Notre Dame Journal of Formal Logic
Volume 25, Number 3, July 1984

# Read on Relevance: A Rejoinder

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The chief difficulty confronting any commentator on "relevance" or "relevant" logic is the Protean character of relevantism, its tendency to assume ever new forms, especially in response to criticism. In a recent note [2] I criticized the original version of relevantism, found in [1] and other joint works of Anderson and Belnap. This version might be called *paleorelevantism* to distinguish it from the various *neorelevantisms* that have sprung up in recent years. My note has provoked replies from Mortensen [5] and Read [6]. I have responded to Mortensen elsewhere [3], and will respond to Read here.

***1 Paleorelevantism*** The Anderson-Belnap paleorelevantist position involved three theses.

1. *Dualism:* There are two senses of disjunction, the truth-functional or "extensional" sense $\vee$, and a non-truth-functional or "intensional" sense $+$. Of argument forms involving disjunction, (A) below is valid for $+$ and invalid for $\vee$; (B) below is valid for $\vee$ and invalid for $+$.

$$\text{(A)}\quad \begin{array}{l} p \text{ or } q \\ \underline{\text{not } p} \\ \therefore q \end{array} \qquad\qquad \text{(B)}\quad \begin{array}{l} q \\ \hline \therefore p \text{ or } q \end{array}$$

2. *Populism:* Dualism is compatible with common sense and involves no revision of the logical practice of ordinary people. In commonsensical arguments of form (A), the disjunction is meant as intensional, while in commonsensical arguments of form (B), the disjunction is meant as extensional.

3. *Objectivism:* The connection of relevance that distinguishes $p + q$ from $p \vee q$ is an objective (e.g., semantical or causal), not a subjective (e.g., epistemological or psychological) matter. (Passages from [1] justifying the attribution of objectivism to Anderson and Belnap were cited in [2]; Read has not challenged this attribution.)

*Received August 22, 1983*

A number of counterexamples to the claims of paleorelevantism have been devised by E. M. Curley, S. A. Kripke, the author, and others (see [2]). The following is fairly typical:

By the regulations of a certain government agency, a citizen $C$ is entitled to a pension if and only if $C$ *either* satisfies certain age requirements *or* satisfies certain disability requirements. An employee $Z$ of the agency is presented with documents establishing that $C$ is disabled. $Z$ transmits to fellow-employee $Y$ the information that $C$ is entitled to a pension (i.e., is either aged or disabled). $Y$ subsequently receives from another source the information that $C$ is not aged, and concludes that $C$ must be disabled.

According to populism, while $Z$'s statement that $C$ is either aged or disabled, being the conclusion of a clearly commonsensical argument of form (B), must be an extensional disjunction, $Y$'s statement that $C$ is either aged or disabled, being the premise of a clearly commonsensical argument of form (A), must be an intensional disjunction.

According to objectivism, the intensional disjunction differs from the extensional disjunction by asserting the existence of an objective connection of relevance between $C$'s not being aged and $C$'s being disabled. But in the first place it is difficult to believe that there is any such objective connection. (Could it be said that if $C$ hadn't been disabled, $C$ would have been aged?) And in the second place, even if there is such an objective connection, $Z$ does not know about it (since $Z$'s statement that $C$ is either aged or disabled is based solely on an argument of form (B)), so $Y$ cannot know about it (since initially $Y$'s information came from $Z$). Hence, $Y$ cannot be in a position to assert the intensional disjunction. Contradiction!

***2 Subjectivism*** One relevantist reaction to such counterexamples might be to abandon objectivism. Supppose $p + q$ is taken to mean "$p \vee q$, and *my* grounds for this assertion are non-truth-functional (i.e., neither grounds for asserting $p$ nor grounds for asserting $q$)". Then the connection of relevance distinguishing $p + q$ from $p \vee q$ becomes a subjective matter. And the counterexamples are blocked! There is no contradiction in assuming that the better-informed $Z$ should have truth-functional grounds for asserting $p \vee q$, while the worse-informed $Y$ should have non-truth-functional grounds. $Y$'s grounds might simply be that $Z$ has asserted $p \vee q$, and that $Z$ is usually reliable.

