Notre Dame Journal of Formal Logic
Volume 24, Number 2, April 1983

Completeness of an Ecthetic Syllogistic

ROBIN SMITH

In this paper I study a formal model for Aristotelian syllogistic which includes deductive procedures designed to model the "proof by ecthesis" that Aristotle sometimes uses and in which all deductions are direct. The resulting system is shown to be contained within another formal model for the syllogistic known to be both sound and complete, and in addition the system is proved to have a certain limited form of completeness.

1 Background This paper follows [4] and [9] in treating Aristotle's syllogistic as a natural deduction system for categorical propositions. In Mates's terminology [7], a *premiss-conclusion argument* (P-c argument) is a set of premisses and a conclusion. If the premisses imply the conclusion, the argument is valid. Aristotle defines a syllogism as "a discourse in which, certain things being posited, something different from the things posited follows of necessity because of their being so" (*Prior Analytics A*.1, 24b18-20). It is clear from this that every syllogism contains a valid P-c argument; also, following Corcoran [3], [4] and Smiley [9], every valid P-c argument is a syllogism. A *perfect* or *complete* syllogism is a discourse which makes it evident that a certain conclusion follows from certain premisses. For some P-c arguments (i.e., the first-figure syllogisms) this is already evident, so that these constitute perfect syllogisms by themselves. A valid P-c argument which is not evidently valid is an *imperfect* or *incomplete* syllogism; if further discourse be added to such an argument which makes its validity evident, then the result is a *perfected* or *completed* syllogism. Thus, a perfect syllogism is a deduction, and the process of completing an imperfect syllogism is the process of constructing a deduction of its conclusion from its premisses. For the details of this terminology and the interpretation of Aristotle which it reflects see [4], pp. 90-94. *Prior Analytics A* 4-7 gives deduction schemata with which to accomplish this for syllogisms in the various Aristotelian moods together with counterexamples to reject other

Received May 8, 1981; revised January 25, 1982

combinations of premisses as 'nonsyllogizing'. On my interpretation (for the details of which see [11]), these schemata, which use letters for terms, amount to metalinguistic deducibility proofs: each shows that premisses of a certain form imply a conclusion of a certain form by showing how to deduce such a conclusion from those premisses. (I agree with [3] and [4] that every syllogism is concrete.)

The principal deduction system used in the *Prior Analytics* has seven rules of inference, corresponding to the four first-figure moods and the three conversion laws, and two types of deductions, viz., "direct" and "indirect". Aristotle also knows that the particular-conclusion moods of the first figure (Darii and Ferio) are derivable from the remaining rules, so that a simpler system is possible. Using plausible formal models for these systems, Corcoran has shown them to be sound and complete [3]. Proof by "ecthesis", or "setting out", is used several times by Aristotle in giving alternative deduction schemata for completing certain syllogistic moods (28a22-26, 28b14-19, 28b20-21); in one case (30a9-13) it is the only procedure used. Commentators have generally discerned ecthesis in another important passage (25a14-17). Aristotle never explains why he includes these alternative deductions and there is some debate about exactly what the procedure is (see [8], [11], and [12]). Most formal models for the syllogistic have not included a means of representing ecthetic proof, although several models for ecthetic proof have been proposed.

I show here that if ecthetic rules are added to a model for the syllogistic, indirect deductions may be dispensed with. That is, I show that for any consistent set S of categorical propositions and any proposition p, if S implies p then p can be deduced from S by a direct deduction in the ecthetic system. It is impossible to say whether Aristotle realized this or not: he does use ecthesis in some cases in which he also gives a *per impossibile* deduction, but this is not uniform.[1] The resulting system may be regarded as conceptually simpler, however, in that no indirect deductions are required. As a further point of interest, the ecthetic system avoids one of the so-called paradoxes of classical logic: if S is inconsistent, it is not in general possible to derive an arbitrary proposition p from S in the ecthetic system. Such a point may have appealed to Aristotle, although from a modern standpoint it indicates that the ecthetic system is weaker than a system including indirect deductions.

