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Axioms for Mereology

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In this paper I shall present some related axioms for mereology and outline deductions from them. I presume an acquaintance with ontology, and I make no explicit references to theses of ontology in my proofs.[1]

Section 1 I shall begin by deriving three theses from an ordinary basis of mereology. That basis consists of four theses:

T1.1 $[AB] : A \in el(B) .\supset. B \in B$

T1.2 $[ABC] : A \in el(B) . B \in el(C) .\supset. A \in el(C)$

D1 $[Aa] :: A \in Kl(a) .\equiv\therefore A \in A \therefore [B] : B \in a .\supset. B \in el(A) \therefore [B] : B \in el(A) .\supset. [\exists CD] . C \in a . D \in el(B) . D \in el(C)$

T1.3 $[Aa] : A \in a .\supset. Kl(a) \in Kl(a)$.[2]

From these we can prove theses D2, T2.1, and T2.3 as follows:

T1.4 $[ABa] : A \in Kl(a) . B \in a .\supset. B \in el(A)$ [D1]

T1.5 $[ABa] : A \in Kl(a) . B \in el(A) .\supset. [\exists CD] . C \in a . D \in el(B) . D \in el(C)$ [D1]

T1.6 $[Ba] : B \in a .\supset. B \in el(Kl(a))$ [T1.3, T1.4]

T1.7 $[Ba] : B \in a .\supset. [\exists D] . D \in el(B)$ [T1.3, T1.6, T1.5]

T1.8 $[AB] : B \in el(A) .\supset. [\exists CD] . C \in el(A) . D \in el(B) . D \in el(C)$ [T1.7]

T1.9 $[A] : A \in A .\supset. A \in Kl(el(A))$ [T1.8, D1]

T1.10 $[Aa] : A \in Kl(a) .\supset. [\exists C] . C \in a$ [T1.7, T1.5]

T1.11 $[ABa] : A \in Kl(a) . B \in Kl(a) .\supset. A \in B$ [T1.10, T1.3]

T1.12 $[AB] : B \in Kl(el(A)) .\supset. A \in B$ [T1.10, T1.1, T1.9, T1.11]

T1.13 $[AB] : B \in el(A) .\supset. B \in el(Kl(A))$ [T1.1, T1.6, T1.2]

T1.14 $[AB] : A \in A . B \in el(Kl(A)) .\supset. [\exists D] . D \in el(B) . D \in el(A)$ [T1.1, T1.5]

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T1.15 $[AB] : A \in A . B \in el(Kl(A)) .\supset. [\exists CD] . C \in el(A) . D \in el(B) . D \in el(C)$ [T1.14, T1.7, T1.2]

T1.16 $[A] : A \in A .\supset. Kl(A) \in Kl(el(A))$ [T1.3, T1.13, T1.15, D1]

T1.17 $[A] : A \in A .\supset. A \in Kl(A)$ [T1.16, T1.12]

T1.18 $[A] : A \in A .\supset. A \in el(A)$ [T1.17, T1.4^3]

T1.19 $[ABC] : C \in el(B) .\supset. C \in el(Kl(A \cup B))$ [T1.1, T1.6, T1.2]

T1.20 $[ABC] :: A \in A \therefore [C] : C \in el(A) .\supset. [\exists D] . D \in el(C) . D \in el(B) \therefore C \in el(Kl(A \cup B)) \therefore\supset. [\exists DE] . D \in el(B) . E \in el(C) . E \in el(D)$.