To refute subjectivized relevantism by counterexample, it would be necessary to find a commonsensical instance of a person arguing by (A) where that person's grounds for the major premise $p$-or-$q$ are truth-functional. But it is impossible to find such an instance, because it is *pointless* to argue in this way: on the one hand, if a person's grounds for asserting $p$-or-$q$ were grounds for asserting $p$, and that person now has grounds for asserting not-$p$, then that person ought to be going back to reconsider his or her premises, not going forward toward new conclusions; on the other hand, if a person's grounds for asserting $p$-or-$q$ are already grounds for asserting $q$, then further argument for $q$ is superfluous.

All this is not to say that subjectivized relevantism is immune to criticism. On the contrary, it can be criticized, not indeed by presenting counterexamples like that of the pensioner above, but by invoking Ockham's Eraser, the

methodological principle of semantics according to which meanings are not to be multiplied beyond necessity, nor ambiguities postulated without compelling evidence.

The mere fact that in commonsensical instances of (A) the major premise *is asserted on non-truth-functional grounds* is no evidence for the hypothesis that it *has non-truth-functionality as part of its very meaning*. For that fact is adequately explained (even on the hypothesis that $p$-or-$q$ always means just $p \vee q$) by the pointlessness considerations adduced above. (For a related observation, see [4].)

The *only* evidence for postulating a non-truth-functional sense of disjunction alongside the truth-functional sense would seem to be that collected long ago by P. F. Strawson and his school, cases where a speaker knows that $q$ but where it would be infelicitous for that speaker to assert that $p$-or-$q$. But such cases have been adequately explained by H. P. Grice (in his William James Lectures): To assert that $p$-or-$q$ would be infelicitous in these cases not because it would be *untrue*, but because it would be *uncooperative* on the part of a speaker who actually knows that $q$. But the points and counterpoints raised by Strawson and Grice in this debate are perhaps too well known to bear repeating here.

***3 Revisionism*** According to Anderson and Belnap, people *ought not* to argue by extensional disjunctive syllogism ($p \vee q$, $\sim p / \therefore q$), and ordinary people *do not* so argue, but classical logicians erroneously *say they do*. According to Brouwer, people *ought not* to argue by double negation elimination ($\sim\sim p / \therefore p$), but people *do* so argue, and classical logicians are (unfortunately) correct when they *say they do*. One relevantist reaction to the counterexamples to paleorelevantism might be to abandon the populist stance of Anderson and Belnap for a revisionist stance like that of Brouwer (while of course continuing to object to extensional disjunctive syllogism, not double negation elimination).

When the issue is thus shifted from factual questions about how people do argue to normative questions about how people ought to argue, it becomes difficult to stage a non-question-begging debate between advocates of opposing positions. (The history of the debate between classical and intuitionistic logicians, from the time of Brouwer to that of Dummett, illustrates this point.) Some have held that in any such debate the burden of proof must be on the classicist, not the relevantist, since the classicist "dogmatically" asserts, while the relevantist "only" skeptically questions, the validity of certain disputed forms of argument. I would hold, on the contrary, that if relevantism is, like intuitionism, in the position of advocating a major revision of long-established logical practices, then the burden of proof must be on the relevantist, to demonstrate some *benefits* to be expected from the proposed revision to outweigh the *costs* that are always involved in any major change of long-established practices. Such a pragmatic case for relevantistic revision of logic could perhaps be made out if the ambitious projects for the advancement of the sciences sketched in some relevantist manifestos were ever realized. But to date those projects remain just projects.

One fallacious pragmatic argument for relevantism is encountered dotted around the literature of relevantism and not infrequently in the oral pronouncements of relevantists. It runs: "Classical logic, especially the classical doctrine

that a contradiction implies anything, is *dangerous*. Suppose, for example, the FBI computer were fed inconsistent information about, say, the color of your car. If it worked by classical logic, it might conclude that you are Public Enemy Number One—with dire consequences for you!''

The answer to this argument is that no danger follows from the mere recognition that a logical relation

(1) $A_1, \ldots, A_n$ imply $B$

obtains, unless one erroneously assumes that this, together with

(2) Each of $A_1, \ldots, A_n$ is vouched for by a usually reliable source of information,

imply

(3) It is wise to act on the assumption that $B$.

