Precisification by Means of Vague Predicates

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My main theses are that (a) it is impossible for all of language to be vague, and (b) some precise predicates can be defined in terms of vague predicates. These propositions are the negations of two used by Bertrand Russell to criticize the suggestion that the problems posed by vague predicates can be solved by precisifying definitions. After reviewing this criticism, I argue for (a) in the second section and (b) in the third. The next section examines the impact of (b) on current theories of vagueness. I argue that while it is bad news for the many-valued approach and the view that vague predicates are incoherent, it is good news for the supervaluational and epistemic theories of vagueness. In the last section I show that Russell's criticism fails for a reason that strengthens Patrick Grim's more recent attack on precisificationism. So I ultimately come to share Russell's pessimism about solving the sorites through precisification.

1 Russell's criticism of precisificationism   Everyone agrees that the source of sorites puzzles, such as the following, lies in the vagueness of the inductive predicate.

1. One minute after noon is noonish.
2. If n minutes after noon is noonish, then so is $n + 1$ minutes.

3. 600 minutes after noon is noonish.

By redefining 'noonish' as 'any time within 15 minutes of noon', we can reject the second step of the argument. Similar precisifying definitions can be offered for other vague predicates playing a crucial role in sorites arguments. Precisificationism is the view that this strategy of redefinition constitutes a solution to the sorites paradox.[1] Those skeptical of this solution sometimes appeal to an objection made by Russell in [8]. Russell argued that since all of language is vague,

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our definientia cannot be precise, hence the definiendum will inherit the vagueness of its definiens. For example, Russell would insist that 'noon' is not precise because of indeterminacies associated with setting temporal standards. Russell's criticism is flawed in two ways of interest to contemporary students of vagueness. The first problem is a self-referential difficulty with the claim that all of language is vague. This difficulty suggests that any language in which the question can be posed must have a precise element.

2 Why language cannot be entirely vague   Let me begin with a rough statement of the self-referential difficulty engendered by the thesis that 'All language is vague'. If all of language is vague, then the predicate 'vague' is vague.[2] A predicate is vague only if it has borderline cases. But if all of language is vague, how can there be any borderline cases of 'vague'? Every linguistic item would be in the extension of 'vague'. 'Vague' would thus qualify as a precise (though general) predicate.

The problem can be clarified with a direct argument in favor of the thesis that not all language is vague:

1. 'Vague' is a predicate.
2. If a predicate is vague, then it has borderline cases.
3. If something is a borderline case of 'vague', then it is a linguistic item.
4. So, 'vague' is vague only if a linguistic item is a borderline case of 'vague'.
5. If all language is vague, then 'vague' applies to every linguistic item.
6. If 'vague' applies to every linguistic item, then no item of language is a borderline cases of 'vague'.
7. Therefore, not all language is vague.

The most controversial premise of this argument is the third. Although most commentators construe vagueness as a linguistic phenomenon, some philosophers are inclined to regard nonlinguistic things as vague. Russell diagnoses people who believe that clouds, trees, and the cosmos are vague objects as victims

> of the fallacy of verbalism—the fallacy that consists in mistaking the properties of words for the properties of things. Vagueness and precision alike are characteristics which can only belong to a representation, of which language is an example. They have to do with the relation between a representation and that which it represents. Apart from representation, whether cognitive or mechanical, there can be no such thing as vagueness or precision; things are what they are, and there is an end of it. ([8], p. 85)

But even if belief in the vagueness of nonrepresentational things is the product of a fallacy, it must be noted that some representational things are not linguistic.

Amongst Russell's list of vague things we find smudged photographs, small-scale maps, distant perceptions, and ordinary beliefs. Given that these nonlinguistic things can be vague, nonlinguistic things could be borderline cases of vague objects. For example, we could have a sequence of photographs ranging from completely clear ones to completely smudged photographs. Given that the smudginess increases very gradually, we may be unclear as to where the clear photographs end and the vague photographs begin. Thus premise 3 might be

denied on the grounds that although no linguistic item is a borderline case of 'vague', some photographs are borderline cases of 'vague'.