I make use here of a formal system S which is equivalent to the D of [3] and [4]. The vocabulary of S consists of a nonempty set of *terms* $\{a, b, c \ldots\}$ and four *constants* A, E, I, O; the wffs of S are all strings of a constant followed by two distinct terms. These wffs are interpreted traditionally: Aab may be read 'All a is b' or 'b belongs to all a', Eab 'no a is b' or 'b belongs to no a', Iab 'some a is b' or 'b belongs to some a' and Oab 'not all a is b' or 'b does not belong to all a'. I use α, β, γ, etc., as metavariables for terms. Formally, an *interpretation* $\mathcal{J}$ of any set Γ of wffs of S is a function defined as follows: if α, β occur in some $p \in \Gamma$, then (1) $\mathcal{J}(\alpha)$, $\mathcal{J}(\beta)$ are each sets $\neq \wedge$; (2) $\mathcal{J}(A\alpha\beta) = T$ iff $\mathcal{J}(\alpha) \cap \mathcal{J}(\beta) = \mathcal{J}(\alpha)$; (3) $\mathcal{J}(E\alpha\beta) = T$ iff $\mathcal{J}(\alpha) \cap \mathcal{J}(\beta) = \wedge$; (4) $\mathcal{J}(I\alpha\beta) = T$ iff $\mathcal{J}(\alpha) \cap \mathcal{J}(\beta) \neq \wedge$; (5) $\mathcal{J}(O\alpha\beta) = T$ iff $\mathcal{J}(\alpha) \cap \mathcal{J}(\beta) \neq \mathcal{J}(\alpha)$; (6) $\mathcal{J}(p) = F$ iff $\mathcal{J}(p) \neq T$. We define $\overline{A\alpha\beta} = O\alpha\beta$, $\overline{I\alpha\beta} = E\alpha\beta$, $\overline{\overline{p}} = p$; it is evident from (1)-(6) that $\mathcal{J}(\overline{p}) = T$ iff $\mathcal{J}(p) = F$. An interpretation *satisfies* a set of wffs Γ iff $\mathcal{J}(p) = T$ for every $p \in \Gamma$. If $\mathcal{J}(p) = T$ for every $\mathcal{J}$ which satisfies Γ, I write $\Gamma \models p$.

The rules for S are as follows:

Bar	$A\alpha\beta, A\beta\gamma \vdash A\alpha\gamma$
Cel	$A\alpha\beta, E\beta\gamma \vdash E\alpha\gamma$
Dar	$I\alpha\beta, A\beta\gamma \vdash I\alpha\gamma$
Fer	$I\alpha\beta, E\beta\gamma \vdash O\alpha\gamma$
EC	$E\alpha\beta \vdash E\beta\alpha$
IC	$I\alpha\beta \vdash I\beta\alpha$
AC	$A\alpha\beta \vdash I\beta\alpha$.

A deduction of p from Γ is a sequence $q_1, \ldots, q_n$ of one of the following sorts:

(1) the sequence begins with a sequence of elements of Γ; each subsequent element is either identical with a previous element or a consequence of previous elements by S1-S4; and the last element is p.

(2) the sequence begins as in (1); then comes $\bar{p}$; then each subsequent element is a consequence of previous elements as above; and the last two elements are $q, \bar{q}$ for some q.

If there is a deduction of p from Γ in S, I write $\Gamma \vdash_S p$. Corcoran has shown S to be complete and to reflect Aristotle's own procedure in dealing with reductions. The procedure of ecthesis concerns only particular (I and O) propositions. Its most characteristic feature is the introduction of a new "term" (the "term set out", *to ektithemenon*) according to the following patterns:

IA If A belongs to some B, then there is a C to which A and B both belong.

IB If A does not belong to some B, then there is a C to which A does not belong and to which B belongs.