Proof: $[ABC] :: Hp(3) \therefore\supset\therefore$

$[\exists DE] :$

4) $D \in A \cup B .$

5) $E \in el(C) . E \in el(D) :$ [3, T1.1, T1.5]

6) $D \in A .\supset. [\exists DE] . D \in el(B) . E \in el(C) . E \in el(D) :$ [5, 2, T1.2, T1.18]

7) $\sim(D \in A) .\supset. [\exists DE] . D \in el(B) . E \in el(C) . E \in el(D) \therefore$ [4, 5, T1.18]

$[\exists DE] . D \in el(B) . E \in el(C) . E \in el(D)$ [6, 7^4]

T1.21 $[AB] :: A \in A \therefore [C] : C \in el(A) .\supset. [\exists D] . D \in el(C) . D \in el(B) \therefore\supset. Kl(A \cup B) \in Kl(el(B))$ [T1.3, T1.19, T1.20, D1]

T1.22 $[AB] :: A \in A \therefore [C] : C \in el(A) .\supset. [\exists D] . D \in el(C) . D \in el(B) \therefore\supset. A \in el(B)$ [T1.21, T1.12, T1.4]

T1.23 $[ABC] : B \in el(A) . C \in el(B) .\supset. [\exists DE] . D \in el(A) . E \in el(C) . E \in el(D)$ [T1.2, T1.18]

T1.24 $[AB] : A \in el(B) . B \in el(A) .\supset. B \in Kl(el(A))$ [T1.2, T1.23, D1]

T1.25 $[AB] : A \in el(B) . B \in el(A) .\supset. A \in B$ [T1.24, T1.12]

T1.26 $[ABCa] : A \in Kl(a) . A \in el(B) . C \in a .\supset. C \in el(B)$ [T1.4, T1.2]

T1.27 $[ABCa] :: A \in Kl(a) \therefore [C] : C \in a .\supset. C \in el(B) \therefore C \in el(A) \therefore\supset. [\exists D] . D \in el(C) . D \in el(B)$ [T1.5, T1.2]

T1.28 $[ABa] :: A \in Kl(a) \therefore [C] : C \in a .\supset. C \in el(B) \therefore\supset. A \in el(B)$ [T1.27, T1.22]

T1.29 $[Aa] :: [B] \therefore A \in el(B) .\equiv: [C] : C \in a .\supset. C \in el(B) \therefore\supset. A \in Kl(a)$

Proof: $[Aa] :: Hp(1) \therefore\supset\therefore$

2) $A \in el(Kl(a))$. [T1.6, 1]

3) $\sim(A \in el(\wedge))$. [T1.1]

4) $Kl(a) \in Kl(a) \therefore$ [1, 3, T1.3]

5) $[C] : C \in a .\supset. C \in el(A) \therefore$ [2, T1.18, 1]

6) $Kl(a) \in el(A)$. [4, 5, T1.28]

$A \in Kl(a)$ [2, 6, T1.25]

T1.30 $[ABa] : A \in el(Kl(a)) . B \in el(A) .\supset. [\exists CD] . C \in a . D \in el(B) . D \in el(C)$ [T1.2, T1.1, T1.5]

T1.31 $[ABa] :: [B] : B \in el(A) .\supset. [\exists CD] . C \in a . D \in el(B) . D \in el(C) \therefore B \in el(A) \therefore\supset. [\exists D] . D \in el(B) . D \in el(Kl(a))$ [T1.6, T1.2]

T1.32 $[Aa] :: A \in A \therefore [B] : B \in el(A) .\supset. [\exists CD] . C \in a . D \in el(B) . D \in el(C) \therefore\supset. A \in el(Kl(a))$ [T1.31, T1.22]

T1.33 $[AB] : A \in el(B) .\equiv. [\exists a] . B \in Kl(a) . A \in a$ [T1.1, T1.9, T1.4]

T1.34 $[Aa] :: A \in Kl(a) .\equiv\therefore [B] \therefore A \in el(B) .\equiv: [C] : C \in a .\supset. C \in el(B)$ [T1.26, T1.28, T1.29[5]]

T1.35 $[Aa] :: A \in el(Kl(a)) .\equiv\therefore A \in A \therefore [B] : B \in el(A) .\supset. [\exists CD] . C \in a . D \in el(B) . D \in el(C)$ [T1.30, T1.32]

