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A Free Logic with Simple and Complex Predicates

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1 Introduction Consider a fragment of colloquial discourse *sans* modal or epistemic operators or psychological verbs. Examples of statements in this fragment are

(1) Vulcan rotates around its axis,[1]

and

(2) Vulcan is self-identical.

Some free logicians regard all such simple statements as asserting *of* the objects to which the constituent singular terms purport to refer that they are things which ______. (Here and elsewhere the blanks are to be filled in by appropriate verbs or verb phrases.) So, for instance, (1) and (2) respectively assert *of* Vulcan that it is a thing which rotates around its axis and, among other possibilities, that it is a thing which is the same as Vulcan.

This kind of linguistic intuition underlies the conviction of many free logicians that all simple statements of the fragment of colloquial discourse in question imply the existence of the purported referents of their constituent singular terms, and hence that if the purported referents fail to exist, the host statement is false. Free logics of this sort are called *negative free logics.*[2]

Other free logicians regard simple statements of the fragment of colloquial discourse in question merely as asserting *that* the objects to which the constituent singular terms purport to refer ______. So, for instance, the sentences (1) and (2) respectively assert *that* Vulcan rotates around its axis and, among other possibilities, is the same as Vulcan. Accordingly, even if all the singular terms are irreferential—as in (2)—the truth-value of the host statement need not be false, and, indeed, is true in the case of (2). (1) is either true or truth-valueless.

This kind of linguistic intuition underlies the conviction of many free logi-

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cians that at least some simple statements do not imply—nor even presuppose (in the Lambert-van Fraassen sense of "presuppose")—the existence of the purported referents of the constituent singular terms, and hence that there are some true simple statements of the fragment of colloquial discourse outlined which contain irreferential singular terms. (2), in fact, is a standard example. Free logics of this kind are called *positive free logics*.[3]

Given that many, if not most, simple statements of our colloquial fragment are equivocal with respect to the kinds of assertion outlined above,[4] the question arises whether there can be a system that accommodates both linguistic intuitions. The answer is yes. Scales' pioneering [7][5] sketched just such a system. In what follows we present an alternative way of accomplishing the same end. The pivotal semantical differences are these. Scales' models included outer domains conceived as sets of nonexistent objects, but we eschew outer domains. (In Scales' terminology, his semantics is Meinongian, but ours is Russellian.) Moreover, Scales' semantics is bivalent, but ours is not, being essentially an amplification of the modified supervaluational approach developed by Bencivenga [1].

Like Scales, we distinguish two kinds of simple statement. There are simple statements consisting of an n-adic *simple predicate* (approximately what Scales calls an n-adic *sentential function*) and n singular terms, where $n \geq 1$, and there are simple statements consisting of an n-adic *complex predicate* and n singular terms, where $n \geq 1$. The former kind of simple statement is the formal counterpart of a simple statement of colloquial English construed as asserting *that* the objects purported to be referred to by the constituent singular terms of that sentence ______. The latter kind of simple statement is the formal counterpart of a simple statement of colloquial English construed as asserting *of* the objects purported to be referred to by the constituent singular terms of that sentence that they are things which ______.

The distinction between simple and complex predicates parallels the distinction between simple singular terms (grammatically proper names) and complex singular terms (for instance, definite descriptions). Just as there can be simple statements containing simple singular terms or complex singular terms, so there can be simple statements containing simple predicates or complex predicates. Moreover, just as in some treatments a simple statement containing a simple singular term can differ in truth-value from a simple statement containing a complex singular term even though the simple singular term and the complex singular term are codesignative,[6] so a simple statement containing a simple predicate can, in the present treatment, differ in truth-value from a simple statement containing a complex predicate even though the simple predicate and the complex predicate are coextensive.

