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Opposition

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In two previous essays ([2] and [3]) I argued that the logical relations represented by the square of opposition hold for all category correct sentences. The ideas there were derived in large measure from the work of Fred Sommers (especially [13], [14], and [15]). During the past fifteen years Sommers has developed a simple calculus for syllogistic which puts the old logic on a competitive footing with the modern predicate calculus. I have traced the development of the new syllogistic in [9]. Sommers' most recent and extensive defense of the new syllogistic [19] provides both opportunity and stimulation to extend my previous remarks on opposition.

In [6] I argued that the general form of an assertion is

every/some (non)*S* is/isn't (non)*P*.

In other words, each such sentence is categorical, consisting of a subject and a predicate. A subject is a universally or particularly quantified term and a predicate is a term affirmed or denied of a subject. Any subject-term and any predicate-term may or may not be negated. Two sentences are "primitive" contradictories of one another whenever one denies of a given subject just what the other affirms of that subject. Consider a sentence of the form

(1) some *S* is *P*.

The primitive contradictory of (1) is

(2) some *S* isn't *P*.

Now (2) must not be confused with

(3) Some *S* is non*P*.

For (3), like (1), is not a denial but an affirmation. It affirms a negative predicate-term of *some S*. The "n't" in (2) forms the contradictory of (1). We could read (2) as 'not: some *S* is *P*', or 'not an *S* is *P*' (cf. "not a creature was

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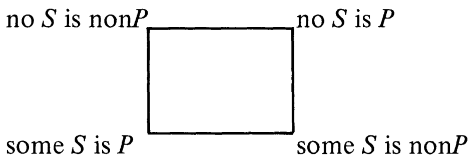
stirring"). In English, such sentences are normally rendered as

(4) no *S* is *P*.

This suggests the following square of opposition, where each contradictory pair is primitive.



The rule governing primitive opposition is the standard law of excluded middle. Since modern logicians recognize only sentential negation, the law is usually rendered as

either *p* or ~*p*.

But, given the general categorical syntax of all assertoric sentences, it is better expressed as

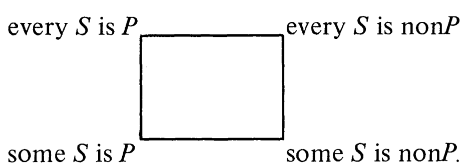
SLEM either some *S* is *P*/non*P* or no *S* is *P*/non*P*.

This Sentential Law of Excluded Middle is relatively uncontroversial and holds universally.

What is controversial is another law, usually confused with SLEM, and one not universally in force. A sentence like (4) is very often taken to be equivalent to one like

(5) every *S* is non*P*

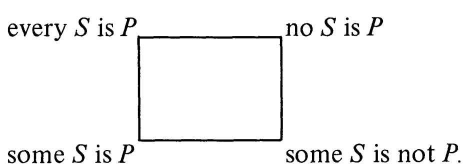
by obversion. Thus (1) and (5) are, like (1) and (4), taken to be contradictory. This suggests a second square of opposition where the contradictory of 'some/every *S* is *P*' is 'every/some *S* is non*P*'. Sommers calls this "proper" or "diagonal" opposition, giving the diagonal square:



Now the law governing this kind of opposition is the Diagonal Law of Excluded Middle.

DILEM either every/some *S* is *P* or some/every *S* is non*P*.

Notice that the traditional square of opposition is a mixture of the primitive and diagonal squares (with 'non' normally replaced by 'not').



What is the relationship between primitive and diagonal opposition? Are the two squares equivalent? The answer is: often, but not always. For, while SLEM is universally applicable, DILEM is not. Sommers has argued very effectively, in Chapter 14 of [19], that there are a variety of kinds of sentences for which DILEM fails. Let us call such sentences *vacuous.* There are, in general, three kinds of vacuous sentences: those with vacuous (empty) subjects, those whose subjects are underdetermined with respect to their predicates, and category mistakes. I have discussed the first and third kinds in [1].

Consider a sentence like (6), which has a vacuous subject.

(6) Some unicorns are blue.

The primitive contradictory of (6) is

(7) No unicorns are blue.