But (1) and (2) simply do not imply (3). Whether it is wise to act on the assumption that $B$ can only be decided after an assessment of *utilities* and *probabilities*: How much is to be gained/lost by acting on the assumption that $B$ if $B$ is/isn't in fact the case? How likely is it that $B$ is the case? The logical relation (1) may be of some use in answering the latter question, since (1) implies that for any coherent assignment of probabilities we have:

(4) $pr(B) \geq 1 - (\Sigma_i(1 - pr(A_i)))$.

And (4) will often provide a useful lower bound on the $pr(B)$—*but not always*. In case $A_1, \ldots, A_n$ are inconsistent, then even if each separately is vouched for by some usually reliable source of information, on any coherent assignment of probabilities we must have:

(5) $\Sigma_i(1 - pr(A_i)) \geq 1$,

which makes (4) useless.

In short, the danger lies not in the FBI's recognizing that ''John Doe's car is red'' and ''John Doe's car is blue'' imply ''John Doe is Public Enemy Number One'', but rather in the possibility that the FBI will make an incoherent assignment of probabilities (perhaps by accepting everything fed into its computers as 100% certain, or perhaps by making the more subtle mistake of assuming that a statement vouched for by a source of information that is right 90% of the time must be assigned probability 90%).

***4 Read on relevance: The first example*** In my note [2] I presented two counterexamples to the paleorelevantist position. The first concerned a card game (illustrating the logical principles of a popular board game, without the distracting features that make the board game popular). One of $n$ black cards is selected randomly, and one of $n$ red cards is selected randomly. The object of the game is to guess the ''mystery pair'' thus selected with the help of clues of the form ''the mystery pair is *not* such-and-such a black card with thus-and-so a red card''.

Read, in his reply [6], seems to have misunderstood the game. For he seems

to suppose that the only way to guess the mystery pair is by an exhaustive process of elimination, collecting $n^2 - 1$ clues ruling out each of the other pairs. He claims that the situation I described in my example, where halfway through the game a player has succeeded in determining one of the mystery cards but not the other, cannot arise. For this reason *he does not treat my example*, but substitutes a simpler example of his own. That Read's claim is false will be seen by anyone who takes the trouble to play the card game as I described it (or—what will be more enjoyable—the board game on which the card game was based, marketed under the trade-name CLUE). Nonetheless, I will oblige Read by accepting the substitution of a simpler example for my original one: To simplify drastically, suppose $n = 2$.

There are now just two black cards, say a spade and a club, exactly one of which is a mystery card, and just two red cards, say a heart and a diamond, exactly one of which is a mystery card. We can let $p$, $\sim p$, $q$, $\sim q$ symbolize the statements that the spade, the club, the heart, and the diamond, respectively, are mystery cards. In this simplified situation, the mystery pair would indeed be guessed by process of elimination. The winning player's argument might be:

$$\begin{array}{l} \sim(p \,\&\, q) \\ \sim(p \,\&\, \sim q) \\ \underline{\sim(\sim p \,\&\, q)} \\ \therefore \sim p \,\&\, \sim q \end{array} .$$

This is a perfectly commonsensical argument, but it is typical of the classically valid arguments that are relevantistically invalid (or "muddle-headed... too artificial to sustain the demands of thought... fairy tales... utterly devoid of rationality" in the rhetoric of [1]). Relevantistically, the argument above can be made valid only by weakening the conclusion to:

$$(\sim p \,\&\, \sim q) \vee (p \,\&\, \sim p) \vee (q \,\&\, \sim q)$$

or else by adding an extra premise, expressing the *relevance* of $p$ to $q$:

(*) $(p \,\&\, q) + (p \,\&\, \sim q) + (\sim p \,\&\, q) + (\sim p \,\&\, \sim q)$.

Read seems to suppose that this extra premise will be available in the card-game example. I would have thought not. For the black mystery card was chosen by one application of a fair random device, the red mystery card by another. In probability and statistics this would be a paradigm of *independence,* and I would have thought it to be a paradigm also of *irrelevance* as understood by Anderson and Belnap: which black card is chosen has nothing whatsoever to do with which red card is chosen.

