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# Mechanizing Logic I: Map Logic Extended Formally to Relational Arguments

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***Formal extensions: Conjunctive and disjunctive terms*** Our concern is to show a new simplified approach to mechanizing logic, including the logic of multiply quantified relational propositions.

First, the diagrammatic method of our earlier paper [11] is used to set up four formal rules which extend the "traditional" or "Aristotelian" logic into the field of arguments with conjunctive, disjunctive, and negative terms. Then with one further central "axiom" (an adaptation of relational conversion) we bring into the same system in a fairly concise and perspicuous fashion arguments involving multiply quantified relational propositions. The limits of this system (we presume it has limits) are not yet known.

It is this extended formal system that will be used in Part II as a base, together with Karnaugh maps, from which to demonstrate the *effective* mechanization of relational arguments by computer.

Arguments such as:

$$\begin{array}{l} A \text{ a } B\cdot C \\ \underline{A \text{ i } D} \\ \therefore \underline{D \text{ i } B} \end{array}$$

(see Appendix 1 for the symbolism)

are frequently cited as being beyond the range of traditional logic, and it is true that unaided syllogistic is unequal to some of them, but in the 1880s Keynes [6]

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showed how to extend the traditional logic to cover such cases, and John Anderson [1] of Sydney University was teaching some of these methods from 1927 to 1958.

In [11] we showed a mechanized effective way of handling such arguments by the Karnaugh map. The first move is to put the argument into Boolean form:

$$\begin{array}{l} A\cdot\bar{B}\vee A\cdot\bar{C}=0 \\ \underline{A\cdot D\neq 0} \\ \therefore \underline{D\cdot B\neq 0}\ . \end{array}$$

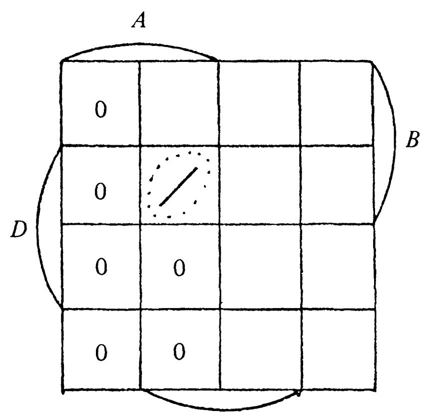


Figure 1a

The premises being correctly entered on the map, the conclusion (enclosed by a dotted line) if valid is literally to be seen on the map. If the conclusion fails to appear, the argument is invalid.

Now, such arguments may also be examined formally by a process requiring only two or four additional rules, which is in appearance like natural deduction except that it depends on the traditional doctrine of distribution (i.e., that the subjects of universal propositions and the predicates of negative propositions are said to be distributed, or universally quantified, while the other terms are undistributed).

***1 Dropdet*** The rule we require for Example 1 is that a conjunct (or *determinant*) may be validly *dropped* from an *undistributed* term.

If '$C$' is thus dropped from the first premise of Example 1 we have a standard valid syllogism left. We name this rule Dropdet and Dropdet separately may be tested by map and seen to be sound.

$$\begin{array}{l} \underline{A \text{ a } B\cdot C} \\ \therefore \underline{A \text{ a } B} \end{array} \qquad \begin{array}{l} \underline{A\cdot\bar{B}\vee A\cdot\bar{C}=0} \\ \therefore \underline{A\cdot\bar{B}=0} \end{array}$$

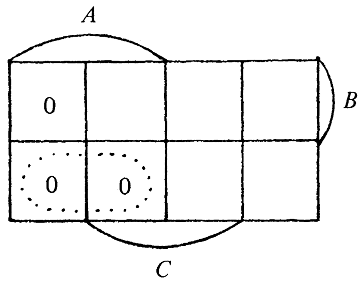


Figure 1b

The attempt to drop from a distributed term is, however, invalid; $A \cdot B$ a $C \therefore A$ a $C$. Figure 1c shows that when the premise is entered the proposed conclusion fails to appear. The rule is Dropdet.