It is doubtful that Russell would have been attracted to this objection since he seems to have believed that all representations are vague. Although he would concede that some photographs are less vague than others, he would deny that any escape vagueness. Nevertheless, other philosophers might find the position attractive, so there is a point to reducing the controversiality of premise 3. A number of strategies are available. Aside from trying a straight refutation of the claim that nonlinguistic things can be vague, it could be claimed that 'vague' is ambiguous. Another alternative is to substitute 'representational item' for 'linguistic item' and rest content with the conclusion that some representational items must be nonvague. However, the best option is to replace premise 3 with the weaker

3'.   If something is a borderline case of 'vague', then some linguistic item is a borderline case of 'vague'.

This new premise only requires that there be a borderline vague linguistic entity if there is a borderline vague entity. It does not require that all borderline vague things be linguistic things.

We can support 3' by first observing that any claim that an object is vague warrants a corresponding claim about the vagueness of predicates used to describe them. For example, those who claim that clouds are vague are impressed with our inability to delimit precisely the spatial and temporal boundaries of a cloud. So they will agree that some propositions of the form 'This cloud is at least n minutes old', 'All clouds occupy a region of at least n cubic millimeters', and 'A is the same cloud as B' are indeterminate. Thus they will agree that 'cloud' is vague. Indeed, the only way they can show that an object is vague is to refer to borderline cases. For the purposes of my argument, we need not insist that talk of vague objects is *confused* recognition of the vagueness of the predicates used to described them. We can be diplomatic and merely request assent to a parallel between the vagueness of things and the vagueness of language. In the case of the sequence of photographs, we can gain assent to the vagueness of 'represents' by noting the indeterminacy of some propositions of the form 'Photograph n represents an F'. The beginning photographs may be clearly representations of flying birds and the last photograph may be a clear non-representation of a flying bird because the smudginess has removed all traces of the figure. In between we have "vague" photographs. But this will only be true if there is a photograph that is a borderline case of 'photograph representing a flying bird'. In addition to these borderline cases, there must be borderline borderline cases. For there is no abrupt transition from nonvague to vague photographs. Thus if photograph n is a borderline case of 'photograph representing a flying bird', there is a photograph $n - m$ which is a borderline case of a vague photograph. So the statement 'Photograph $n - m$ represents a flying bird' will be a borderline case of a vague statement. Since statements are linguistic items, we have an example of a linguistic item that is a borderline case of 'vague'. The example follows from the assumption that photographs can be vague. Hence, we have an illustration of how the parallel between vague objects and vague language generates a parallel between borderline vague objects and borderline vague

language. It is this second parallel that warrants acceptance of 3′ by proponents of vague objects.

Once the incompatibility between full generality and vagueness is appreciated, a more direct refutation of the thesis that all language is vague becomes available. For the thesis can be refuted by counterexamples featuring predicates with maximal generality. Everything is in the extension of 'self-identical', so it cannot have any borderline cases. Thus it dramatically illustrates the distinction between generality and vagueness. Other examples depend on one's metaphysical views. A monist about F's can allow no borderline cases of F's because he takes F to apply to everything. Thales can allow nothing to be a borderline case of 'water', materialists must disallow borderline cases of 'material thing', while idealists need to insist that 'mental' is free of borderline cases. In addition to predicates with maximal generality, predicates known to have no generality are counterexamples.[3] Since contradictory predicates such as 'round square' cannot have anything in their extensions, recognition of their contradictoriness eliminates any doubt whether the predicate applies to anything. So they have no borderline cases. Our mathematical vocabulary contains many precise predicates of intermediate generality, such as 'prime number' and 'square'.

3   The failure of compositionality   The second flaw in Russell's criticism is his assumption that vagueness is strongly compositional;

(SC)   If some component of the definiens is vague, then the definiendum is vague.

In order to preclude the possibility that precise predicates could be defined from a stock of vague ones. Russell must maintain that any vagueness in the definiens is always inherited by the definiendum. However, there are counterexamples to this claim.

The first class of counterexamples to the compositionality thesis relies on the precision of one of the defining clauses. Since 'self-identical' applies to everything, it is perfectly precise. Disjoining it with any vague expression leaves its precision intact. Just as everything is in the extension of 'self-identical', everything is in the extension of 'is self-identical or a heap of sand'. Since this disjunction can serve as an albeit circular definition of 'self-identical', we have our first counterexample to the strong compositionality thesis. Conjoining any vague term with a contradictory expression such as 'round-square' or 'largest prime' will not expand their extensions. Just as nothing falls under 'round-square', nothing falls under 'large round-square'. A third type of counterexample involves an initial extension that is neither universal nor empty. The expression 'is a prime between 40 and 46' has {41,43} as its extension. The same extension is enjoyed by 'is a prime between 40 and 46, and is near 42'.