An example may help to fix the main ideas. Let "v" abbreviate "Vulcan", "R" abbreviate "rotates around its axis", "$=v$" abbreviate "is the same as Vulcan", and "$\lambda x(x = v)$" abbreviate "is a thing which is the same as Vulcan". Then each of

(3) Rv

(4) $v = v$

(5) $\lambda x(x = v)v$

is a simple statement, the first pair containing simple predicates, and the third a complex predicate. Moreover, in the treatment to follow, (3) is truth-valueless, and (4) not only is true, but is logically true, while (5) is false even though "$=v$" and "$\lambda x(x = v)$" are coextensive. (4) and (5) represent the pair of assertions associated with the equivocal (2).

We turn now to a precise statement of the system FL^{sc}.

2 The syntax of FL^{sc} The *vocabulary* of FL^{sc} is as follows:

(i) individual variables: $x, y, z, \ldots$
(ii) individual constants (singular terms): $a, b, c, \ldots$
(iii) simple predicates: $P, Q, R, \ldots$
(iv) connectives: $\sim$, &
(v) quantifier: $\forall$
(vi) abstractor: λ
(vii) existence sign: E!
(viii) identity sign: $=$

The symbols in (i)–(viii) are autonymous. Since no confusion can result, we will also use symbols of category X as metavariables for symbols of category X. If s and t are variables or singular terms, As/t is the result obtained from substituting s for all free occurrences of t in A (possibly after some relettering to avoid scope confusions). The definition of *predicate* and *statement* is by simultaneous recursion as follows:

(a) simple predicates are predicates
(b) if P is an n-adic simple or complex predicate, $Pa_1, \ldots, a_n$ is a statement
(c) E!a and $a = b$ are statements
(d) if A, B are statements, so are $\sim A$, A & B and $\forall x Ax/a$
(e) if A is a statement, $\lambda x_1 \ldots \lambda x_n (Ax_1/a_1 \ldots x_n/a_n)$ is an n-adic predicate
(f) nothing else is a predicate or a statement.

(The definitions of $\vee$, $\supset$, $\equiv$, and $\exists$ are as usual.) The simple statements of FL^{sc} are isolated by clauses (b) and (c) of the above definition.

The *intelim rules* for FL^{sc} are any adequate set of intelim rules for the classical logic of statements and the following:

FUE

(m)	$\forall xA$ (or E!a)
	.
	.
	.
(n)	E!a (or $\forall xA$)
	.
	.
	.
(p)	Aa/x

FUI

(m) | | $E!a$
| | ——
| | .
| | .
| | .
(n) | | Aa/x provided a does not occur in $(n + 1)$ or any of the premises of the derivation
$(n + 1)$ | $\forall xA$

FIE

(m) | $a = b$ (or A)
| .
| .
| .
(n) | A (or $a = b$)
| .
| .
| .
(p) | Ab/a

FII

| .
| .
| .
(m) | $a = a$

FAbE

(m) | $\lambda x_1 \ldots \lambda x_n (Ax_1/a_1 \ldots x_n/a_n) b_1 \ldots b_n$
| .
| .
| .
(n) | $A(b_1/a_1 \ldots b_n/a_n)$ & $(E!b_1 \, \& \ldots \& \, E!b_n)$

FAbI

(m) | $A(b_1/a_1 \ldots b_n/a_n)$ & $(E!b_1 \, \& \ldots \& \, E!b_n)$
| .
| .
| .
(n) | $\lambda x_1 \ldots \lambda x_n (Ax_1/a_1 \ldots x_n/a_n) b_1 \ldots b_n$

Some important categorically derived statements of FL^{sc} are instances of the following:

CD1 $(E!b_1 \, \& \ldots \& \, E!b_n) \supset (\lambda x_1 \ldots \lambda x_n (Ax_1/a_1 \ldots x_n/a_n) b_1 \ldots b_n \equiv Ab_1/a_1 \ldots b_n/a_n)$

CD2 $\lambda x_1 \ldots \lambda x_n (Ax_1/a_1 \ldots x_n/a_n) b_1 \ldots b_n \equiv ((E!b_1 \, \& \ldots \& \, E!b_n) \, \& \, (Ab_1/a_1 \ldots b_n/a_n))$

CD3 $E!b \equiv (\lambda x(Ax/a)b \vee \lambda x(\sim Ax/a)b)$

CD4 $\lambda y(Ay/x)b \supset \exists xA$

CD5 $\forall x(\lambda y(Ay/a)x \equiv \lambda y(By/a)x) \supset (\lambda y(Ay/a)b \equiv \lambda y(By/a)b)$.

