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# On Measures and Distinguishability

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*1* To what, if anything, are speakers of English referring when they speak of two liters of water? When there are two liters of water in a bottle, what is being numbered? Of what, if anything, are there two?

Suppose we desire to arrive at a formalized language analysis of the sentence

(1) There are exactly two liters of water in the bottle.

As a first try we might utilize Frege's method for symbolizing number adjectives and paraphrase (1) as something like

(2) There exists a pair $x$ and $y$ of distinct liters of water in the bottle, and for any liter of water $z$ in the bottle, $z$ is identical to either $x$ or $y$.

Is this an acceptable analysis?

In approaching this question we must first arrive at a clear understanding of the meaning of (1). Suppose Bob and Carol each pours exactly one liter of water into the bottle, thereby making (1) true. Thereafter, Ted and Alice each draws out exactly one liter from the bottle, thereby emptying it, and each then pours the liter back. Bob talks about the liter of water he poured into the bottle, Carol talks about the liter of water she poured into the bottle, and so on. They all agree that unless an extraordinary coincidence has occurred, Bob's liter of water is nonidentical with Ted's liter of water, Carol's is nonidentical with Alice's, and so altogether there are four nonidentical liters of water in the bottle.

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According to this argument there is a sense of the phrase 'liter of water' in which it is true that

(3) There are at least four liters of water in the bottle.

Given this sense, of course, there are a good many more than four; perhaps Bob's and Ted's liters of water differ by no more than a solitary molecule. However this may be, Bob, Carol, and company might still maintain that there is a sense of the phrase 'liter of water' in which (1) is true. Indeed, they would probably recognize that in ordinary usage this sense is overwhelmingly the more usual one.

Suppose that we use 'liter-of-water$_1$' to designate the sense which makes (1) true and 'liter-of-water$_2$' to designate the sense which makes (3) true.[1] What, now, shall we say about (2)? In the first place, (2) is clearly false given the liter-of-water$_2$ reading. But might not

(2′) There exists a pair $x$ and $y$ of distinct liters-of-water$_1$ in the bottle, and for any liter-of-water$_1$ $z$ in the bottle, $z$ is identical to either $x$ or $y$

be an acceptable paraphrase of (1) (on its true reading)?

Here we must ask what (2′) means. There is something exceedingly puzzling in talking about liters-of-water$_1$ being identical or distinct. Bob and Ted can meaningfully assert that the physical quantities they poured into the bottle are nonidentical; it is very easy to determine in principle whether two liters-of-water$_2$ are the same or different. But liters-of-water$_1$ do not seem to be physical quantities. Indeed, it is tempting to suppose that 'liter-of-water$_1$' is not even a referring term. If so, (2′) is certainly false. At best, however, (2′) is puzzling. Thus, as a paraphrase for (1), which is quite easy to understand, (2′) seems not to have much to recommend it.

At the same time

(2″) There exists a pair $x$ and $y$ of distinct liters-of-water$_2$ in the bottle, and for any liter-of-water$_2$ $z$ in the bottle, $z$ is identical to either $x$ or $y$

looks like a plausible paraphrase for

(4) There are exactly two liters-of-water$_2$ in the bottle.

Of course, both are false, but (2″) appears to nicely capture the logical force of (4). It exploits the fact that 'liter-of-water$_2$' picks out physical quantities of water and that these quantities are capable of being clearly distinguished.

Can a parallel way be found to capture

(4′) There are exactly two liters-of-water$_1$ in the bottle

in formalized terms? Ordinary users of language probably understand (4′) to be shorthand for something like

(5) If a person were to take a liter container and transfer the water out of the bottle, then he would fill the liter container exactly two times (if it were empty to start with, if he filled it up all the way, etc.).

Understood according to this counterfactual construal, the question, "Of what are there two when there are two liters of water in the bottle?" seems to have no clear answer. The adjective 'two' seems to modify possible actions rather than actual objects.

Suppose, however, we momentarily forget (4′) and direct our attention to

(6) There are exactly two gross of paper clips in the container.

