1T1 $[AB] : A\ \varepsilon\ el(B) .\supset. B\ \varepsilon\ B$ [B_1A1]

B_1T2 $[Aa] : A\ \varepsilon\ a .\supset. A\ \varepsilon\ el(A)$

Proof: $[Aa] \therefore \mathrm{Hp}(1) .\supset:$

(2) $A\ \varepsilon\ el(A) .\vee. [\exists D] . A\ \varepsilon\ el(D) . \sim(A\ \varepsilon\ el(D)) :$ [B_1A1; 1]

$A\ \varepsilon\ el(A)$ [2]

B_1T3 $[Fa] :: F\ \varepsilon\ a .\supset\therefore [A] : A\ \varepsilon\ Kl(a) .\equiv: [B] : [\exists C] . C\ \varepsilon\ el(A) . C\ \varepsilon\ el(B) .\equiv. [\exists DE] . D\ \varepsilon\ a . E\ \varepsilon\ el(B) . E\ \varepsilon\ el(D)$ [B_1D1; B_1T2; B_1T1]

B_1T4 $[AB] \therefore A\ \varepsilon\ el(B) .\supset: [\exists a] : B\ \varepsilon\ a : [D] : B\ \varepsilon\ el(Kl(a)) . B\ \varepsilon\ el(D) .\supset. A\ \varepsilon\ el(D)$

Proof: $[AB] :: \mathrm{Hp}(1) .\supset::$

$[\exists a] ::$

(2) $B\ \varepsilon\ a \because$

(3) $[CD] \because [E] \therefore E\ \varepsilon\ C .\equiv: [F] : [\exists G] . G\ \varepsilon\ el(E) . G\ \varepsilon\ el(F) .\equiv. [\exists HI] . H\ \varepsilon\ a . I\ \varepsilon\ el(F) . I\ \varepsilon\ el(H) :: B\ \varepsilon\ el(C) . B\ \varepsilon\ el(D) ::\supset. A\ \varepsilon\ el(D) ::$

(2)–(3): [B_1A1; 1]

(4) $[E] \therefore E \,\varepsilon\, Kl(a) .\equiv: [F] : [\exists G] . G \,\varepsilon\, el(E) . G \,\varepsilon\, el(F) .\equiv.$
$[\exists HI] . H \,\varepsilon\, a . I \,\varepsilon\, el(F) . I \,\varepsilon\, el(H) ::$ $[B_1T3; 2]$

(5) $[D] : B \,\varepsilon\, el(Kl(a)) . B \,\varepsilon\, el(D) .\supset. A \,\varepsilon\, el(D) \therefore$ $[3; 4]$

$[\exists a] : B \,\varepsilon\, a : [D] : B \,\varepsilon\, el(Kl(a)) . B \,\varepsilon\, el(D) .\supset. A \,\varepsilon\, el(D)$ $[2; 5]$

B_1T5 $[ABCDa] \because B \,\varepsilon\, a : [E] : B \,\varepsilon\, el(Kl(a)) . B \,\varepsilon\, el(E) .\supset. A \,\varepsilon\, el(E) \therefore$
$[E] \therefore E \,\varepsilon\, C .\equiv: [F] : [\exists G] . G \,\varepsilon\, el(E) . G \,\varepsilon\, el(F) .\equiv. [\exists HI] .$
$H \,\varepsilon\, a . I \,\varepsilon\, el(F) . I \,\varepsilon\, el(H) :: B \,\varepsilon\, el(C) . B \,\varepsilon\, el(D) ::\supset. A \,\varepsilon\, el(D)$

Proof: $[ABCDa] \because \mathrm{Hp}(5) ::\supset::$

(6) $[E] \therefore E \,\varepsilon\, Kl(a) .\equiv: [F] : [\exists G] . G \,\varepsilon\, el(E) . G \,\varepsilon\, el(F) .\equiv.$
$[\exists HI] . H \,\varepsilon\, a . I \,\varepsilon\, el(F) . I \,\varepsilon : el(H) ::$ $[B_1T3; 1]$

(7) $[E] : E \,\varepsilon\, C .\equiv. E \,\varepsilon\, Kl(a) \therefore$ $[3; 6]$

(8) $B \,\varepsilon\, el(Kl(a)) .$ [Extensionality; 7; 4]

$A \,\varepsilon\, el(D)$ $[2; 8; 5]$

B_1T6 $[ABa] \therefore B \,\varepsilon\, a : [D] : B \,\varepsilon\, el(Kl(a)) . B \,\varepsilon\, el(D) .\supset. A \,\varepsilon\, el(D) :\supset.$
$A \,\varepsilon\, el(B)$

Proof: $[ABa] :: \mathrm{Hp}(2) :\supset::$

(3) $[CD] \because [E] \therefore E \,\varepsilon\, C .\equiv: [F] : [\exists G] . G \,\varepsilon\, el(E) . G \,\varepsilon\, el(F) .\equiv.$
$[\exists HI] . H \,\varepsilon\, a . I \,\varepsilon\, el(F) . I \,\varepsilon\, el(H) :: B \,\varepsilon\, el(C) . B \,\varepsilon\, el(D) ::\supset.$
$A \,\varepsilon\, el(D) ::$ $[B_1T5; 1; 2]$