These theses, which correspond very closely to expressions found in Aristotle's text, (cf. 28a24-25, 28b21, 30a9-10) may be given two interpretations according to the semantical category of the term C introduced. If we regard C as a syllogistic term (i.e., a universal), then *IA* asserts that there is a C to *all* of which A and B both belong (i.e., A and B have a common part), while *IB* asserts that there is a C to *none* of which A belongs and to *all* of which B belongs (i.e., some part of A is disjoint from B). In what follows, I shall make use of this latter interpretation, which has been advocated in recent times by Łukasiewicz [8] and Patzig [9] (although it is found in Alexander [1] at least as an alternative). However, it is also possible to regard C as an individual (i.e., if A and B are classes, as a member of those classes), in which case no quantifiers need be added to *IA* and *IB*. Thom [12] shows that ecthesis may be formally modelled as a restricted rule of inference somewhat analogous to existential instantiation, and I so treat it. I define the system *SE* as follows. *SE* contains all the rules of S and in addition the following:

I-Int	$A\gamma\alpha, A\gamma\beta \vdash I\alpha\beta$	
O-int	$A\gamma\alpha, E\gamma\beta \vdash O\alpha\beta$	
I-elim	$I\alpha\beta \vdash A\gamma\alpha, A\gamma\beta$	restriction: γ occurs neither in the premisses nor previously in the deduction.

O-elim $O\alpha\beta \vdash A\gamma\alpha, E\gamma\beta$ restriction: γ occurs neither in the premisses nor previously in the deduction.

The definition of 'deduction of p from Γ in *SE*' is identical to that for 'direct deduction' in *S* with the following additions:

(1) add I-int and O-int to the two-premiss rules
(2) add I-elim and O-elim to the one-premiss rules
(3) add the restriction: no term introduced by I-elim or O-elim occurs in the conclusion.

I use $\Gamma \vdash_{SE} p$ with an obvious sense.

Several deduction schemata will illustrate the operation of these rules. I give here possible completions for Baroco, Bocardo, and Disamis.

Baroco	1. $A\alpha\beta$	premisses
	2. $O\gamma\beta$	
	3. $A\delta\gamma$	2, O-elim
	4. $E\delta\beta$	
	5. $E\beta\delta$	4, EC
	6. $E\alpha\delta$	1, 5, Cel
	7. $E\delta\alpha$	6, EC
	8. $I\gamma\delta$	3, AC
	9. $O\gamma\alpha$	7, 8, Fer
Bocardo	1. $O\beta\alpha$	premisses
	2. $A\beta\gamma$	
	3. $A\delta\beta$	1, O-elim
	4. $E\delta\alpha$	
	5. $A\delta\gamma$	2, 3, Bar
	6. $O\gamma\alpha$	4, 5, O-int
Disamis	1. $I\beta\alpha$	premisses
	2. $A\beta\gamma$	
	3. $A\delta\beta$	1, I-elim
	4. $A\delta\alpha$	
	5. $A\delta\gamma$	2, 3, Bar
	6. $I\gamma\alpha$	3, 5, O-int.

The rules Dar and Fer are derivable in the system as the moods Darii and Ferio, as illustrated by the following schema for Ferio:

Ferio	1. $E\beta\gamma$	premisses
	2. $I\alpha\beta$	
	3. $A\delta\alpha$	2, I-elim
	4. $A\delta\beta$	
	5. $E\delta\gamma$	1, 4, Cel
	6. $O\alpha\gamma$	3, 5, O-int.

[John Corcoran has called my attention to the fact that Galen ([5], IX.6, X.8) gives completions very similar to those given above for Baroco and Bocardo.]

One final point should be observed concerning the relationship of ecthesis to other modes of inference. I have said that I take the *Prior Analytics* to be giving deduction schemata, so that his letters are really syntactic metavariables. However, the term introduced by ecthesis (that is, by I-elim or O-elim) does not correspond to any concrete term, in any actual syllogism. For this reason, it might be desirable to treat the term set out in the ecthesis as of a distinct semantic category and introduce some new type of symbol for these terms, as is often done in natural-deduction systems for the terms introduced by applying existential instantiation. I prefer to regard these terms as "unassigned names" (see [11]) and to keep the syntax simpler in order to keep the proof of the theorem simpler.