Read's position is that it is "a deep and important mistake" to attempt, as I have just done, to *begin* by identifying whether a relevant connection exists and *end* by judging on that basis whether an argument is relevantistically valid. Rather, one must *begin* by judging—by commonsense standards, presumably—whether the argument is valid, and *end* by judging on that basis whether a relevant connection is present. *Since* it is just common sense to argue: "*These* are all the possible cases, so if all but one are ruled out, that one must obtain", *therefore* the extra premise (*) above must hold, however little one may

be able to identify an objective connection of relevance between which black card is chosen and which red card is chosen. So Read argues.

As indicated in Section 2 above, a relevantist can indeed block not only the card game example but all the counterexamples suggesting a contradiction between relevantism and common sense, provided he is willing to abandon the thesis that connections of relevance should be objectively identifiable. But if this is Read's strategy, then he ought to acknowledge that I already clearly indicated in my earlier note ([2], p. 102) that the examples there were not directed against the subjectivist position, but against Anderson and Belnap. In any case, as indicated in Section 2, the subjectivist still faces the problem of providing some *evidence* for the hypothesis of an extensional/intensional ambiguity of disjunction.

***5 Read on relevance: The second example*** The second example in my note [2] was a mathematical one. It was in two parts, of which Read treats only one. The consideration of mathematical examples in an attempt to refute paleo-relevantism is legitimate, because Anderson and Belnap committed themselves to the compatibility of relevantism not only with common sense, but also with traditional and current mathematical practice. (Pertinent passages from [1] were cited in [2]; Read does not challenge my interpretation of them.)

I described a hypothetical situation where I claimed a number theorist would be in a position to assert $A(n) \vee B(n)$ but not to assert $A(n) + B(n)$. Read seems to accept this claim. I further claimed that nonetheless it would be standard mathematical practice, both currently and traditionally, for such a number theorist to argue:

(*) $A(n) \vee B(n), \sim A(n) / \therefore B(n).$

Read *does not treat the question* whether a number theorist *would* so argue. He merely claims that a number-theorist *ought not* to argue thus, and that a valid argument could only be obtained by weakening the conclusion:

(**) $A(n) \vee B(n), \sim A(n) / \therefore B(n) \vee (A(n) \ \& \ \sim A(n)).$

Now I submit that it is no part of standard mathematical practice, currrently or traditionally, to argue as in (**). Open any issue of the *Annals of Mathematics* and you will find that mathematicians never pepper their theorems with caveats of the form "unless $n$ is both perfect and not perfect" or "unless there are both more and fewer than $n$ solutions to the equation" or "unless $n$ is a composite prime", as (**) suggests.

Indeed, recent work of R. K. Meyer (unpublished) on "relevant arithmetic" demonstrates beyond doubt that standard mathematical arguments cannot be formalized relevantistically, just as I was claiming in my second example in [2]. For half of the most famous theorems of elementary number theory (including the theorem that every integer is a sum of four squares) it seems that no relevantistically acceptable proof is known, for all Meyer's work. For the other half of the most famous theorems of elementary number theory (including the theorem that no cube is a sum of two cubes) relevantistic proofs are available, but they

cannot be regarded as formalizations of standard proofs. They involve carrying along caveats of the form "unless 0 = 1", which one would never have heard from the lips of Fermat or Euler or Lagrange or Gauss or Liouville, and eliminating these caveats at the end of the proof by various manipulations. Meyer's manipulations are undeniably clever, but the very need for such cleverness demonstrates that relevantism conflicts with standard mathematical practice.

It seems that a relevantist *must* be a revisionist, at least so far as mathematics is concerned, and that Read is prepared to accept this. But if Read's strategy is to adopt a revisionist position, then he ought to acknowledge that I clearly indicated in my earlier note ([2], p. 104) that I was concerned there with the question of the agreement of relevantism with common sense and standard mathematical practice. My examples were directed against Anderson and Belnap, and not against those who are prepared to say, "Of course, mathematicians *do* argue that way; I just say they *oughtn't* to". In any case, as indicated in Section 4 above, the revisionist still faces the problem of providing some *motivation* for the proposal to change our long-established practices.

To conclude, Read makes no serious attempt to defend the original Anderson-Belnap version of relevantism, which is what I was attacking in [2]. In his treatment of my first example, he seems to tend towards subjectivism, while in his treatment of my second example, he tends towards revisionism. Both these neorelevantist positions face problems which Read does little to solve.

## REFERENCES

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