$$\frac{A \cdot B \cdot \bar{C} = 0}{\therefore \underline{A \cdot \bar{C} = 0}}$$

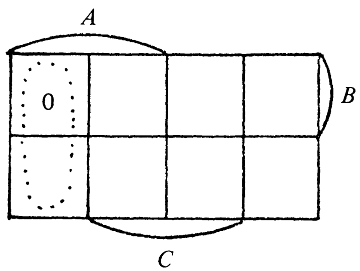


Figure 1c

## 2 *Addet*

$$\begin{array}{l} A \text{ a } B \cdot D \\ \underline{B \text{ a } C} \\ \therefore \underline{A \text{ a } C} \end{array}$$

It is valid to *add a determinant* (conjunct) to a *distributed* term. This move may be lawfully carried out with the subjects of universal affirmatives or negatives and with the predicates of all negative propositions whether universal or particular.

So if we add '$D$' to the subject of the second premise, again we have a standard valid syllogism. Separately, $B$ a $C \therefore B \cdot D$ a $C$, $B\bar{C} = 0 \therefore BD\bar{C} = 0$ (see Figure 2a).

To add a determinant to an undistributed term is, however, invalid. Figure 2b shows the proposed conclusion does not appear. $A$ a $B \therefore A$ a $B \cdot C$, or in Boolean form $A \cdot \bar{B} = 0 \therefore A \cdot \bar{B} \vee A \cdot \bar{C} = 0$.

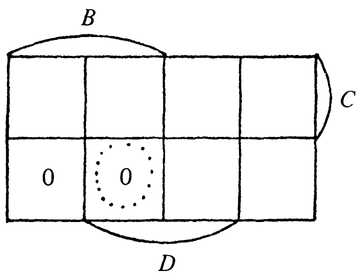


Figure 2a

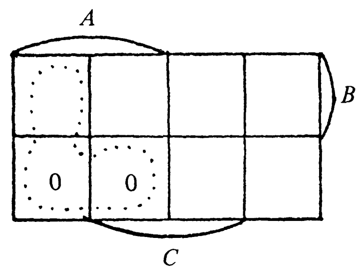


Figure 2b

The rule is Addet.

*3 Add SC* It is valid to add a superfluous conjunct which is a term or part of a term *already present* in the proposition to the other term of the proposition. So in premise 2 (below) the '$C$' of the predicate may be added to the subject, thus:

$$\begin{array}{lcl} A \text{ a } B\cdot C & \qquad & A \text{ a } B\cdot C \\ \underline{B \text{ a } C} & & \underline{B\cdot C \text{ a } C} \\ \therefore \underline{A \text{ a } C} & & \therefore \underline{A \text{ a } C} \end{array}$$

and again we have a valid syllogism. The rule Add SC (superfluous conjunct) has the *restriction* that an SC may not be added to the subject of an 'o' proposition. A map confirms the restriction and the valid rule.

We may if we wish add the '$C$' of premise one to the subject '$A$'.

$$\begin{array}{lcl} \underline{A \text{ a } B\cdot C} & \qquad & \underline{A\bar{B} \vee A\bar{C} = 0} \\ \therefore \underline{A\cdot C \text{ a } B\cdot C} & & \therefore \underline{AC\bar{B} = 0} \end{array}$$

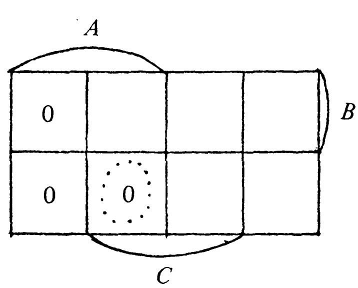


Figure 3a

But the case is different with the 'o' proposition:

$$\begin{array}{lcl} \underline{A \text{ o } B\cdot C} & \qquad & \underline{A\bar{B} \vee A\bar{C} \neq 0} \\ \therefore \underline{A\cdot C \text{ o } B\cdot C} & & \therefore \underline{AC\bar{B} \neq 0} \end{array}$$

which is invalid.