As before, it is possible to distinguish between 'gross-of-paper-clips$_1$' and 'gross-of-paper-clips$_2$'. Suppose we set out to capture

(6′) There are exactly two gross-of-paper-clips$_1$ in the container.

In approaching this question it is worth noting that a gross-of-paper-clips$_2$ is a set of physical objects, and so any two are identical just in case neither contains (at any time) a paper clip not contained by the other. Thus, a container holding two gross-of-paper-clips$_1$ holds an exceedingly large number of gross-of-paper-clips$_2$. Suppose $m$ is short for this number. Then

(7) There are exactly $m$ gross-of-paper-clips$_2$ in the container

is logically equivalent to (6′). And since (7) is susceptible to treatment by Frege's method, we can paraphrase (6′) as

(8) There exist distinct gross-of-paper-clips$_2$ $x_1, \ldots, x_m$ in the container, and for any gross-of-paper-clips$_2$ $z$ in the container, $z$ is identical to one of $x_1, \ldots, x_m$.

A much less cumbersome solution, however, is possible. Let us say that two gross-of-paper-clips$_2$ are 'disjoint' just in case they have no paper clips in common. Now consider:

(9) There exist two disjoint gross-of-paper-clips$_2$ $x$ and $y$ in the container, and for any gross-of-paper-clips$_2$ $z$ in the container, $z$ is either $x$ or $y$, or $z$ is not disjoint with $x$ and $z$ is not disjoint with $y$.

It is my claim that (9) is equivalent to (8). The idea is that if (6) is true, then $x$ and $y$ form a mutually exclusive and exhaustive division of the paper clips in the container. More generally,

(10) There are exactly $k$ gross-of-paper-clips$_1$ in the container

can be paraphrased as

(11) There exist $k$ disjoint gross-of-paper-clips$_2$ $x_1, \ldots, x_k$ in the container, and for any gross-of-paper-clips$_2$ $z$ in the container, $z$ is either one of $x_1, \ldots, x_k$ or $z$ is not disjoint with at least two of $x_1, \ldots, x_k$.

It may be objected that we are quantifying in (8), (9), and (11) over collections of objects with no guarantee that these collections are themselves objects. What is to prevent us from quantifying over gross-of-paper-clips$_2$ each of which is widely scattered throughout portions of the American continent, for example? Surely these are not objects.

I have no desire to insist that these are objects, but neither do I see problems in permitting variables to range over them. There is a clear criterion of identity for them, and it is difficult to see what bearing spatial location has upon the legitimacy of quantification over them. If it is granted that the paper clips themselves exist it seems difficult to deny that there are collections of them.

*2* So far I have argued that (6′) can be treated by paraphrasing it as (9). I wish now to argue that the same can be done with (4′) by paraphrasing it as

(12) There exist two disjoint liters-of-water$_2$ $x$ and $y$ in the bottle, and for any liter-of-water$_2$ $z$ in the bottle, $z$ is either $x$ or $y$, or $z$ is not disjoint with $x$ and $z$ is not disjoint with $y$.

Here we are quantifying over physical quantities of water rather than sets of physical objects, and there is a great difference between substances like water and discrete objects like paper clips. Nevertheless, philosophers have frequently noted an important analog between sets and quantities of stuff: Both are defined according to what they contain. Just as two sets are the same just in case neither contains (at any time) anything not contained by the other, so two liters-of-water$_2$ are the same just in case neither contains (at any time) any water not contained by the other. Accordingly, neither sets nor quantities are capable of growing or shrinking over time.

Again, by quantifying over liters-of-water$_2$ I do not wish to suggest that they are objects, or individuals, or particulars (neither do I wish to deny they are these). I wish only to argue that they exist, that we can speak of them ("The liter of water Bob poured into the bottle"), and that, like sets, we can distinguish them from one another according to what they contain.