The above examples use one precise component. They show that the vagueness of one clause of a definition need not infect the definiendum. If we picture the counterexamples as being built by the addition of a vague clause to a preexisting definition, they are bound to appear artificial because the vague clauses do not alter the meaning of the definiendum. This reaction is not quite as pronounced if we picture the construction process in reverse order, that is, as the

addition of a precise clause to a vague one. Yet it remains the case that the vague clause is made redundant by the precise one. Whichever direction we picture the construction, the result is an inefficient and hence artificial predicate.

However, there is no shortage of efficient counterexamples. Conjoining 'is a little less than 1002' with 'has {7,11,13} amongst its divisors' yields a precise conjunctive predicate. Since its extension, {1001}, is distinct from the extensions of its components, the resulting predicate is efficient.

In addition to this group of counterexamples, there is a more interesting set of cases which shows that precision can be increased by adding vague clauses. For example, 'is a child or an adolescent or an adult' is less vague than 'is a child'. Both predicates suffer from the vagueness of 'human being' but the disjunctive predicate is free from borderline cases lying between 'child' and 'adolescent'. In light of the residual vagueness of the disjunctive predicate, it is not a genuine counterexample to the compositionality of vagueness. Nevertheless, it does suggest the existence of cases in which the addition of vague clauses goes beyond vagueness reduction and achieves vagueness elimination. Unlike the previous cases, these would be counterexamples to the weak compositionality principle:

(WC)   If all the components of the definiens are vague, then the definiendum is vague.

Let us first consider a case involving conjunction. Although 'is an integer somewhat greater than 104' is vague and 'is an integer somewhat less than 106' is vague, the conjunction 'is an integer somewhat greater than 104 and somewhat less than 106' is absolutely precise.

Vagueness can also be eliminated disjunctively. Although 'small integer', 'medium integer', and 'large integer' are each vague, 'is either a small, medium, or large integer' is not vague. An integer that is a borderline small integer and a borderline medium integer is not a borderline 'small or medium integer'.

The above examples illustrate two ways in which vagueness can be reduced. First, we can superimpose a predicate that is definite everywhere the first predicate is indefinite. This second predicate can itself be vague as long as the first predicate definitely covers the second's indefinite cases. Thus 'is an integer somewhat greater than 104' conjoins with 'is an integer somewhat less than 106' to yield a precise predicate because each covers the other's indefinite cases. Whereas the cover-up method covers indefinites with definites, the exhaustion method turns distributively indefinite cases into collectively definite ones. An integer that is borderline between being a small and a medium integer is an indefinite case for both 'small integer' and 'medium integer'. But it is not borderline between 'is either small or medium' and 'is neither small nor medium'. Thus disjoining the predicates removes the vagueness. Notice that this method only works when the predicates are exhaustive for the individual in question. If someone is a borderline case of a fat man and a borderline case of a bald man, he can still be a borderline case of 'bald or fat man'.

The noncompositionality of vagueness shows that even if language were entirely vague, precisifying definitions could still produce some precise predicates from an initial stock of vague ones. Can any precise predicate be defined in terms of vague ones? If inefficient definitions are allowed, the answer is affir-

mative. For we can merely add to any adequate definition of a precise predicate the disjunctive clause 'or is a small round square'. The answer is probably still affirmative once we add an efficiency requirement. Each natural number can be defined with definiens of the form 'is somewhat greater than n and is somewhat less than $n + 2$'. 'Triangle' can be defined as a more or less enclosed figure composed of somewhat fewer than four sides and somewhat greater than two each of which has as its degree of curvature and number of gaps an integer somewhat greater than -1 and somewhat less than 1. These definitions satisfy the requirement that each clause make a difference to the extension. Of course, one may wish to invoke a stricter efficiency requirement to follow out the suspicion that these definitions surreptitiously invoke precise predicates. For the above examples simply coat initially precise predicates with a vague exterior and then strip the coat to reveal the original precision. My conjecture is that stricter efficiency requirements will only serve to complicate the definitional task rather than render it impossible. Those sympathetic to concept empiricism should find this conjecture plausible. For the concept empiricist maintains that our theoretical vocabulary can be defined in terms of our observational predicates. Since our observational vocabulary is vague, the concept empiricist is committed to defining whatever precise predicates we have in terms of vague observational predicates.