CD1 is the restricted form of the *principle of abstraction*. CD4 shows the *principle of particularization* for simple statements with complex predicates, CD2 shows the conditions under which complex predicates are eliminable from statements containing singular terms, and CD5 depicts an important *substitutivity principle for complex predicates.* As will be clear in what follows the analogues of CD4 and CD5 for simple predicates do not hold.

3 The semantics of FL^{sc} A *model M* is a pair $\langle D, g\rangle$, where D is a set, and g a function totally defined on predicates but partially defined on singular terms such that

(i) $g(a) \in D$ when $g(a)$ is defined
(ii) $g(P) \in \mathcal{P}(D^n)$ when P is n-adic.

We introduce the widely adopted simplification that every element of D has a label as follows. We say that M is *full* if for every $d \in D$ there is a singular term a such $g(a) = d$. The first such singular term in some alphabetical order will be designated $\bar{d}$.

A *completion* M' of $M = \langle D, g\rangle$ is a model $\langle D', g'\rangle$ such that

(i) $D \subseteq D'$
(ii) $g'(a) = g(a)$ if $g(a)$ is defined
(iii) $g(P) \subseteq g'(P)$
(iv) $g(a)$ is defined for all a
(v) M' is full (possibly in a larger language).

Care must be taken to distinguish a full model from one which is the completion of another model. In the former, every element of D has a label, but some singular terms may be irreferential; in the latter, every singular term is referential.

We introduce the *extension function* e_M and the *fact function* f_M for a full model $M = \langle D, g\rangle$ by simultaneous recursion as follows:

(i) if $g(P)$is defined, then $e_M(P) = g(P)$
(ii) if all of $g(a_1), \ldots, g(a_n)$ are defined, then
(a) $f_M(Pa_1 \ldots a_n) = \mathrm{T}$ if $\langle g(a_1), \ldots, g(a_n)\rangle \in e_m(P)$, and
(b) $f_M(Pa_1 \ldots a_n) = \mathrm{F}$ otherwise
(iii) if $g(a)$ and $g(b)$ are defined, then
(a) $f_M(a = b) = \mathrm{T}$ if $g(a) = g(b)$, and
(b) $f_M(a = b) = \mathrm{F}$ otherwise
(iv) if exactly one of $g(a)$, $g(b)$ is defined, then $f_M(a = b) = \mathrm{F}$
(v) (a) $f_M(\mathrm{E}!a) = \mathrm{T}$ if $g(a)$ is defined, and
(b) $f_M(\mathrm{E}!a) = \mathrm{F}$ otherwise
(vi) (a) $f_M(\sim A) = \mathrm{T}$ if $f_M(A) = \mathrm{F}$, and
(b) $f_M(\sim A) = \mathrm{F}$ if $f_M(A) = \mathrm{T}$
(vii) if $f_M(A)$ and $f_M(B)$ are defined then
(a) $f_M(A \,\&\, B) = \mathrm{T}$ if $f_M(A) = f_M(B) = \mathrm{T}$, and
(b) $f_M(A \,\&\, B) = \mathrm{F}$ otherwise
(viii) if $f_M(Aa/x)$ is defined whenever $g(a)$ is defined, then
(a) $f_M(\forall xA) = \mathrm{T}$ if $f_M(Aa/x) = \mathrm{T}$ whenever $g(a)$ is defined, and
(b) $f_M(\forall xA) = \mathrm{F}$ otherwise

(ix) $e_M(\lambda x_1 \ldots \lambda x_n(Ax_1/a_1 \ldots x_n/a_n)) = \{\langle d_1, \ldots, d_n \rangle : d_1, \ldots, d_n \in D$ and $f_M(A\bar{d}_1/a_1 \ldots \bar{d}_n/a_n) = \mathrm{T}\}$

(x) e_M and f_M are not otherwise defined.