Section 2 The theses D2, T2.1, and T2.2 form an adequate axiom system for mereology:

D2 $[AB] : A \in el(B) .\equiv. [\exists a] . B \in Kl(a) . A \in a$

T2.1 $[Aa] :: A \in Kl(a) .\equiv\therefore [B] \therefore A \in el(B) .\equiv: [C] : C \in a .\supset. C \in el(B)$

T2.2 $[Aa] :: A \in el(Kl(a)) .\equiv\therefore A \in A \therefore [B] : B \in el(A) .\supset. [\exists CD] . C \in a . D \in el(B) . D \in el(C)$.

We see that these theses correspond to T1.33, T1.34, and T1.35, which we derived from the axiom system in Section 1. To show that the two systems are equivalent, we have only to derive theses corresponding to T1.1, T1.2, D1, and T1.3 from D2, T2.1, and T2.2.

T2.3 $[AB] : A \in el(B) .\supset. B \in B$ [D2]

T2.4 $[ABC] : C \in el(A) . A \in el(B) .\supset. C \in el(B)$ [D2, T2.1]

T2.5 $[A] : A \in A .\supset. A \in Kl(A)$ [T2.1]

T2.6 $[A] : A \in A .\supset. A \in el(A)$ [T2.5, D2]

T2.7 $[Aa] : A \in a .\supset. A \in el(Kl(a))$ [T2.6, T2.2]

T2.8 $[Aa] : A \in a .\supset. Kl(a) \in Kl(a)$ [T2.7, T2.3]

T2.9 $[Aa] : A \in Kl(a) .\supset. Kl(a) \in Kl(a)$.

Proof: $[Aa] \therefore Hp(1) .\supset:$

2) $\sim(A \in el(\wedge)) :$ [T2.3]

3) $[\exists C] . C \in a :$ [T2.1, 1, 2]

$Kl(a) \in Kl(a)$ [3, T2.8]

T2.10 $[ABa] : A \in Kl(a) . B \in a .\supset. B \in el(A)$ [D2]

T2.11 $[Aa] : A \in Kl(a) .\supset. A \in el(Kl(a))$ [T2.9, T2.6]

T2.12 $[ABa] : A \in Kl(a) . B \in el(A) .\supset. [\exists CD] . C \in a . D \in el(B) . D \in el(C)$ [T2.11, T2.2]

T2.13 $[ABCa] :: [B] : B \in a .\supset. B \in el(A) \therefore A \in el(B) . C \in a \therefore\supset. C \in el(B)$ [T2.4]

T2.14 $[ABa] :: A \in A \therefore [B] : B \in el(A) .\supset. [\exists CD] . C \in a . D \in el(B) . D \in el(C) \therefore [C] : C \in a .\supset. C \in el(B) \therefore\supset. A \in el(B)$

Proof: $[ABa] :: Hp(3) \therefore \supset .$

4) $A \in el(Kl(a))$.	[1, 2, T2.2]
5) $Kl(a) \in Kl(a)$.	[4, T2.3]
6) $Kl(a) \in el(B)$.	[5, 3, T2.1]
$A \in el(B)$	[4, 6, T2.4]

T2.15 $[Aa] :: A \in A \therefore [B] : B \in a . \supset . B \in el(A) \therefore [B] . B \in el(A) . \supset . [\exists CD] . C \in a . D \in el(B) . D \in el(C) \therefore \supset . A \in Kl(a)$ [T2.13, T2.14, T2.1]

T2.16 $[Aa] :: A \in Kl(a) . \equiv \therefore A \in A \therefore [B] : B \in a . \supset . B \in el(A) \therefore [B] : B \in el(A) . \supset . [\exists CD] . C \in a . D \in el(B) . D \in el(C)$ [T2.10, T2.12, T2.15]

Since T2.3, T2.4, T2.16, and T2.8 correspond to the axioms of Section 1, the system of theses D2, T2.1, and T2.2 is adequate for the development of mereology. The structure of the system suggests new single axioms for mereology, thesis T3.1 among others. By proving a correspondent of T3.1 from D2, T2.1, and T2.2, we show that it can be proved in ordinary systems of mereology.