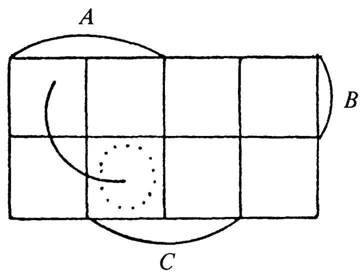


Figure 3b

'≠0' is shown by a '/' for one cell or a curved line for several cells. The conclusion if valid would require a definite '/' showing the nonemptiness of the cell with the dotted circle, but in fact all we have is that at least one of the three cells (we know not which) is nonempty—the argument is invalid—hence the restriction on adding an SC to the subject of an 'o' proposition. Disregarding the restriction could permit, e.g., 'Some cows are not brown animals' (true), ∴ 'Some brown cows are not brown animals' (false).

The Add SC performed in the first example of Add SC, above, cannot actually be mapped: $B$ a $C \therefore B \cdot C$ a $C$. This conclusion is 'unmappable' because it is a tautology. It is always on the map—the move is sound. $B\bar{C} = 0 \therefore BC\bar{C} = B \cdot 0 = 0$. The rule is Add SC.

*4 Drop SC*

$$\frac{A \cdot C \text{ e } C \cdot B}{\therefore A \text{ e } C \cdot B} \quad \text{or} \quad \underline{A \cdot C \text{ e } B}$$

It is valid to drop a superfluous conjunct from one term when it is also present as or as part of the other term, with the *restriction* that it is always invalid to drop an SC from the subject of an 'a' proposition.

There is no need to map, the Boolean form is the same for all three propositions above, $ACB = 0$.

But note the *restriction*:

$$\frac{A \cdot C \text{ a } C \cdot B}{\therefore A \text{ a } C \cdot B} \qquad \frac{AC\bar{B} = 0}{\therefore A\bar{C} \vee A\bar{B} = 0} \qquad \text{invalid}$$

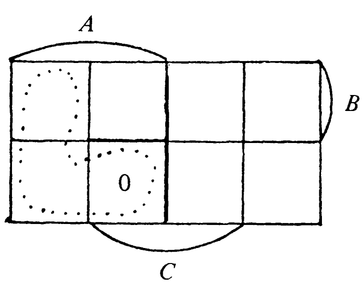


Figure 4

If the conclusion were valid, the three cells containing the 'dotted line' would all hold zeros.

Disregarding the restriction could permit, e.g., 'All brown cows are brown animals (true), ∴ 'All cows are brown animals' (false). The rule is Drop SC.

Now these four rules occasionally supplemented by a Boolean law are sufficient to cover a wide range of arguments involving conjunctive terms. We could set up a parallel set of rules for disjunctive terms, but De Morgan's law will change our disjuncts to conjuncts subject to the four rules above. One or two examples of such a change should be sufficient.

$$\frac{A \vee B \text{ a } C}{\therefore A \text{ a } C} = \frac{\bar{C} \text{ a } \bar{A}\cdot\bar{B}}{\therefore \bar{C} \text{ a } \bar{A}} \qquad \text{valid Dropdet}$$

i.e., by contraposition (using De Morgan's law) it can be seen that valid dropping a disjunct from a distributed term is equivalent to valid dropping a determinant from an undistributed term.

$$\frac{A \text{ o } B}{\therefore C \vee A \text{ o } B} = \frac{\bar{B} \text{ o } \bar{A}}{\therefore \bar{B} \text{ o } \bar{C}\cdot\bar{A}} \qquad \text{valid Addet}$$

Valid adding a disjunct to an undistributed term is equivalent to valid adding a determinant to a distributed term. It will also be shown that Addet and Dropdet can be dispensed with, i.e., the extension could be worked with only two rules, for Addet and Dropdet can be derived from syllogisms.

Tacit use of some of the laws of Boolean algebra, i.e., De Morgan's law, double negation, etc., has long been made in traditional logic.