The preceding definition stipulates what the extensions of simple and complex predicates are in a full model *M*, and reflects the intuitive idea that some statements in *M* are made true (or false) by the facts therein. But the facts dominate only in the presence of referential singular terms. So the question arises how the truth-values of statements containing irreferential singular terms are to be computed given the "factual" information supplied by *M*. The answer is supplied by the following two definitions:

If *M* is a full model, and *M′* a completion of *M*, the *valuation for M′ from the point of view of M* is the function $V_{M'(M)}$ such that:

(i) if A is of the form $\lambda x_1 \ldots \lambda x_n(Bx_1/a_1 \ldots x_n/a_n)b_1 \ldots b_n$ then
  (a) $V_{M'(M)}(A) = \mathrm{T}$ if $f_M(A) = \mathrm{T}$, and
  (b) $V_{M'(M)}(A) = \mathrm{F}$ otherwise

(ii) if A is of the form $Pa_1, \ldots, a_n$, $a = b$ or $\mathrm{E}!a$, then
  (a) $V_{M'(M)}(A) = f_M(A)$ if $f_M(A)$ is defined, and
  (b) $V_{M'(M)}(A) = f_{M'}(A)$ otherwise

(iii) $V_{M'(M)}(\sim A) = \mathrm{T}$ if $V_{M'(M)}(A) = \mathrm{F}$, and $= \mathrm{F}$ otherwise

(iv) $V_{M'(M)}(A \mathbin{\&} B) = \mathrm{T}$ if $V_{M'(M)}(A) = V_{M'(M)}(B) = \mathrm{T}$, and $= \mathrm{F}$ otherwise

(v) $V_{M'(M)}(\forall xA) = \mathrm{T}$ if $V_{M'(M)}(Aa/x) = \mathrm{T}$ for all a such that $V_{M'(M)}(\mathrm{E}!a) = \mathrm{T}$, and $= \mathrm{F}$ otherwise.

To illustrate the key conditions (i) and (ii) in this definition, consider the simple statements "$\lambda x(x = a)a$" and "$a = a$," and assume that $g(a)$ is undefined (a is irreferential). Then (i) forces the falsity of "$\lambda x(x = a)a$" in any completion *M′* of *M* in virtue of the facts of *M*, but, in contrast, (ii) declares "$a = a$" true in *M′* because that value is the value it gets in *M′*.

We define the *supervaluation for a full model M* as the function S_M such that

(i) $S_M(A) = \mathrm{T}$ if $V_{M'(M)}(A) = \mathrm{T}$ for all completions *M′* of *M*

(ii) $S_M(A) = \mathrm{F}$ if $V_{M'(M)}(A) = \mathrm{F}$ for all completions *M′* of *M*

(iii) $S_M(A)$ is undefined otherwise.

Supervaluations settle the truth-values of statements relative to *M*, about which the facts may be silent in *M*, by looking at all of its completions. A statement of the form $\lambda x(Ax/a)b$, where $g(b)$ is undefined, will always be false—that is, $S_M(\lambda x(Ax/a)b) = \mathrm{F}$—because it is false in all of the completions of *M*, a consequence of the facts of *M*. In contrast, an atomic statement containing irreferential singular terms but no complex predicate can be true, false, or truth-valueless. In particular, returning to (3)-(5), it can now be confirmed that $S_M(3)$ is undefined, $S_M(4) = \mathrm{T}$, and $S_M(5) = \mathrm{F}$, under suitable choice of *M*. Indeed, as earlier intimated, (4) is *logically true*—that is, $S_M(v = v) = \mathrm{T}$ for all *M*.