2 ***The completeness theorem for SE*** I show that *SE* is consistent and (in a restricted sense) complete by proving the following:

Theorem *If* Γ *is consistent,* $\Gamma \vdash_S p$ *iff* $\Gamma \vdash_{SE} p$.

Proof: Here 'Γ is consistent' means '$\Gamma \vdash p$ and $\Gamma \vdash \bar{p}$ for no p'. It will be convenient to take advantage of a further result of [3]: S is equivalent to the system RS obtained by deleting Dar and Fer (Aristotle virtually proves this in *An. Pr. A* 7). I will show that RS is equivalent to the corresponding RSE produced by deleting these rules from SE. First, note that every direct deduction in $S(RS)$ is a deduction in $SE(RSE)$. Therefore, if there is a direct deduction of p from Γ in $S(RS)$, there is a deduction of p from Γ in $SE(RSE)$. Next, since RS is known (cf. [3]) to be sound and complete on the usual interpretation it will be sufficient, to show that $\Gamma \vdash_{SE} p \Rightarrow \Gamma \vdash_S p$, if we show that all the rules of SE are sound, i.e., that if $\Gamma \vdash_{SE} p$ then every interpretation which satisfies Γ makes p true. All rules other than I-elim, O-elim, I-int, and O-int are sound since S is sound, and I-int and O-int are derivable in S (as Darapti and Felapton). It remains to show that O-elim and I-elim are sound. For any interpretation which satisfies all of Γ, let a *δ-extension of* $\mathcal{I}$ (where δ does not occur in Γ; I write '$\mathcal{I}^\delta$') be defined as follows: for all λ which occur in Γ, $\mathcal{I}^\delta(\lambda) = \mathcal{I}(\lambda)$. (In other words, $\mathcal{I}^\delta$ includes $\mathcal{I}$ and is defined at another point.) For $(\mathcal{I}^\delta)^\varepsilon$ I write $\mathcal{I}^{\delta\varepsilon}$. Now consider any sequence $\langle q_1, \ldots, q_n\rangle$ constituting a deduction of q_n from Γ in SE. Let q_i, q_{i+1} be the first propositions inferred by I-elim or O-elim. Then for all q_j, $j < i$, if $\mathcal{I}(\Gamma) = T$ then $\mathcal{I}(q_j) = T$. Suppose q_i, q_{i+1} follow from some q_j by I-elim. Then $q_j = I\alpha\beta$, $q_i, q_{i+1} = A\gamma\alpha$, $A\gamma\alpha$ (γ not found in Γ). Define $\mathcal{I}^\gamma(\gamma) = \mathcal{I}(\alpha) \cap \mathcal{I}(\beta)$. (Since $\mathcal{I}(I\alpha\beta) = T$, we know that $\mathcal{I}(\alpha) \cap \mathcal{I}(\beta) \neq \wedge$). Then obviously $\mathcal{I}^\gamma(A\gamma\alpha) = \mathcal{I}^\gamma(A\gamma\beta) = T$. Similarly, if q_i, q_{i+1} follow by O-elim from $q_j = O\alpha\beta$, they are $A\gamma\alpha$, $E\gamma\beta$. Define $\mathcal{I}^\gamma(\gamma) = \mathcal{I}(\alpha) \sim (\mathcal{I}(\alpha) \cap \mathcal{I}(\beta))$ (since $\mathcal{I}(O\alpha\beta) = T$ and thus $\mathcal{I}(\alpha) \cap \mathcal{I}(\beta) \neq \mathcal{I}(\alpha)$, this is not null). Then clearly $\mathcal{I}^\gamma(A\gamma\alpha) = \mathcal{I}^\gamma(A\gamma\beta) = T$. We continue in this manner, extending $\mathcal{I}^\gamma$ to $\mathcal{I}^{\gamma\gamma'}$, $\mathcal{I}^{\gamma\gamma'\gamma''}$, etc., at each new use of I-elim or O-elim. Thus $\mathcal{I}^{\gamma\cdots\gamma^n}(q_n) = T$. But if $\langle q_i\rangle$ is a deduction of q from Γ, none of the $\gamma \ldots \gamma^n$ can occur in q_n. Therefore, $\mathcal{I}(q_n) = T$, and $\Gamma \vDash q_n$. Therefore, since RS is complete, $\Gamma \vdash_{RS} q_n$.