$A \,\varepsilon\, el(B)$ $[B_1A1; 1; 3]$

B_1T7($=B_1$A1.1) $[B_1T4; B_1T6]$

B_1T8 $[Aa] : A \,\varepsilon\, a .\supset. A \,\varepsilon\, el(Kl(a))$ $[B_1T6]$

B_1T9 $[Aa] : A \,\varepsilon\, a .\supset. Kl(a) \,\varepsilon\, Kl(a)$ $[B_1T8; B_1T1]$

B_1T10 $[Aa] : A \,\varepsilon\, a .\supset. A \,\varepsilon\, Kl(A)$ $[B_1D1]$

B_1T11 $[Aa] : A \,\varepsilon\, a .\supset. A = Kl(A)$ $[B_1T9; B_1T10]$

B_1T12 $[ABC] : A \,\varepsilon\, el(B) . B \,\varepsilon\, el(C) .\supset. A \,\varepsilon\, el(C)$

Proof: $[ABC] \because \mathrm{Hp}(2) .\supset::$
$[\exists a] \therefore$

(3) $B \,\varepsilon\, a :$
(4) $[D] : B \,\varepsilon\, el(Kl(a)) . B \,\varepsilon\, el(D) .\supset. A \,\varepsilon\, el(D) \therefore$ } $[B_1T4; 1]$

(5) $B \,\varepsilon\, el(Kl(a)) ::$ $[B_1T8; 3]$

$A \,\varepsilon\, el(C)$ $[4; 5; 2]$

B_1T13 $[ABCDa] \because A \,\varepsilon\, el(B) \therefore [E] \therefore E \,\varepsilon\, D .\equiv: [F] : [\exists G] . G \,\varepsilon\, el(E) .$
$G \,\varepsilon\, el(F) .\equiv. [\exists HI] . H \,\varepsilon\, a . I \,\varepsilon\, el(F) . I \,\varepsilon\, el(H) :: B \,\varepsilon\, el(B) .$
$B \,\varepsilon\, el(C) . C \,\varepsilon\, a ::\supset. A \,\varepsilon\, el(D)$

Proof: $[ABCDa] \because \mathrm{Hp}(5) ::\supset::$

(6) $[E] \therefore E \,\varepsilon\, Kl(a) .\equiv: [F] : [\exists G] . G \,\varepsilon\, el(E) . G \,\varepsilon\, el(F) .\equiv. [\exists HI] .$
$H \,\varepsilon\, a . I \,\varepsilon\, el(F) . I \,\varepsilon\, el(H) ::$ $[B_1T3; 5]$

(7) $[E] : E \,\varepsilon\, D .\equiv. E \,\varepsilon\, Kl(a) \therefore$ $[2; 6]$

(8) $A \,\varepsilon\, el(C) .$ $[B_1T12; 1; 4]$

(9) $C \,\varepsilon\, el(Kl(a)) .$ $[B_1T8; 5]$

(10) $A \,\varepsilon\, el(Kl(a)) .$ $[B_1T12; 8; 9]$

$A \,\varepsilon\, el(D)$ [Extensionality; 7; 10]

B_1T14 $[AB] :: B \,\varepsilon\, B \because [CDa] \because [E] \therefore E \,\varepsilon\, D .\equiv: [F] : [\exists G] . G \,\varepsilon\, el(E) .$
$G \,\varepsilon\, el(F) .\equiv. [\exists HI] . H \,\varepsilon\, a . I \,\varepsilon\, el(F) . I \,\varepsilon\, el(H) :: B \,\varepsilon\, el(B) .$
$B \,\varepsilon\, el(C) . C \,\varepsilon\, a ::\supset. A \,\varepsilon\, el(D) \therefore\supset. A \,\varepsilon\, el(B)$

Proof: $[AB] :: \mathrm{Hp}(2) \therefore\supset::$

(3) $B \,\varepsilon\, el(B) \therefore$ $[B_1T2; 1]$

(4) $[E] \therefore E \,\varepsilon\, Kl(B) \,.\!\equiv\!:\, [F] : [\exists G] \,.\, G \,\varepsilon\, el(E) \,.\, G \,\varepsilon\, el(F) \,.\!\equiv.$
$[\exists HI] \,.\, H \,\varepsilon\, B \,.\, I \,\varepsilon\, el(F) \,.\, I \,\varepsilon\, el(H) ::$ [B_1T3; 1]
(5) $A \,\varepsilon\, el(Kl(B))$. [B_1T10; 1]
(6) $B = Kl(B)$. [B_1T11; 1]
$A \,\varepsilon\, el(B)$ [5; 6]
B_1T15(=BA1) [B_1T1; B_1T13; B_1T14]

It is evident from B_1T15 that any thesis derivable within the framework of System $\mathfrak{B}$ can be derived within the framework of System $\mathfrak{B}_1$, and this completes the proof that the two systems are inferentially equivalent.

REFERENCE

[1] Lejewski, C., "A note concerning the notion of mereological class," *Notre Dame Journal of Formal Logic*, vol. 19 (1978), pp. 251-263.

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