Finally, if *M* is a nonfull model, add to the language new individual constants and assign each d for which there is no $\bar{d}$ to some new constant. A supermodel M' of M is thereby generated. Define $S_{M'}$ according to the above instructions. S_M is the restriction of $S_{M'}$ to the original language. This definition does not depend on the nature of M'.

The *adequacy* of FL^{sc} in the sense that A is categorically derivable in FL^{sc} if and only if A is logically true follows directly from the ensuing facts. First, the system FL^s is obtained by deleting the abstractor λ, the accompanying clauses involving λ in the formation rules, and FAbE and FAbI. The semantics for FL^s is obtained by appropriate deletions in the definitions and conventions of those clauses concerning expressions of the form $\lambda x_1 \ldots \lambda x_n (Ax_1/a_1 \ldots x_n/a_n)$. FL^s (in effect) has been proved adequate in the above sense, relative to the amended semantics, by Bencivenga [1]. Second, a translation of FL^{sc} into FL^s in the manner suggested by CD2 is forthcoming, such that, where A^* is the transform in FL^s of A in FL^{sc},

(i) A is categorically derivable in FL^{sc} if and only if A^* is categorically derivable in FL^s,

and

(ii) A is logically true in the semantics of FL^{sc} if and only if A^* is logically true in the semantics of FL^s.

(i) and (ii) are established by straightforward inductive arguments.

4 Concluding remarks We end with four remarks of philosophical interest.

First, as is well known, the symbol E! in FL^{sc} (or in FL^s) is redundant given the inclusion in the vocabulary of the symbol $=$. Indeed, where a is a variable or singular term, E!a can everywhere be replaced by $\exists x(x = a)$. But if $=$ is dropped from the language of FL^s, then, as Meyer et al. [4] have recently shown, E!a is not eliminable given the resources of that system. But the addition of complex predicates to FL^s *sans* identity does permit the elimination of E! from the primitive basis of the language. In FL^{sc} this is reflected in CD3 and shows one benefit of the greater expressive power of a language with complex predicates. Moreover, the present result has implications for Williams' view in [10] that existence is not a first-order predicate. Part of Williams' argument trades on the allegation that E! seems only to be definable in terms of a "meaningless" complex predicate involving identity, But FL^{sc} provides an alternative meaningful and nontrivial definition of E!.

Second, the inclusion of abstractors in the language FL^{sc} enables conformity with an ideal of logical laws, namely, that they be general. To see what is at issue consider a version of negative free logic by Burge [2]. Burge's system, which is devoid of abstractors, requires him to distinguish between atomic and complex statements *vis à vis* the existence of the purported designata of their constituent singular terms as necessary conditions. Thus, he has an axiom scheme to the effect that $Aa/b \supset$ E!a, provided A is atomic. But this axiom scheme seems to violate the ideal that the laws of logic be general, that they hold for *all* statements A. The introduction of abstractors allows one to capture the Burge intuition and write it in a more acceptable way. Such is reflected in the categorically derived principle of FL^{sc},

CD6 $\lambda x_1 \ldots \lambda x_n(Ax_1/a_1 \ldots x_n/a_n)b_1 \ldots b_n \supset (\mathrm{E}!b_1 \,\&\ldots\&\, \mathrm{E}!b_n)$.

Note that A can be *any* statement atomic or complex. Here is yet another benefit of the greater expressive power of FL^{sc}.

The final pair of considerations display the capacity of FL^{sc} to yield important distinctions. First, consider the statement

(6) $\forall x(x = x \equiv (\mathrm{E}!x \,\&\, x = x)) \supset (a = a \equiv (\mathrm{E}!a \,\&\, a = a))$.