The four rules Addet, Dropdet, Add SC, and Drop SC are shown in action on moderately complicated examples using all four of them.

Example 1:

| | |
|---|---|
| 1. $A$ e $\bar{B} \vee \bar{C}$ | |
| 2. $A\cdot C$ i $D$ | / ∴ $D\cdot C$ i $B$ |
| 3. $A\cdot C$ i $D\cdot C$ | (2 Add SC) |
| 4. $A$ i $D\cdot C$ | (3 Drop SC) |
| 5. $A$ a $B\cdot C$ | (1 Obvert) |
| 6. $D\cdot C$ i $B\cdot C$ | (4, 5 Syll.) |
| 7. $D\cdot C$ i $B$ | (6 Drop SC) |

Example 2:

| | |
|---|---|
| 1. $A$ a $B\cdot C$ | |
| 2. $B$ e $D$ | / ∴ $A\cdot C$ e $D$ |

This can be solved using Addet on each premise but it may also be solved by Add SC and Drop SC; these are often used together in one line and the justification may be labeled, e.g., (1 Add, Drop SC). In effect, the $C$ is moved from the predicate to the subject or vice versa, subject to the restrictions discussed above.

| | |
|---|---|
| 3. $A\cdot C$ a $B$ | (1 Add, Drop SC) |
| 4. $A\cdot C$ e $D$ | (2, 3 Syll.) |

Example 3 (Adapted from Copi [4]):

| | | |
|---|---|---|
| 1. | $M \cdot C \cdot G$ e $D$ | |
| 2. | $M \cdot R$ a $C$ | |
| 3. | $C \cdot \bar{D}$ a $E$ | |
| 4. | $M \cdot E$ a $C \cdot G$ | $/ \therefore M \cdot R \cdot G$ a $E$ and $M \cdot R \cdot E$ a $G$ |
| 5. | $M \cdot R \cdot G$ a $C$ | (2 Addet) |
| 6. | $M \cdot R \cdot G$ a $C \cdot G$ | (5 Add SC) |
| 7. | $M \cdot R \cdot G$ a $M \cdot C \cdot G$ | (6 Add SC) |
| 8. | $M \cdot C \cdot G$ a $\bar{D}$ | (1 Obvert) |
| 9. | $M \cdot C \cdot G$ a $C \cdot \bar{D}$ | (8 Add SC) |
| 10. | $M \cdot C \cdot G$ a $E$ | (3, 9 Syll.) |
| 11. | $M \cdot R \cdot G$ a $E$ | (7, 10 Syll.) *first conclusion* |
| 12. | $M \cdot E$ a $G$ | (4 Dropdet) |
| 13. | $M \cdot R \cdot E$ a $G$ | (12 Addet) *second conclusion* |

These rules will sometimes be needed in working relational arguments.

***Complex terms and syllogism interrelated*** Given tautologies like '$B \cdot C$ a $B$' even 'Dropdet' and 'Addet' may be brought under syllogistic rules; however, we see no way of carrying out all the Superfluous Conjunct moves without adding to the system the special rules for adding and dropping superfluous conjuncts.

Dropdet syllogistically:

1. $\underline{A \text{ a } B \cdot C}$
$\therefore \underline{A \text{ a } B}$

Syllogistic proof:

| | | |
|---|---|---|
| 1. | $A$ a $B \cdot C$ | |
| 2. | $\underline{(B \cdot C \text{ a } B)}$ | (Tautology) |
| | $\therefore A$ a $B$ | (1,2 Syll.) |

Addet syllogistically:

| | | |
|---|---|---|
| 1. | $A$ a $B$ | |
| 2. | $\underline{(A \cdot C \text{ a } A)}$ | (Tautology) |
| | $\therefore A \cdot C$ a $B$ | (1, 2 Syll.) |

We offer these two demonstrations as an indication of the consistency of syllogistic and complex term rules. We propose, however, to avail ourselves of the special rules 'Addet' and 'Dropdet' and 'Add SC' and 'Drop SC' [2], [10].