To show that if $\Gamma \vdash_S p$, $\Gamma \vdash_{SE} p$ for consistent Γ we need only consider the indirect case, since any direct deduction in S is a deduction in SE. Let P be an indirect deduction of p from Γ in RS. Then we may without loss of generality suppose P to be ordered as follows:

1. All premisses from Γ used in the deduction
2. $\bar{p}$
3. The lines of the deduction
4. q
5. $\bar{q}$.

We show how to construct a parallel deduction of p from Γ in *SE*. First, note that since Γ is supposed consistent, $\bar{p}$ must be appealed to in deducing q or $\bar{q}$ (or possibly both): otherwise we should have $\Gamma \vdash q, \bar{q}$. There are four cases, according to the type of p: p may be $A\alpha\beta$, $E\alpha\beta$, $I\alpha\beta$, $O\alpha\beta$. The following lemma will be useful:

Lemma *Let an α-β chain be a set of A propositions $A\alpha\lambda_1, A\lambda_1\lambda_2, \ldots, A\lambda_n\beta$. Then: (1) there is a direct deduction of $A\alpha\beta$ from Γ iff Γ contains an α-β chain; (2) there is a direct deduction of $E\alpha\beta$ from Γ iff Γ contains some $E\gamma\delta$ and either (a) an α-γ chain (unless $\alpha = \gamma$) and a β-δ chain, (unless $\beta = \delta$) or (b) an α-δ chain (unless $\alpha = \delta$) and a β-γ chain (unless $\beta = \gamma$).*

Deductive Steps

1. Suppose first that $p = I\alpha\beta$. Then $\bar{p} = E\alpha\beta$. Now, the only rules of inference having E premisses are Cel and EC. The latter would yield $E\beta\alpha$; the former, with an A premiss $A\gamma\alpha$, would yield $E\gamma\beta$. Successive applications of these rules still will yield only further E propositions. In fact, if $\Gamma \nvdash_S q$ and if $\Gamma + E\alpha\beta \vdash_S q$ by direct deduction, then q must be of the form $E\gamma\delta$. Therefore, at least one of $q, \bar{q}$ is of the form $E\gamma\delta$. In that case, however, the other one of the pair must be of the form $I\gamma\delta$. Now, obviously no E proposition occurs essentially in a direct proof of an I proposition; hence, P contains a deduction of one of $q, \bar{q}$ not utilizing $E\gamma\delta$. Suppose without loss of generality that $q = E\gamma\delta$; then $\bar{q} = I\gamma\delta$, and $\Gamma \vdash I\gamma\delta$, $\Gamma \nvdash E\gamma\delta$, $\Gamma + E\alpha\beta \vdash E\gamma\delta$. Now by the lemma, since $\Gamma \nvdash E\gamma\delta$ and $\Gamma + E\alpha\beta \vdash E\gamma\delta$, there exist two (perhaps zero-length) chains $\langle\gamma, \ldots, \beta\rangle$, $\langle\delta, \ldots, \alpha\rangle$ in Γ. From these chains, by successive uses of Bar, we have $\Gamma \vdash_{\overline{SE}} A\gamma\alpha$, $\Gamma \vdash_{\overline{SE}} A\delta\beta$ or $\Gamma \vdash_{\overline{SE}} A\delta\alpha, A\gamma\beta$. We also have $\Gamma \vdash I\gamma\delta$. The deduction of $I\gamma\delta$ must be direct; hence, $\Gamma \vdash_{\overline{SE}} I\gamma\delta$. Thus, $\Gamma \vdash_{\overline{SE}} A\gamma\alpha, A\delta\beta, I\gamma\delta$ or $\Gamma \vdash_{\overline{SE}} A\gamma\beta, A\delta\alpha, I\gamma\delta$. Consider the first case. To a sequent composed successively of deductions of $A\gamma\alpha, A\delta\beta, I\gamma\delta$ from Γ we add the following:

1. $A\lambda\gamma$	(λ not in Γ)	$I\gamma\delta$, I-elim
2. $A\lambda\delta$		
3. $A\lambda\alpha$		1, $A\gamma\alpha$, Bar
4. $A\lambda\beta$		2, $A\delta\beta$, Bar
5. $I\alpha\beta$		3, 4, I-int

This then completes a deduction of $I\alpha\beta$ from Γ in *SE*. The remaining case requires only some uses of IC.