T2.17 $[ABa] \because [b] \therefore A \in el(Kl(b)) . \equiv : [C] : C \in a . \supset . C \in el(Kl(b)) :: [C] : C \in a . \supset . C \in el(B) :: \supset . A \in el(B)$ [T2.7, T2.2, T2.14]

T2.18 $[AB] : A \in el(B) . \supset . Kl(A) \in el(B)$ [T2.5, T2.8]

T2.19 $[ABCa] \because [b] \therefore A \in el(Kl(b)) . \equiv : [C] : C \in a . \supset . C \in el(Kl(b)) :: A \in el(B) . C \in a :: \supset . C \in el(B)$ [T2.7, T2.18, T2.4]

T2.20 $[Aa] :: [b] \therefore A \in el(Kl(b)) . \equiv : [C] : C \in a . \supset . C \in el(Kl(b)) \therefore \supset . A \in Kl(a)$ [T2.17, T2.19, T2.1]

T2.21 $[Aa] :: A \in Kl(a) . \equiv \therefore [b] \therefore A \in el(Kl(b)) . \equiv : [C] : C \in a . \supset . C \in el(Kl(b))$ [T2.1, T2.20]

T2.22 $[BEab] :: [C] : C \in a . \supset . C \in el(Kl(b)) \therefore E \in a . B \in el(E) \therefore \supset . [\exists CD] . C \in b . D \in el(B) . D \in el(C]$ [T2.2]

T2.23 $[Cab] :: [BE] : E \in a . B \in el(E) . \supset . [\exists CD] . C \in b . D \in el(B) . D \in el(C) \therefore C \in a \therefore \supset . C \in el(Kl(b))$ [T2.2]

T2.24 $[ab] \therefore [C] : C \in a . \supset . C \in el(Kl(b)) : \equiv : [BE] : E \in a . B \in el(E) . \supset . [\exists CD] . C \in b . D \in el(B) . D \in el(C)$ [T2.22, T2.23]

T2.25 $[Aa] :: A \in Kl(a) . \equiv \therefore [b] \therefore A \in el(Kl(b)) . \equiv : [BE] : E \in a . B \in el(E) . \supset . [\exists CD] . C \in b . D \in el(B) . D \in el(C)$ [T2.21, T2.24]

T2.26 $[Aaf] :: A \in Kl(a) \therefore [AB] : A \in f(B) . \equiv . [\exists a] . B \in Kl(a) . A \in a \therefore \supset \therefore [b] \therefore A \in f(Kl(b)) . \equiv : [BE] : E \in a . B \in f(E) . \supset . [\exists CD] . C \in b . D \in f(B) . D \in f(C)$ [D2, T2.25]

T2.27 $[Aa] :: A \in A \because [f] :: [AB] : A \in f(B) . \equiv . [\exists a] . B \in Kl(a) . A \in a \therefore A \in Kl(A) \therefore \supset \therefore [b] \therefore A \in f(Kl(b)) . \equiv : [BE] : E \in a . B \in f(E) . \supset . [\exists CD] . C \in b . D \in f(B) . D \in f(C) \because \supset . A \in Kl(a)$ [T2.5, D2, T2.25]

T2.28 $[Aa] :: A \in Kl(a) \,.\!\equiv\!\because\, A \in A \because [f] :: [AB] : A \in f(B) \,.\!\equiv.\, [\exists a] \,.\, B \in Kl(a) \,.\, A \in a \therefore A \in Kl(A) \therefore\!\supset\!\therefore [b] \therefore A \in f(Kl(b)) \,.\!\equiv:\, [BE] : E \in a \,.\, B \in f(E) \,.\!\supset.\, [\exists CD] \,.\, C \in b \,.\, D \in f(B) \,.\, D \in f(C)$ [T2.26, T2.27]