***Darapti*** Darapti and comparable arguments may be handled as in the following example.[1]

| | | |
|---|---|---|
| 1. | Whales a mammals | |
| 2. | Whales a sea-creatures | |
| 3. | (Whales i whales) | (Premise supplied since whales are admittedly existent) |
| 4. | Whales i sea-creatures | (2, 3 Syll.) |
| 5. | Sea-creatures i mammals. | (1, 4 Syll.) |

The argument with its three premises also maps without difficulty [3].

***Earlier forms of diagrams*** The treatment by diagram of arguments having conjunctive and disjunctive terms was included by John Venn in his *Symbolic Logic* a hundred years ago, but little attention has been paid to this, probably because Venn's system could not be usefully extended beyond four variables. Moreover, Venn's diagrams were not fully iconic in that they suggest that the area representing the conjunction of the complements of all variables is not on the same footing as the other compartments of the diagram. The Karnaugh map remedies this defect. In concentrating on circles and ellipses Venn may have been too influenced by Euler.

Lewis Carroll was drawn to rectangular figures, but he unfortunately employed too many principles of geometrical division to cover increases in the number of variables. His fancy embraced overlapping rectangles within a rectangle, diagonals, and crosses,[2] producing cells of varied shapes as against the mechanized uniformity of Karnaugh map growth where cells are symmetrically divided alternately by horizontal lines followed by vertical lines and back to horizontal lines, ad infinitum.

Marquand, a contemporary of Venn, may be said to have partly anticipated the Karnaugh map but, like Veitch, huddled all his variable labels together on only two sides of the rectangular figure with resultant loss of perspicuity. Furthermore, his custom of alternating labels for a term and its complement along the whole of a side of the figure instead of keeping them together whenever possible, as Karnaugh does, adds to the difficulty of 'reading' his diagrams. On the eight-variable Karnaugh map only two variables appear on each side of the figure. It would appear that Karnaugh's achievement was far from obvious.

***Relational arguments – Extending the system*** There is no well-known way of symbolizing a second, third, . . . or *n*th quantifier in traditional logic. We now have to present a well-tested, working method for achieving this which will be shown to be easy to handle in deductive arguments. The distinction between:

Every student fears every examination

and

Every student fears some examination (or other)

can be symbolized simply and effectively (using initial letters of the key words in the examples) as

$$S \text{ a } F\text{–}E_d$$

and

$$S \text{ a } F\text{–}E_u$$

These are seen as universal affirmative propositions with complicated relational predicates.

In '$F$–' the hyphen serves to mark the fact that '$F$' is a relation and the following '$E$' is its relatum. The subscript '$d$' stands for 'distributed' and marks the quantity of the relatum. In this context, the 'every' points to the universal quantification of the relatum $E$, and the '$u$' (undistributed) here records the particular quantification of the $E$ in the second example.

This notation can be extended into more and more complicated cases involving quantifiers within quantifiers; e.g., 'Some beetle attacks are reported by every orchardist growing some fruits' appears as:

$$B \text{ i } R-(O \cdot G-F_u)_d$$

where brackets are needed to show the scope of the final subscript quantifier.

Apart from these small notational additions, which result in forms that are concise and fairly perspicuous and close to normal English, we shall require one new axiom or Immediate Inference for stating relational propositions in equivalent forms interchanging subjects and relata. We have found this simple extension to be of wide application and surprising utlity and it will culminate in relational arguments being diagrammed and computerized for both valid and invalid arguments.

Relational Conversion [9] (henceforth RC) appears in the literature at least as early as Aristotle. One of his examples is, in effect, if '$A$ is 3 times $B$' then '$B$ is $\frac{1}{3}$ of $A$'. Moderns favor examples like '$A$ is a parent of $B \equiv B$ is the child of $A$'. We have extended this key idea to multiply quantified propositions like 'Every student fears every exam', $S$ a $F-E_d \equiv$ 'Every exam is feared by every student', $E$ a $F'-S_d$. This approach to RC renders accessible to extended traditional logic a large range of arguments from which, without RC, it was cut off.