2. Suppose next that $p = O\alpha\beta$, so that $\bar{p} = A\alpha\beta$. Now, the deductions of q and $\bar{q}$ must both be direct. Therefore, neither q nor $\bar{q}$ can be of the form $O\gamma\delta$, since no direct rule has an O conclusion. Therefore, suppose without loss of generality that $q = E\gamma\delta$, $\bar{q} = I\gamma\delta$. Again, since Γ is consistent, $A\alpha\beta$ must be used

in the deduction of at least one of these. There are three cases:

(a) $\Gamma \vdash E\gamma\delta$ and $\Gamma \nvdash I\gamma\delta$
(b) $\Gamma \nvdash E\gamma\delta$ and $\Gamma \vdash I\gamma\delta$
(c) $\Gamma \nvdash E\gamma\delta$ and $\Gamma \nvdash I\gamma\delta$.

In Case (a), $I\gamma\delta$ must be inferred either from some $A\delta\gamma$ by AC or from some $A\gamma\delta$ by AC and IC. Since by hypothesis $\Gamma \nvdash I\gamma\delta$, therefore $\Gamma \vdash A\gamma\delta$ or $\Gamma \vdash A\delta\gamma$. Now, $A\gamma\delta$ follows from a set Γ iff some chain $\langle\gamma, \ldots, \delta\rangle \subseteq \Gamma$. Thus, no γ-δ chain is in Γ, but such a chain is in $\Gamma + A\alpha\beta$; therefore, Γ must contain a γ-α chain and a β-δ chain. We then have the following:

1.	$\Gamma \vdash A\gamma\alpha$	
2.	$\Gamma \vdash E\gamma\delta$	
3.	$\Gamma \vdash A\beta\delta$	
4.	$\Gamma \vdash E\delta\gamma$	EC, 2
5.	$\Gamma \vdash E\beta\gamma$	3, 4, Cel
6.	$\Gamma \vdash E\gamma\beta$	5, EC
7.	$\Gamma \vdash O\alpha\beta$	1, 6, O-int

A similar proof obtains if instead $\Gamma \vdash A\delta\gamma$. In Case (b), $\Gamma + A\alpha\beta$ must contain some $E\gamma'\delta'$ and γ-γ', δ-δ' chains. But since $\Gamma \vdash E\gamma\delta$, some one of these conditions must fail in Γ. It cannot be $E\gamma'\delta'$; thus, $A\gamma\beta$ must complete one of the chains γ-γ', δ-δ'. Let it be γ-γ'. Then Γ contains a γ-α chain and a β-γ' chain. We also have $\Gamma \vdash I\gamma\delta$. Thus, we have the following:

1. $\Gamma \vdash A\gamma\alpha$
2. $\Gamma \vdash A\delta\delta'$
3. $\Gamma \vdash A\beta\gamma'$
4. $\Gamma \vdash E\gamma'\delta'$
5. $\Gamma \vdash I\gamma\delta$

We now complete the deduction of $O\alpha\beta$ as follows.

6.	$A\lambda\gamma$	I-elim (λ not in Γ)
7.	$A\lambda\delta$	I-elim (λ not in Γ)
8.	$A\lambda\delta'$	2, 7, Bar
9.	$E\delta'\gamma'$	4, EC
10.	$E\lambda\gamma'$	8, 9, Cel
11.	$E\gamma'\lambda$	10, EC
12.	$E\beta\lambda$	3, 11, Cel
13.	$E\lambda\beta$	12, EC
14.	$A\lambda\alpha$	1, 6, Bar
15.	$O\alpha\beta$	13, 14, O-int.