Section 3 I shall now show that the thesis

T3.1 $[Aa] :: A \in Kl(a) \,.\!\equiv\!\because\, A \in A \because [f] :: [AB] : A \in f(B) \,.\!\equiv.\, [\exists a] \,.\, B \in Kl(a) \,.\, A \in a \therefore A \in Kl(A) \therefore\!\supset\!\therefore [b] \therefore A \in f(Kl(b)) \,.\!\equiv:\, [BE] : E \in a \,.\, B \in f(E) \,.\!\supset.\, [\exists CD] \,.\, C \in b \,.\, D \in f(B) \,.\, D \in f(C)$

is an adequate sole axiom for mereology. The functor '*el*' is introduced by the definition

D3 $[AB] : A \in el(B) \,.\!\equiv.\, [\exists a] \,.\, B \in Kl(a) \,.\, A \in a.$

Since T3.1 and D3 correspond to T2.28 and D2 in Section 2, which we proved equivalent to standard mereology, we have only to derive theses corresponding to the axioms of Section 2 from T3.1 and D3.

T3.2 $[Aa] \because [f] :: [AB] : A \in f(B) \,.\!\equiv.\, [\exists a] \,.\, B \in Kl(a) \,.\, A \in a :\supset\therefore [b] \therefore A \in f(Kl(b)) \,.\!\equiv:\, [BE] : E \in a \,.\, B \in f(E) \,.\!\supset.\, [\exists CD] \,.\, C \in b \,.\, D \in f(B) \,.\, D \in f(C) ::\equiv\therefore [b] \therefore A \in el(Kl(b)) \,.\!\equiv:\, [BE] : E \in a \,.\, B \in el(E) \,.\!\supset.\, [\exists CD] \,.\, C \in b \,.\, D \in el(B) \,.\, D \in el(C)$ [D3 [6]]

T3.3 $[Aa] \because A \in Kl(a) \,.\!\equiv:: A \in A :: A \in Kl(A) \,.\!\supset\therefore [b] \therefore A \in el(Kl(b)) \,.\!\equiv:\, [BE] : E \in a \,.\, B \in el(E) \,.\!\supset.\, [\exists CD] \,.\, C \in b \,.\, D \in el(B) \,.\, D \in el(C)$ [T3.1, T3.2]

T3.4 $[A] : A \in A \,.\!\supset.\, A \in Kl(A)$ [T3.3]

T3.5 $[A] : A \in A \,.\!\supset.\, A \in el(A)$ [T3.4, D3]

T3.6 $[BEa] : E \in a \,.\, B \in el(E) \,.\!\supset.\, [\exists CD] \,.\, C \in a \,.\, D \in el(B) \,.\, D \in el(C)$ [T3.5]

T3.7 $[Aa] :: A \in Kl(a) \,.\!\supset\therefore [b] \therefore A \in el(Kl(b)) \,.\!\equiv:\, [BE] : E \in a \,.\, B \in el(E) \,.\!\supset.\, [\exists CD] \,.\, C \in b \,.\, D \in el(B) \,.\, D \in el(C)$ [T3.4, T3.3]

T3.8 $[Aa] :: [b] \therefore A \in el(Kl(b)) \,.\!\equiv:\, [BE] : E \in a \,.\, B \in el(E) \,.\!\supset.\, [\exists CD] \,.\, C \in b \,.\, D \in el(B) \,.\, D \in el(C) \therefore\!\supset.\, A \in Kl(a)$ [T3.6, T3.3]

T3.9 $[ABa] : A \in el(Kl(a)) \,.\, B \in el(A) \,.\!\supset.\, [\exists CD] \,.\, C \in a \,.\, D \in el(B) \,.\, D \in el(C)$ [T3.4, T3.7]