If it were argued that such a relational pair consists only of two verbal forms for the *one* proposition, then one *could* argue similarly, for example, for contraposition (A11 $A$ are $B \equiv$ All $\bar{B}$ are $\bar{A}$). We think it reasonable to treat these as distinct though equivalent propositions with formal differences of importance, as should emerge from our illustrations of their use.

Similarly we treat 'Every student fears some exam or other' ($S$ a $F-E_u$) as formally distinct from though equivalent to 'Some exam or other is feared by every student' ($E$ i $F'-S_d$), which is not to be confused with the singular proposition 'Some *one* unspecified exam is feared by every student' ($UE$ a $F'-S_d$). The latter implies the former but is not implied by them:

| | | |
|---|---|---|
| 1. | $UE$ a $F'-S_d$ | |
| 2. | ($UE$ a $E$) | (Tautology assumed) |
| 3. | $E$ i $F'-S_d$ | (1, 2 syllogism) (Given '$UE$ i $UE$') |
| 4. | $S$ a $F-E_u$. | (3 RC) |

In 1 and 2 the '$U$' is a term indicating an unspecified but in principle specifiable object.

RC apparently resembles quantifier order-change, but has important differences. This emerges when we see, e.g., that '$D-C_u$ a $D-C_u$' (All drawing some circles are drawing some circles) is an unmappable tautology, whereas its RC, '$C$ i $D'-(D-C_u)_d$' (Some circles are drawn by all beings drawing some circles) does not have the formal properties of tautology and is mappable or diagrammable and hence mechanizable. Our rules permit the last two propositions to be distinct and equivalent. This is an independent system with its own criteria [10]. Notice we make formal use of correlates.

RC is the interchange of subject and main relatum together with moving from relation to its correlate in an equivalent proposition.

In what we are calling RC, e.g.,

$$(1)\ S \text{ a } F{-}E_u \qquad (2)\ E \text{ i } F'{-}S_d\ ,$$

notice:

(a) $S$ and $E$ change places: $E$ is formally a *relatum* (a fragment of a term) in (1), but an *independent nonrelational* subject-term in (2). Note that the 'some or other' of $E$ in (1) carries over unchanged to $E$ in (2).
(b) The relation-particle, '$F-$', becomes '$F'-$', its correlate.
(c) The 'i' copula of (2) is determined by the '$u$' subscript ('some') of (1).
(d) The universal 'a' copula of (1) determines the subscript of $S$ in (2) ('every').
(e) 'Quality' is unchanged: affirmative propositions stay affirmative; negative negative.

Modern uses of Relational Conversion are to be found in Sommers [12] and earlier in De Morgan's work [5]. Our own background knowledge of the notion comes mainly from Stebbing [13]. But none of these logicians views RC quite the way we conceive it.

***The system at work*** First, two very familiar examples:

*A. Inference by Complex Conception (CC).* The name 'Inference by Complex Conception' was bestowed on this type of argument by an older generation of traditional logicians who, however, regarded it as an immediate inference.[3] It is also exemplified by the well-known 'Horse's Head' argument. Our example is taken from Quine [8]:

All circles are figures.
∴ All who draw circles draw figures.

$D-$ = (beings) drawing; $D'-$ = (things) drawn by.

| | | |
|---|---|---|
| 1. | $C \text{ a } F$ | |
| 2. | $(D{-}C_u \text{ a } D{-}C_u)$ | (assumed, based on subject of the conclusion) |
| | $/ \therefore D{-}C_u \text{ a } D{-}F_u$ | |
| 3. | $C \text{ i } D'{-}(D{-}C_u)_d$ | (2 RC) (Some circles are drawn by all drawing some circles) |
| 4. | $F \text{ i } D'{-}(D{-}C_u)_d$ | (3, 1 Syllogism) |
| 5. | $D{-}C_u \text{ a } D{-}F_u$ | (4 RC) valid |

*B. Demonstration of A Fortiori:*

All *A*s are greater than any *B*s. All *B*s are greater than any *C*s. Therefore all *A*s are greater than any *C*s.