Given that (6) is false and the SLEM holds (i.e., the primitive contradictory of a false sentence is true), (7) must be true. As well, since (6) is false because it has a vacuous subject, so is

(8) Some unicorns are nonblue,

making

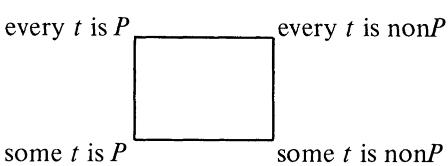
(9) No unicorns are nonblue.

true. Now (7) and (9) are the primitive contradictories of (6) and (8), respectively. The diagonal contradictories of (6) and (8) are

(10) Every unicorn is nonblue.
(11) Every unicorn is blue.

Like (6) and (8), the vacuousity of the subject makes (10) and (11) false. So DILEM cannot hold.

Before continuing our examination of opposition for various kinds of vacuous sentences it would be helpful to look briefly at how sentences with singular terms fit the squares. Given any general term, *T*, a sentence with universally quantified *T* implies the corresponding sentence with particularly quantified *T*. In other words, 'every *T* is *P*' implies 'some *T* is *P*'. This is the familiar syllogistic principle of subalternation. What is unique about singular terms is that not only does the universal imply the particular but the particular implies the universal as well. Given a singular term *t*, 'every *t* is *P*' is logically equivalent to 'some *t* is *P*'. This means, in effect, that singular subject-terms *are*, contrary to surface appearances, logically quantified. But it doesn't matter which quantity. Sommers calls such singular subjects "wild" in quantity (see [4], [5], [7], [8], [11], [12], [16]-[18], and [20]). The notion of wild quantity for singulars was first noted by Leibniz (see [10]). What this means, then, is that a square like



when t is singular, collapses into just

every/some t is P_______ every/some t is non$P$.

Now consider the vacuous sentence

(12) The present king of France is bald.

Contemporary logicians take

(13) The present king of France is not bald.

to be the contradictory of (12). But (13), is, as Russell saw, ambiguous between

(13.1) The present king of France is nonbald.

and

(13.2) The present king of France isn't bald.

(Admittedly, Russell attributed the ambiguity to the scope rather than the sense of 'not'.) Sentence (13.2) is the primitive contradictory of (12). But (13.1) is merely the diagonal contradictory of (12). To see this remember that singular subjects are logically wild in quantity. We can take them to be either universal or particular as we wish. Let (12) have the logical form

(12′) some K is B.

The diagonal contradictory of (12′) is

(14) every K is non$B$

We can take (14) to be the logical form of (13.1). Since SLEM always holds, either the present king of France is bald or he isn't; but his failure to exist means that he is neither bald nor is he nonbald (i.e., DILEM doesn't hold).

We have thus far seen that SLEM holds in all cases, but that DILEM fails for sentences with vacuous (singular or general) subjects. Sommers has argued that DILEM fails as well for sentences whose subjects are underdetermined with respect to their predicates. Consider

(15) Some men will live on Venus in the next century.

The subject of (15) is underdetermined (indeed, undetermined) with respect to the predicate. We just don't know yet. The same, of course, holds for

(16) Some men will fail to live on Venus in the next century.

(where 'fails to live . . .' is the term negation of 'live . . .'). What now of

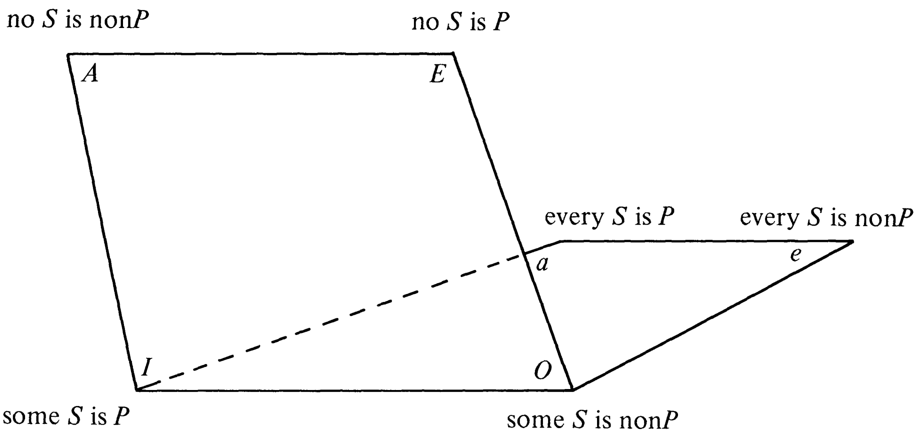
(17) Every man will live on Venus in the next century.

and

(18) Every man will fail to live on Venus in the next century.