Whether or not all precise predicates are definable in terms of vague ones, the fact that some are definable vitiates Russell's view that logic applies to all and only precise predicates. Since disjunctions and conjunctions of vague predicates can be precise, logic will apply to them. But if it applies to the disjunctions and conjunctions, logic must also apply to the vague disjuncts and conjuncts.

4 Implications for current theories of vagueness   Like Russell, contemporary proponents of the compositionality thesis have tended to assume the thesis rather than argue for it. Nevertheless, a couple of contemporary positions on vagueness do imply the weak form of the compositionality thesis.

The many-valued approach reflects the fact that x is a borderline case of an F by assigning 'Fx' a real number between 0 and 1 as an intermediate truth value.[4] The standard many-valued rule for evaluating disjunctions is to assign the disjunction the highest value of its disjuncts. The rule for conjunction is to assign the lowest value of the conjuncts. These rules ensure that vagueness will fail to be compositional just when one of the disjuncts has a truth value of 1 or when one of the conjuncts has a truth value of 0. So according to this position, there should be no counterexamples to the weak compositionality thesis. For if the disjuncts and conjuncts are all vague, the disjunctions and conjunctions must have intermediate truth values.

Weak compositionality is also implied by the view that vague predicates are incoherent,[5] for disjunctions and conjunctions of incoherent predications are themselves incoherent. This is true regardless of whether 'incoherent' is read as 'inconsistent' or 'meaningless'. Disambiguation is necessary when we ask about strong compositionality, however. Under the inconsistency reading, strong compositionality fails just when a consistent predication is disjoined with an inconsistent predication. Since it is unclear whether disjoining a meaningless

"predication" with a meaningful one results in a meaningful disjunction, reading 'incoherent' as 'meaningless' does not deliver a clear answer to the question of whether strong compositionality is denied.

If the many-valued approach and the incoherence position were the only theories of vagueness on the market, we would have some reason to take their support of weak compositionality as grounds for questioning the apparent counterexamples. But since the market is crowded with theories that lack this implication, we should accept appearances.

Indeed, theories which equate vagueness with partial definition imply the falsehood of weak compositionality. Although we find Kit Fine saying "an expression is made more precise *only* through making its simple terms more precise" ([3], p. 275), his brand of supervaluationism opens the possibility of defining precise predicates by means of vague ones, for it is possible to define a term completely with partially defined definiens. For example, we could define 'somewhat greater than n' in terms of the necessary condition of being larger than n and the sufficient condition of equaling $n + 1$. This leaves it an open question as to whether successors of $n + 1$ qualify. The predicate 'somewhat less than n' can receive a similar partial definition in terms of the necessary condition of being less than n and the sufficient condition of equaling $n - 1$. The conjunctive predicate 'is an integer somewhat greater than 104 and somewhat less than 106' is fully defined given the partial definitions of its definiens.

The failure of weak compositionality is also predicted by the epistemic approach to vagueness.[6] According to this approach, vagueness is a sort of ignorance; there is an answer to the question of where to draw the line, but we cannot know it. A number that is borderline between being small and medium leaves us in ignorance as to whether 'It is small' and in ignorance as to whether 'It is medium'. However, we will not be ignorance as to whether 'It is either small or medium'. Vagueness would only be compositional if the following principle held: $(-Kp \& -Kq) \supset -K(p \lor q)$. Since there are obvious counterexamples to this principle, we are left with two theories of vagueness that are strengthened by the compositionality counterexamples.

5   A better objection to precisificationism   I have argued that Russell's objection to precisificationism fails because (a) it is impossible for all of language to be vague, and in any case, (b) some precise predicates can be defined in terms of vague predicates. Nevertheless, Russell's objection fails for a reason that strengthens a more recent objection to precisificationism.

Patrick Grim has observed that precisificationists cannot allow a precise predication to entail a vague one.[7] For example, if 'gizzle gick' is to be our precise counterpart of 'swizzle stick', it would be natural to assent to the following conditionals:

(1) If something is a gizzle gick, then it is a swizzle stick.
(2) If something is a gizzle gick, then something is a swizzle stick.
(3) If something is a gizzle gick, then something is a small object.

If the precisificationist accepted one of these conditionals, his commitment to the existence of gizzle gicks would commit him to a nonempty extension for a

vague predicate. This would renew his vulnerability to sorites arguments. To avoid the sorites, the precisificationist must assign an empty extension to all vague predicates and, so, deny every bridge conditional.