$G$ = greater than; $G'$ = less than, smaller than, etc.

As usual, the argument is regarded as an enthymeme. The assumption of the transitivity of 'greater than' is premise 3: 'All things greater than some things greater than any *C*s are greater than any *C*s', based on the predicate of the conclusion.

1. $A$ a $G{-}B_d$
2. $B$ a $G{-}C_d$
3. $(G{-}(G{-}C_d)_u$ a $G{-}C_d)$ (Assumption of transitivity) / ∴ $A$ a $G{-}C_d$
4. $B$ a $G'{-}A_d$ (1 RC)
5. $G{-}C_d$ i $G'{-}A_d$ (4, 2 Syll.) (Given $B$ i $B$)
6. $A$ a $G{-}(G{-}C_d)_u$ (5 RC)
7. $A$ a $G{-}C_d$ (3, 6 Syll.) valid

The general pattern of the transitivity premise is '$R{-}(R{-}X_d)_u$ a $R{-}X_d$', where $R$ is a relation and $X$ its relatum and '$R{-}X_d$' is the predicate of the conclusion (as in 3 above). Intransitivity is the same pattern with an 'e' copula. Asymmetry is indicated by "$R{-}X_d$ e $R'{-}X_d$". Naturally in applying these forms to particular instances we must here have the restriction that applies to all enthymemes; that the supplied premise should be true.

***Representative samples of arguments involving relations*** The operations performed at discretion in the following three examples will later be shown to be performed by the Karnaugh map which carries out analogues of these operations without the need for discretion to intercede. The examples differ in complexity and will bring out between them the various features of our systems at work as well as examples of symbolizing varieties of English expression.

Example 1 (Brief and introductory):

Argument: Some botanists are eccentric women.
Some botanists do not like any eccentric person.
Therefore, some botanists are not liked by all botanists. [7]

1. $B$ i $E \cdot W$
2. $B$ o $L{-}E_d$ / ∴ $B$ o $L'{-}B_u$
3. $E$ e $L'{-}B_u$ (2 RC)
4. $B$ i $E$ (1 Dropdet)
5. $B$ o $L'{-}B_u$ (3, 4 Syll.) valid

Example 2 (Relations in both subject and predicate and shows usefulness of SC):

Argument: Everyone reading some good biology books is interested in every good biology book.
All zoologists read some good biology books.
All Darwin's main works are good biology books.
All good biology books interesting every zoologist contain some firmly established doctrines.
All firmly established doctrines are seminal.
∴ All Darwin's main works contain something seminal.

1. $R{-}B_u$ a $I{-}B_d$
2. $Z$ a $R{-}B_u$

3. $D$ a $B$
4. $B\cdot(I'-Z_d)$ a $C-F_u$
5. $F$ a $S$ $/\therefore D$ a $C-S_u$
6. $Z$ a $I-B_d$ (1, 2 Syll.)
7. $B$ a $I'-Z_d$ (6 RC)
8. $B$ a $B\cdot(I'-Z_d)$ (7 Add SC)
9. $B$ a $C-F_u$ (4, 8 Syll.) or (4, 8, 3 Sorites)
10. $D$ a $C-F_u$ (3, 9 Syll.)
11. $F$ i $C'-D_d$ (10 RC)
12. $S$ i $C'-D_d$ (11, 5 Syll.)
13. $D$ a $C-S_u$ (12 RC) valid

Example 3 (Shows relation with conjunctive relatum and a particular conclusion):

Argument: All contributions to this publication are reports of original research work. Anyone who produces a report of original research work is hard-working or nonconformist. Some very obscure persons produced some of the material which is a contribution to this publication. Therefore, (a) some very obscure persons are hard-working or nonconformist, and (b) some reports of original research work are produced by some very obscure persons. ([4], p. 150, no. 7)