(6) is not logically true in FL^{sc}. One might be tempted thus to conclude that FL^{sc} is nonextensional. But this conclusion is not imminent. Traditionally extensionality, *qua* condition on languages, concerns the interchangeability *salva veritate* of *statements* of the same truth-value, and of coextensive *terms*, singular and *general.* But (6) above reflects no breakdown in the principle of the interchangeability *salva veritate* of coextensive general terms; "$x = x$" and "$\mathrm{E}!x \,\&\, x = x$" are open *statements*, not general terms. What others[7] call *general terms* are called here complex predicates, and the interchangeability principle reflected in CD5 shows that FL^{sc} is general term extensional. Since it is easy to see that the other interchangeability principles mentioned above hold in FL^{sc}, this system is completely extensional—at least given the traditional interpretation of extensionality. All this despite the feel that (6) is not logically true. This suggests that arguments based on alleged failure of extensionality, arguments which have played such a central and forceful part in the philosophical logic of the past few decades, are in need of precise restatement and reassessment.

Finally, consider again the sort of argument that impressed many of the early pioneers of positive free logics, the argument that the classical inference pattern from

a is so and so,

to

There exists something that is
so and so,

is invalid when *a* is irreferential. The judgement seems warranted by the argument from

Vulcan is self-identical,

to

There exists something that is
identical with Vulcan,

an argument with an apparently true premise and a false conclusion. But the lesson to be learned from FL^{sc} is that *if* the classical inference pattern is construed (in its simplest form) as having the structure

$$\frac{\lambda y(Ay/b)a}{\text{Therefore, } \exists x(\lambda y(Ay/b)x)},$$

then the counterexample fails because the premise is false. On the other hand when construed as having the structure

$$\frac{Aa/x}{\text{Therefore, } \exists xA},$$

the premise could be true, and the counterexample thus succeeds. Both argument patterns have some call on the name *Existential Generalization*. Hence the debate between positive free logicians and others over the status of *Existential Generalization* may often be only verbal, and it is one of the benefits of FL^{sc} to have made this clear.

NOTES

1. Here Vulcan is the putative planet, not the god.
2. For an example of such a logic, see [7].
3. For an example of such a logic, see [5].
4. See [7], p. 104. The basis of this distinction in ordinary language has been detailed by Scales. Additionally, one may think of the distinction applied here to all simple statements as an extension of a similar distinction logicians in the Leśniewski tradition are fond of making in the case of simple identity statements.
5. See chapter IV. Scales' treatment antedates Stalnaker's treatment in [9] by almost a decade.
6. An example is provided in [8]. Let "R" abbreviate the set name "the Russell set." If s and t are codesignative just in case s designates whatever individual t designates, then "R" and the definite description "$(Ix)(x = R)$" are codesignative. But whereas "$R = R$" is true, "$R = (Ix)(x = R)$" is false in Scott's treatment.
7. For example, Leonard [3] and Quine [6].

REFERENCES

[1] Bencivenga, Ermanno, "Free semantics," *Boston Studies in the Philosophy of Science*, vol. 47 (1981), pp. 31–48.

[2] Burge, Tyler, "Truth and singular terms," *Noûs*, vol. 8 (1974), pp. 309–325.

[3] Leonard, Henry, "Essences, attributes and predicates," *Proceedings and Addresses of the American Philosophical Association*, vol. 37 (1964), pp. 25–51.

[4] Meyer, Robert K., Ermanno Bencivenga, and Karel Lambert, "The ineliminability of E! in free quantification theory without identity," *Journal of Philosophical Logic*, vol. 11 (1982), pp. 229–231.

[5] Meyer, Robert K. and Karel Lambert, "Universally free logic and standard quantification theory," *The Journal of Symbolic Logic*, vol. 33 (1968), pp. 8–26.

[6] Quine, W. V., *Theories and Things*, Belknap Press, Cambridge, Massachusetts, 1981.

[7] Scales, Ronald, *Attribution and Existence*, University of Michigan Microfilms, Ann Arbor, 1969.

[8] Scott, Dana, "Advice on modal logic," pp. 143–173 in *Philosophical Problems in Logic*, ed., Karel Lambert, Reidel, Dordrecht, 1970.

[9] Stalnaker, Robert, "Complex predicates," *The Monist*, vol. 60 (1977), pp. 327–340.

[10] Williams, C. J. F., *What is Existence?*, Clarendon Press, Oxford, 1981.

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