This analogy with sets notwithstanding, some philosophers have challenged the notion that quantities of water can be distinguished from one another. Henry Laycock, one such person, writes:

> The water in a bottle, so described, gets its distinctness by proxy from the bottle; but not so described it has no criterion of distinctness at all. . . . There is no natural way of thinking of the water in two distinct bottles as two distinct things and thus of the water in one bottle as one distinct thing. ([8], pp. 31-32)

> Water has not the built-in structure in virtue of which we will be able to pick out and distinguish some of it . . . and to lack such built-in structure is to lack the traditional criterion of distinctness and individuation. ([9], p. 436)

According to this view there is no clear sense in which the liter of water Bob poured into the bottle is different from that which Ted poured or which Carol poured. More generally, to talk about liters-of-water$_2$ as though they can be distinguished from one another is misleading. It appears furthermore to be Laycock's view that quantifying over liters-of-water$_2$ is objectionable.[2]

It is very difficult to present an outright refutation of this view, but it is likewise difficult to see how this view is capable of doing justice to our ordinary talking and thinking about the world. It is certainly true that water lacks the sort of built-in structure that paper clips have, but ordinary users of English do find it convenient or even necessary at times to distinguish one quantity of water from the rest of the world's water. Very often we pick out and distinguish some of it, analyze it, and continue to talk about it when it is no longer physically separated from other substances. Sometimes we even talk about a quantity of water which may never have been so separated in the first place ("The water above the red line"). All of this makes perfectly good sense even to young children.

Thus, if we fail to countenance the existence of such purported things as the liter of water Bob poured into the bottle, it is hard to see how to explain many apparent phenomena in the world around us. Are we to say there is nothing Bob poured into the bottle? Are we to say that Bob poured something into the bottle which existed but passed out of existence as soon as it became mixed with other water? Both of these alternatives seem implausible.

Laycock's own view seems to be something like the following: The water of the world exists, and it exists in what Bob poured into the bottle. What Bob poured, as such, has no existence, but since water itself is something which does exist, it is mistaken or at least misleading to infer that there is nothing Bob poured.

I am not convinced that this is a completely coherent view, but suppose for the sake of argument that it is. The problem is that we are still left with no obvious way to formalize sentences which speak of the water Bob poured into the bottle. Since this liter of water fails to exist in its own right, we may not permit it to be the value of a bound variable. Thus, for example, Russell's standard approach is not an option. But it is hard to see if quantification over liters-of-water$_2$ is entirely ruled out, what alternative approach one might take. Clearly, this matter has implications for a large portion of ordinary discourse.

It might be objected that we are missing in all of this the central thrust of Laycock's reservations. Perhaps the real problem comes in the assumption that the comparison between quantities and sets can yield results of philosophical interest. In particular, it might be objected that our criterion for the individuation of liters-of-water$_2$ is not really clear. It might be clear what is meant by saying that a paper clip belonging to one set fails to belong to another, but how are we to decide whether there is "any water" belonging to one quantity rather than another?

In the first place, of course, there really are measures of water (e.g., moles) which are distinguished in precisely the way sets are. But, generally speaking, the answer is that we are at liberty to choose any method we wish for deciding this question. It would be natural to divide liters of water at the molecular level, but we might also come to decide that 119 molecules is the cut-off point.[3] In the case of mixtures like wine a more imaginative approach may be called for. The point is that there is no officially agreed upon method for distinguishing liters of water, but from this consideration it surely doesn't follow that liters of water cannot be distinguished. Moreover, if we should rule out quantification over liters of water on these grounds, we ought to do the same for mountains and buildings.

Nor, I believe, should we stop at recognizing the existence of the world's liters-of-water$_2$. If we wish to do justice to ordinary English there is good reason to countenance the existence of all of the world's quantities of water. It would make little sense to countenance only those quantities for whose volume we have terms like 'liter' and 'cup'.

Quantities of water are distinguished in precisely the same fashion as liters-of-water$_2$: Any two are the same just in case neither contains any water not contained by the other. And so, if we elect to divide water at the molecular level, there will be a vast number of them in the universe. This number, however, will be finite (if there are $n$ molecules of water, there are $2^n - 1$ quantities).

Here we must also heed an important word of warning: A phrase such as 'The quantity of water Bob poured into the bottle' is not a true definite description. Bob poured one liter but a multitude of nonidentical quantities. For purposes of formalization we must uniquely identify the quantity as, for example, the largest quantity of water Bob poured into the bottle.