A similar proof obtains if $A\alpha\beta$ completes the δ-δ' chain; likewise, if EC must be used to derive $E\gamma\delta$ from $E\delta'\gamma'$.

Case (c) is impossible. For in this case $A\alpha\beta$ must close two different chains. For instance, Γ may contain some α-γ and β-δ chains (for $A\gamma\delta$) and also some $E\gamma'\delta'$ with $A\alpha\beta$ closing a γ-γ' chain and with a δ-δ' chain. In this case Γ contains a β-γ' chain, a β-δ chain, a δ-δ' chain, and $E\gamma'\delta'$: hence $\Gamma \vdash A\beta\gamma'$,

$A\beta\delta'$, $E\gamma'\delta'$, and hence $E\beta\delta'$; but by *AC* and *IC*, $\Gamma \vdash I\beta\delta'$, and Γ is thus inconsistent, contradicting the hypothesis. All other possibilities are similarly ruled out.

3. Suppose now that $p = A\alpha\beta$, so that $\bar{p} = O\alpha\beta$. Since no direct rule uses an O premiss, $O\alpha\beta$ is not used either to deduce q or to deduce $\bar{q}$. Hence $\Gamma \vdash q$, $\Gamma \vdash \bar{q}$, contradicting the hypothesis.

4. If $p = E\alpha\beta$: reasoning identical to Case 3 except that IC uses an I premiss. However, a survey of the rules shows that an I premiss cannot be used to deduce a universal proposition, so one of q, $\bar{q}$ is derived directly from Γ. Moreover, if $I\alpha\beta$ is used in the derivation of either q or $\bar{q}$, then since IC can yield only $I\alpha\beta$ and $I\beta\alpha$, either q or $\bar{q} = I\alpha\beta$. But then $\Gamma \vdash E\alpha\beta$ directly.

Proof of Lemma: (1) if Γ contains an α-β chain, then $\Gamma \vdash_S A\alpha\beta$ by repeated applications of Bar. If $\Gamma \vdash_S A\alpha\beta$ directly, then since only Bar has an A conclusion, $\Gamma \vdash_S A\alpha\gamma$, $\Gamma \vdash_S A\gamma\beta$ for some γ. If both $A\alpha\gamma, A\gamma\beta \in \Gamma$, then we have an α-β chain $\langle A\alpha\gamma, A\gamma\beta\rangle$; if either Γ, applying the previous arguments yields some $\delta(\delta')$ such that $A\alpha\delta$, $A\delta\gamma(A\gamma\delta', A\delta'\beta)$, thus giving an α-β chain $A\alpha\delta$, $A\delta\gamma$, $A\gamma\beta(A\alpha\gamma$, $A\gamma\delta'$, $A\delta'\beta)$, etc. Since deductions are always finite, we eventually reach an α-β chain of propositions in Γ. (2) The "if" clause is obvious by repeated applications of Bar, Cel and (in Case (b)) EC. To prove the "only if" case, note that if $E\alpha\beta \in \Gamma$ or $E\beta\alpha \in \Gamma$, the theorem is trivial. Suppose therefore that $E\alpha\beta, E\beta\alpha \notin \Gamma$. Then $E\alpha\beta$ in the deduction is inferred by Cel from $A\alpha\gamma$, $E\gamma\beta$ or by Cel, EC from $A\beta\gamma$, $E\gamma\alpha$. $\Gamma \vdash_S A\alpha\gamma(A\beta\gamma)$ iff Γ contains an α-$\gamma(\beta$-$\gamma)$ chain, by (1): if $E\gamma\beta(E\gamma\alpha) \in \Gamma$, the theorem is proved. If not, applying the foregoing arguments to $E\gamma\beta(E\gamma\alpha)$ produces either $A\gamma\delta$, $E\delta\beta$ or $A\beta\delta$, $E\delta\gamma(A\gamma\delta$, $E\delta\alpha$ or $A\alpha\delta$, $E\delta\gamma)$ from which $E\gamma\beta(E\gamma\alpha)$ is inferred by Cel and possibly EC. In the first case, $\Gamma \vdash_S A\gamma\delta$ iff Γ contains a γ-δ chain; adding this to the α-$\gamma(\beta$-$\gamma)$ chain which must be present if $\Gamma \vdash_S A\alpha\gamma(A\beta\gamma)$ yields an α-$\delta(\beta$-$\delta)$ chain. In the second case, $\Gamma \vdash_S A\beta\delta(A\alpha\delta)$ iff Γ contains a β-$\delta(\alpha$-$\delta)$ chain. If $E\delta\beta$ or $E\delta\gamma(E\delta\alpha$ or $E\delta\gamma)$ is not in Γ, repeating the argument again will discover an extension to either the α-$\gamma(\beta$-$\gamma)$ or the β-$\delta(\alpha$-$\delta)$ chain and a new E proposition. Since all deductions are finite, we eventually reach some E proposition in Γ, the terms of which are linked to α, β by chains, QED (The proof of this lemma is based on arguments Aristotle uses in *Posterior Analytics A* 20-21.)