1. $C$ a $R$
2. $P-R_u$ a $(H\vee N)$
3. $O$ i $P-(M\cdot C)_u$ $/\therefore O$ i $(H\vee N)$ and $R$ i $P'-O_u$
4. $M\cdot C$ i $P'-O_u$ (3 RC)
5. $C$ i $P'-O_u$ (4 Dropdet)
6. $R$ i $P'-O_u$ (5, 1 Syll.) valid (conclusion (b))
7. $O$ i $P-R_u$ (6 RC)
8. $O$ i $(H\vee N)$ (7, 2 Syll.) valid (conclusion (a))

We have tested the formal system over a decade on a wide range of arguments with multiply quantified relational propositions taken mainly from [4] and [8]. We believe that the reader will find it stands comparison with other systems for perspicuity and conciseness.

We have discovered, moreover, that relational arguments can also be tested by Karnaugh map and may be incorporated as a natural extension in the same *effective* system for syllogisms and sorites with complex terms that we demonstrated in [11].

The map system is based on the same fundamental principles as the formal system. In addition to demonstrating the three valid conclusions just shown, the system will be shown rejecting certain *invalid* conclusions for the same arguments.

***Appendix 1 Main symbols or abbreviations*** The main symbols and abbreviations are shown in Table 1.

Table 1

| Symbol | Meaning | Example | In Symbols | Boolean Form |
|---|---|---|---|---|
| a | All. . . are. . . | All $X$ are $Y$. | $X$ a $Y$ | $X \cdot \bar{Y} = 0$ |
| e | No. . . are. . . | No $X$ are $Y$. | $X$ e $Y$ | $X \cdot Y = 0$ |
| i | Some. . . are. . . | Some $X$ are $Y$. | $X$ i $Y$ | $X \cdot Y \neq 0$ |
| o | Some. . . are not. . . | Some $X$ are not $Y$. | $X$ o $Y$ | $X \cdot \bar{Y} \neq 0$ |
| $A$, $B$, $C$. . . | Terms (subjects, predicates) and relata. | Some pupils dislike every exam. | $P$ i $D{-}E_d$ | $P \cdot D{-}E_d \neq 0$ |
| $\bar{A}$, $\bar{B}$, $\bar{C}$. . . | Non-$A$, logical opposite or complement of $A$, etc. | Neither igneous nor granitic. | $\overline{I \vee G}$ | $\bar{I} \cdot \bar{G}$ |
| . | . . . and. . . | All $X$ are male children. | $X$ a $(M \cdot C)$ | $(X \cdot \bar{M}) \vee (X \cdot \bar{C}) = 0$ |
| ∨ | . . . or. . . (nonexclusive) | All $X$ are male or children. | $X$ a $(M \vee C)$ | $X \cdot \bar{M} \cdot \bar{C} = 0$ |
| $R-$ | Relating. . . (Relation) | Disliking any exam. | $D{-}E_d$ | |
| $R'-$ | Related to. . . (Correlate) | Disliked by some pupils. | $D'{-}P_u$ | |
| $d$ | All (every, any, each) | Disliking any exam. | $D{-}E_d$ | |
| $u$ | Some or other | Disliked by some pupils. | $D'{-}P_u$ | |

Subscripts '$u$' and '$d$' are relata-quantifiers.

*Appendix 2 Singular propositions* We follow the usual traditional approach in treating singular terms as if they were for formal purposes class terms.

Although this procedure runs counter to linguistic habits in English (but not in Chinese) it should be clear that class arguments remain valid (or invalid) if appropriate singular terms are substituted for class terms.

In ordinary language singulars are almost always indeterminate propositions like 'men are winged' where we have to use our discretion to decide the quantity.

We are, however, tempted by the view in an interesting article by Sommers [12] that singular propositions are indifferently universal or particular.

## NOTES

1. We have to acknowledge the advice of the referee on this topic.
2. Diagonals are introduced by Carroll when he expands his diagram to fit five variables, and crosses are brought in for the sixth.
3. It introduces a relation-functor into the subject and the predicate of categorical propositions. Care has to be taken, of course, to avoid ambiguity in the two occurrences of the relation-functor.

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