It is worth noting that quantification need not even be restricted to liters-of-water$_2$ or quantities-of-water$_2$. Speakers of English frequently attach ordinary language quantifiers such as 'all', 'some', and 'any' directly to mass terms as in, "Some water in the bottle was added since yesterday," and it would appear that a natural formalization of this sentence would be of the form, '$(\exists x)$ (Water $(x)$ & . . . )'. Elsewhere ([10], pp. 167ff), I have proposed in outline form a semantical theory for mass terms according to which clear sense can be made of such formulas by placing certain restrictions upon the formation rules of the language. Within the bounds of this theory it becomes possible to quantify over water itself, as it were, in addition to liters-of-water$_2$ and quantities-of-water$_2$. However, because of the special nature of the restricted formal language, ontological commitments of the usual sort are not created, and hence the concerns of Laycock do not become a relevant issue.

Finally, count terms, such as 'quantity', 'portion', and 'liter' play a special role in ordinary language, their special function being that of attaching to mass terms and forming complex count terms (Burge calls them "grammatical lackeys for mass terms"). In particular, measure terms are designed to convert mass terms into complex terms for the purpose of applying the concept of number to those substances named by the mass terms. We might regard expressions such as 'liter-of-' as functions whose domain consists of mass terms and whose range consists of complex count terms. On the view I have proposed these functions operate, semantically speaking, in a very straightforward fashion: The water of the world is mapped (by the 'liter-of' function) onto the class of liters-of-water$_2$ (not all of which are disjoint), the kerosene of the world to the class of liters-of-kerosene$_2$, and so forth.

*3* In this discussion I have advanced some suggestions concerning the logic of the expression 'two liters of water'. It would be natural to ask whether these suggestions are appropriate for all expressions of the form 'two ______s of ________', where we fill in a volume, area, or linear measure term for the first blank and a concrete mass term for the second.

Here I will make only the briefest of remarks. I do not believe my suggestions are appropriate for all such expressions, not even if we somehow manage to exclude those which are nonsensical ("two inches of soup"); but it is not at all clear how one might identify the class of such expressions for which my suggestions are appropriate. There seem to be a number of issues which must first be settled. For example, what shall we say about liters of trash? There is a sense in which water is a homogeneous substance and trash is not, and this would seem to be relevant as to whether it makes sense to speak of liters-of-trash$_2$.

And what about succotash (to use Sharvy's example)? Is there a certain ratio of lima beans to corn which must be consistently maintained when we set out to determine how many cups-of-succotash$_2$ there are in a bowl of succotash? There is also some uncertainty as to what we should count as measure terms. Shall we include such terms as 'heap', 'flock', 'truckload', or 'roomful'? These and many other questions must be taken up prior to developing a full-blown theory of measure terms.

## NOTES

1. It appears that 'liter-of-water$_2$' is itself ambiguous. It might be thought to designate either: (i) a liter of space which happens to currently have water occupying it, or (ii) a quantity of water which happens to fill exactly a liter of space. In what follows we will understand 'liter-of-water$_2$' in terms of (ii).

2. Laycock [9], p. 441. Evidently Laycock acknowledges, however, that the water of the world may serve as the value of a bound variable.

3. It might be pointed out that by replacing the water one atom at a time (or 118 molecules at a time) the transitivity of identity eventually forces us to say of two disjoint liters that they are identical. This I grant, but since it also holds true of paper clips, chairs, and ships afloat, this point does not specifically relate to any of the above proposals. An interesting alternative (which bypasses this difficulty) would be to hold that two liters-of-water$_2$ are distinct just in case there is no space occupied by one which is not occupied by the other. Presumably, on this approach, there are uncountably many liters-of-water$_2$ in the bottle if (1) is true, and our ontological commitments are affected enormously. But this approach might recommend itself on the grounds of providing a uniform treatment for compounds like water and mixtures like wine. At any rate, (12) is neutral with respect to the method of individuating liters-of-water$_2$ and can be used with whatever approach is taken.

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