It is worth noting that each of the deduction strategies given here uses exactly one application of I-elim or O-elim and one application of I-int or O-int. Since all other deductions can be accomplished directly in *RS* (and thus with no applications of the ecthetic rules), it follows that every deduction can be accomplished with at most one use of one of each pair of ecthetic rules. Thus, "iterated" or "nested" uses of these rules are redundant.

NOTE

1. Another reason should be noted: in [8], p. 156, Patzig says that Aristotle needs ecthesis for the completions of Baroco and Bocardo with necessary premisses in *An. Pr. A* 8 (he is following Alexander: cf. [1], 121, 2-9). Aristotle's difficulty is that he cannot use

indirect deductions here because the denial of the desired conclusion "Necessarily some C is not A" is "Possibly all C is A", and syllogisms with one necessary and one contingent premiss have not been studied at chapter 8. In the case of Bocardo, however, the *reductio* yields Barbara with necessary minor and contingent major premisses, which in 36a2-7 Aristotle says give a "perfect syllogism" with a contingent conclusion. This detail, like most concerning Aristotle's modal syllogistic, may not bear up under careful scrutiny.

REFERENCES

[1] Alexander of Aphrodisias, *In Primum Aristotelis Analyticorum Priorum Commentaria. Commentaria in Aristotelem Graeca* II, 1.

[2] Clark, Michael, *The Place of Syllogistic in Logical Theory,* University of Nottingham Press, Nottingham, 1980.

[3] Corcoran, John, "Completeness of an ancient logic," *The Journal of Symbolic Logic,* vol. 37 (1973), pp. 696-702.

[4] Corcoran, John, "Aristotle's natural deduction system," in *Ancient Logic and Its Modern Interpretations,* ed., J. Corcoran, D. Reidel, Dordrecht, 1974, pp. 85-131.

[5] Galen, *Institutio Logica* (Εἰσαγωγὴ Διαλεκτική), ed. C. Kalbfleisch, B. G. Teubner, Leipzig, 1896.

[6] Łukasiewicz, Jan, *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic,* second edition, Clarendon Press, Oxford, 1957.

[7] Mates, Benson, *Stoic Logic.* University of California Press, Berkeley and Los Angeles, 1961.

[8] Patzig, Günther, *Aristotle's Theory of the Syllogism,* tr. Jonathan Barnes, D. Reidel, Dordrecht, 1968.

[9] Smiley, T. J., "What is a syllogism?" *Journal of Philosophical Logic,* vol. 2 (1973), pp. 136-154.

[10] Smith, Robin, "The syllogism in *Posterior Analytics I*," *Archiv für Geschichte der Philosophie*, vol. 64 (1982), pp. 113-135.

[11] Smith, Robin, "What is Aristotelian ecthesis?," *History and Philosophy of Logic,* vol. 3 (1982), pp. 113-127.

[12] Thom, Paul, "Ecthesis," *Logique et analyse,* vol. 74-76 (1976), pp. 299-310.

Department of Philosophy
Kansas State University
Manhattan, Kansas 66506