T3.10 $[BEab] :: [C] : C \in a \,.\!\supset.\, C \in el(Kl(b)) \therefore E \in a \,.\, B \in el(E) \therefore\!\supset.\, [\exists CD] \,.\, C \in b \,.\, D \in el(B) \,.\, D \in el(C)$ [T3.9]

T3.11 $[Aa] :: A \in A \therefore [B] : B \in el(A) \,.\!\supset.\, [\exists CD] \,.\, C \in a \,.\, D \in el(B) \,.\, D \in el(C) \therefore\!\supset.\, A \in el(Kl(a))$ [T3.4, T3.7]

T3.12 $[Cab] :: [BE] : E \in a \,.\, B \in el(E) \,.\!\supset.\, [\exists CD] \,.\, C \in b \,.\, D \in el(B) \,.\, D \in el(C) \therefore C \in a \therefore\!\supset.\, C \in el(Kl(b))$ [T3.11]

T3.13 $[Aa] :: A \in Kl(a) \,.\!\supset\therefore [b] \therefore A \in el(Kl(b)) \,.\!\equiv:\, [C] : C \in a \,.\!\supset.\, C \in el(Kl(b))$ [T3.7, T3.10, T3.12]

T3.14 $[Aa] :: [b] \therefore A \in el(Kl(b)) .\equiv: [C] : C \in a .\supset. C \in el(Kl(b)) \therefore\supset. A \in Kl(a)$ [T3.10, T3.12, T3.8]

T3.15 $[Ca] : C \in a .\supset. C \in el(Kl(a))$ [T3.6, T3.12]

T3.16 $[Aa] : A \in Kl(a) .\supset. A \in el(Kl(a))$ [T3.15, T3.13]

T3.17 $[Aa] : A \in Kl(a) .\supset. [\exists C] . C \in a$ [T3.16, T3.5, T3.9]

T3.18 $[AB] : A \in el(B) .\supset. Kl(B) \in Id(B)$

Proof: $[AB] : Hp(1) .\supset.$

2) $B \in B .$ [1, D3]
3) $B \in Kl(B) .$ [2, T3.4]
4) $B \in el(Kl(B)) .$ [2, T3.15]
5) $Kl(B) \in Kl(B) .$ [4, D3]
$Kl(B) \in Id(B)$ [3, 5]

T3.19 $[ABa] :: A \in Kl(a) \therefore [C] : C \in a .\supset. C \in el(B) \therefore\supset. A \in el(B)$

Proof: $[ABa] :: Hp(2) \therefore\supset\therefore$

$[\exists C] .$
3) $C \in a .$ [1, T3.17]
4) $C \in el(B) :$ [3, 2]
5) $Kl(B) \in Id(B) \therefore$ [4, T3.18]
6) $[C] : C \in a .\supset. C \in el(Kl(B)) \therefore$ [2, 5]
7) $A \in el(Kl(B)) .$ [1, 6, T3.13]
$A \in el(B)$ [5, 7]

T3.20 $[ABCa] : A \in Kl(a) . A \in el(B) . C \in a .\supset. C \in el(B)$ [T3.18, T3.13]

T3.21 $[Aa] :: A \in Kl(a) .\equiv\therefore [B] \therefore A \in el(B) .\equiv: [C] : C \in a .\supset. C \in el(B)$ [T3.19, T3.20, T3.14]

T3.22 $[Aa] :: A \in el(Kl(a)) .\equiv\therefore A \in A \therefore [B] : B \in el(A) .\supset. [\exists CD] . C \in a . D \in el(B) . D \in el(C)$ [T3.9, T3.11]

Since D3, T3.21, and T3.22 correspond to D2, T2.1, and T2.2, we see that T3.1 is adequate as a sole axiom for mereology, and that T3.1 and D3 are equivalent to any standard axiom system.