Grim uses this point to criticize the precisificationist in two ways. First, denial of the conditionals is highly counterintuitive. Indeed, the counterintuitiveness is comparable to the counterintuitiveness of denying the induction step of the sorites argument. So the precisificationist has not made a significant advance over those who simply deny a crucial premise of the puzzling argument. Grim's second objection is that once the conditionals are denied it is hard to make sense of the claim that the precisified predicates are *replacing* the vague ones. In order to replace something, something must be preserved. The usual sort of precising definition preserves some logical connections or at least some of the reference of the vague predicate. But since vague predicates have been denied reference and logical connections, the "precisifying definitions" seem to be purely stipulative ones. Indeed, we seem well on our way to requiring a brute swap of one whole language for another. Grim emphasizes that our precise language would have been purchased at a high price, and that once bought it may turn out to be unlearnable.

The failure of the compositionality of vagueness puts us in a position to supplement Grim's attack with a third criticism. As Grim's bridge conditionals illustrate, the precisificationist is committed to an *eliminative* reduction of vague predicates. Vague predicates are being assimilated to 'phlogiston', 'N-rays', 'humours', 'aether', 'poly-water', and 'witch'. But if vague predicates have empty extensions, conjunctions of vague predicates must have empty extensions. Yet we have seen that some predicates can be conjoined to define precise predicates that have nonempty extensions. Thus precisificationism mistakenly implies that no nonempty precise predicate can be defined in terms of the conjunction of two vague predicates. Like the incoherence theorist, the precisificationist is committed to weak compositionality. Thus Grim's attack on precisificationism is strengthened by the same point that undermined Russell's.

NOTES

1. Sympathy to precisificationism is displayed in the first chapter of [2]. Carnap proposes that prior to formalization, qualitative expressions should be replaced by comparative ones, or better yet quantitative ones. Susan Haack supports Carnap in the sixth chapter of [5].

2. I agree that 'vague' is vague. I only take issue with Russell's way of reaching this conclusion. I suggest another way in [10].

3. Bertil Rolf points out that logically false statements employing vague predicates are precise in [7].

4. The many-valued approach to vagueness originates with [13]. It seems to have attracted the allegiance of a plurality of the commentators on the sorites paradox. Well developed versions of this position are presented in [6] and [9].

5. The clearest examples of incoherence theorists are [11] and [12]. But one can also find support for the incoherence thesis in the works of W. V. Quine, Michael Dummett, and Bertil Rolf.

6. James Cargile was the first to defend the epistemic position in [1].

7. The fullest presentation of Grim's criticism appears in [4].

REFERENCES

[1] Cargile, J., "The sorites paradox," *The British Journal for the Philosophy of Science*, vol. 20 (1969), pp. 193–202.

[2] Carnap, R., *Logical Foundations of Probability*, University of Chicago Press, Chicago, 1950.

[3] Fine, K., "Vagueness, truth, and logic," *Syntheses*, vol. 30 (1975), pp. 265–300.

[4] Grim, P., "Taking sorites arguments seriously: some hidden costs," *Philosophia*, vol. 14 (1984), pp. 251–272.

[5] Haack, S., *Deviant Logic: Some Philosophical Issues*, Cambridge University Press, London, 1974.

[6] Machina, K., "Truth, belief, and vagueness," *Journal of Philosophical Logic*, vol. 5 (1976), pp. 47–78.

[7] Rolf, B., "A theory of vagueness," *Journal of Philosophical Logic*, vol. 9 (1980), pp. 315–325.

[8] Russell, B., "Vagueness," *Australasian Journal of Philosophy*, vol. 1 (1923), pp. 84–92.

[9] Sanford, D., "Borderline logic," *American Philosophical Quarterly*, vol. 12 (1975), pp. 29–39.

[10] Sorensen, R., "An argument for the vagueness of 'vague'," *Analysis*, vol. 45 (1985), pp. 134–137.

[11] Unger, P., "There are no ordinary things," *Synthese*, vol. 41 (1979), pp. 117-154.

[12] Wheeler, S., "On that which is not," *Synthese*, vol. 41 (1979), pp. 155–173.

[13] Zadeh, L., "Fuzzy sets," *Information and Control*, vol. 8 (1965), pp. 338–353.

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