These are the diagonal opposites of (15) and (16). Sommers has argued that sentences like (15) and (16) are false and that their diagonal contradictories are therefore undefined. Thus DILEM cannot hold in such cases. But SLEM still does hold. While we cannot now know that some men will live on Venus in the next century or every man will fail to live on Venus in the next century, we do know now that either some men will or no men will.

Sentences with vacuous subject or subjects underdetermined with respect to their predicates have primitive contradictories, but their diagonal contradictories are undefined. They satisfy SLEM and can be displayed on primitive squares, but they do not satisfy DILEM, and so cannot be displayed on diagonal squares. These facts can be illustrated with the following:



Here *AEIO* is a primitive square and *aeIO* is a diagonal square. When *I* and *O* are nonvacuous, both squares apply and, indeed, are identical. For nonvacuous sentences $A = a$ and $E = e$ (note that when a nonvacuous sentence has a singular subject $A = a = I$ and $E = e = O$). For vacuous sentences only the primitive, *AEIO*, square holds. Again, the primitive square is constructed according to SLEM, the diagonal square according to DILEM.

We have yet to consider the third kind of vacuous sentence: category mistakes. Let '*x* is /*P*/' read '(non)*P* can be sensibly, category correctly, predicated of *x*' (see [13]). A sentence like

(19) every/some *S* is (non)*P*

is category correct if and only if

CLEM some *S* is /*P*/

is true. Sommers calls this the Categorial Law of Excluded Middle. It holds only for category correct sentences. CLEM must not be confused with another law: the Predicative Law of Excluded Middle.

PLEM either every/some *S* is *P* or every/some *S* is non*P*

PLEM implies CLEM, but is not implied by CLEM. Consider

(20) Kripke will live on Venus in the next century.

Sentence (20) is category correct. It has the form

(20′) *K* is *V*.

Since it is not a category mistake (Kripke is the sort of thing that can sensibly be said to live on Venus, etc.) (21) is true.

(21) *K* is /*V*/.

Nonetheless, both (20) and

(22) Kripke will fail to live on Venus in the next century.

are false (since for now, presumably, Kripke is undetermined with respect to his planet of residence in the next century). PLEM holds for nonvacuous sentences. CLEM holds for nonvacuous sentences and for vacuous but category correct sentences too. Thus

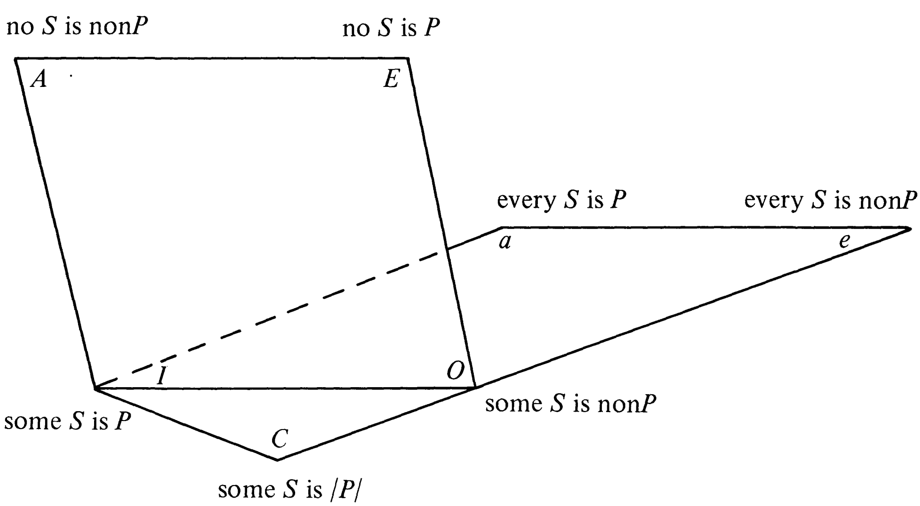
(23) 2 will live on Venus in the next century.

is a category mistake since CLEM does not hold, i.e.,

(24) 2 is /*V*/

is false.

A final summary now by use of the following diagram:



When *C* is true (i.e., CLEM) then either *I* or *O* (i.e., PLEM). When PLEM holds then DILEM holds. SLEM always holds.

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