Section 4 The most important of the above results is thesis T3.1, which is only twelve ontological units long. So far as I know, no other single axiom for the primitive term '*Kl*' has been published which contains fewer than thirteen ontological units.[7] It is canonic, organic, ontologically uniform, and internally independent.[8] However, it is neither externally independent nor as elegant as those axioms which lack functor variables.[9] There are many other axioms related to T3.1 and to the system D2, T2.1, and T2.2, including one the same length as T3.1 but inorganic:

T4.1 $[Aa] \because A \in Kl(a) .\equiv:: [f] :: [AB] : A \in f(B) .\equiv. [\exists a] . B \in Kl(a) . A \in a \therefore [A] : A \in A .\supset. A \in Kl(A) \therefore\supset\therefore [b] \therefore A \in f(Kl(b)) .\equiv: [BE] : E \in a . B \in f(E) .\supset. [\exists CD] . C \in b . D \in f(B) . D \in f(C).$

The number of defective axioms of the same length suggests there may be a yet shorter axiom with '*Kl*' as its primitive term, but I have been unable to find such a thesis.

No known single axiom of mereology is as 'self-evident' as the systems of several shorter axioms such as that used in Section 1. The pedagogical and perhaps philosophical virtues of short axioms have been sadly ignored. The system D2, T2.1, and T2.2 has a certain elegance and is easier to grasp than T3.1 and the other axioms discussed above. Theses D2, T2.1, and T2.2 have three, four, and six ontological units respectively. I know at least one system for '*el*' and '*Kl*' in which the longest thesis has five units and the others four or fewer:

T4.2 $[AB] : A \in el(B) \,.\!\supset\!.\, B \in Kl(el(B))$

T4.3 $[Aa] : A \in a \,.\!\supset\!.\, A \in el(Kl(a))$

T4.4 $[Aa] : A \in el(Kl(a)) \,.\!\supset\!.\, [\exists BC] \,.\, B \in a \,.\, C \in el(A) \,.\, C \in el(B)$

T4.5 $[Aa] :: A \in Kl(a) \,.\!\equiv\!\therefore\, [B] \therefore A \in el(B) \,.\!\equiv\!:\, [C] : C \in a \,.\!\supset\!.\, C \in el(B)$

T4.6 $[AB] :: A \in A \therefore [C] : C \in el(A) \,.\!\supset\!.\, [\exists D] \,.\, D \in el(C) \,.\, D \in el(B) \therefore\!\supset\!.\, A \in el(B).$

(If one wishes one of the theses to be a definition, one may replace T4.2 by D2.) I do not know of any system using only '*el*' and '*Kl*' in which the longest thesis has four units, nor do I know how to prove that five are necessary.

NOTES

1. Some of the important theses of ontology used in this paper are given in [3], pp. 44–45.

2. The axiom system T1.1, T1.2, D1, and T1.3 differs only slightly from that in [1], p. 471.

3. The proof of T1.18 is indebted to that in [1].

4. The proof of T1.20 and consequently of T1.22 is based on that given in [2], pp. 560–561.

5. Thesis T1.34 is a further simplification of Professor Lejewski's simplification of a theorem first proved by Tarski. The theorem was first published in [5] as theorem CCIX, p. 90, and Leśniewski says Tarski proved it in 1921 (see [5], p. 87, statement c). Lejewski's theorem is mentioned in [2], p. 559, and was proved in 1960 according to [6], p. 421. Thesis A21 in [2], p. 561, corresponds to T1.34; it is not clear whether Clay or Lejewski discovered it.

6. I omit the proof of T3.2, which is strictly ontological. One may prove it with an auxiliary definition by means of extensionality, or by elementary methods alone, though this takes rather longer.

7. See [3]. The axiom appears on p. 43 as E2, and it is proved equivalent to an axiom for '*el*' on pp. 45–47.

8. The terms here used are informally defined and discussed in [7].

9. On eliminating variable functors from mereological axioms, see [4], p. 135 and pp. 138–139. See also the brief remarks in [7], p. 62.

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