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Syllogisms with Statistical Quantifiers

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1 Introduction In this paper I will develop a system of categorical syllogisms, with square of opposition and rules of validity, for categorical statements having statistical quantifiers. Rules for the development of syllogistic systems having an indefinitely large number of quantifiers have already been worked out by Peterson and Carnes [3]. More recently, Brown [1] has provided a syntax and a semantics for quantifiers of all types. My approach differs from the work of Peterson and Carnes in two important respects:

1. Their approach was to develop infinitely many separate systems, each with a finite set of quantifiers defined within it. My approach in this paper will be to develop a single system within which infinitely many quantifiers are defined.

2. Their approach gave the normally vague quantifiers 'few' and 'many' a precise meaning within each system (through that meaning could vary from one system to the next). This was accomplished by selecting a pair of quantifiers within each system with which 'few' and 'many' could be arbitrarily identified. My approach will be to treat 'few' and 'many' as strictly associated with the base quantifier "100% of". They will, however, retain their vagueness, and this will make it necessary to add an arbitrary element elsewhere in the system.

2 Percentile quantifiers To express quantities other than those recognized for classical and intermediate categorical propositions, Peterson and Carnes use ratios, such as "$\frac{2}{3}$ of", rather than using quantifiers expressed as percentages. In this system we will take the alternative approach. We will recognize an infinite number of quantifiers for categorical propositions, all of the form "n% of", where 'n' is any real number such that $0 \leq n \leq 100$. This basic form may be modified by adding the word "almost", to produce another infinitely large set of quantifiers of the form "Almost n% of".

Both kinds of quantifiers receive minimal interpretation. In [5] I explain that a quantifier receives "minimal" interpretation when it means *at least* that

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quantity, and possibly more; a quantifier receives "maximal" interpretation when it means *no more than* that quantity, and possibly less. Thus the statement, "25% of S are P" is, of course, true if the actual percentage of S's that are P's is exactly 25%, but it is also true if the actual percentage of S's that are P's is 37%, 69%, or even 100%. The statement "Almost 33% of S are P", which might, of course, be true when in fact only 32% of S are P, would also be true when 47% of S are P. That is, it is true for quantities *slightly less* than 33%, and for any quantities greater than that. (The world "almost" is, of course, vague, so that the question how much less counts as "slightly" less is not a question that can be given a definitive answer. This creates some problems for our rules of validity, which will be discussed later.)

We will also recognize three types of quantifiers which may be defined in terms of the quantifiers which we already have:

1. Quantifiers of the form "More than n% of", where "More than n% of S are P" $=_{df}$ It is not the case that "$(100 - n)$% of S are not P".

2. Quantifiers of the form "Many more than n% of", where "Many more than n% of S are P" $=_{df}$ It is not the case that "Almost $(100 - n)$% of S are not P".

3. Quantifiers of the form "Less than n% of", where "Less than n% of S are P" $=_{df}$ "More than $(100 - n)$% of S are not P".

Quantifiers of the form "Less than n% of" are given *maximal* interpretation, i.e., they are understood to mean *no more than* that quantity, and possibly less. As with all maximally interpreted quantifiers, this gives them *negative* meaning. A categorical proposition of the form "Less than n% of S are not P" is, therefore, a *double* negative, and must be treated as an affirmative statement.

Quantifiers of the form "More than n% of" and those of the form "Many more than n% of" naturally receive minimal interpretation.

It should be noted that these definitions permit some rather anomolous constructions. It may make sense to say, for example, that prices have increased by more than 100%, but it is not possible for some statement to be true of more than 100% of a class. Hence, any time the quantifier constructions "More than 100% of" and "Many more than 100% of" appear in a proposition, that proposition will necessarily be false. Conversely, due to the concept of "minimal interpretation", the quantifier constructions "0% of" and "Almost 0% of", which must be taken to mean *at least* that quantity and possibly more, will invariably make the propositions in which they occur true. (This is not, of course, to say that such propositions are necessarily true when they occur in ordinary English. In the context of ordinary language we usually interpret these constructions as "100%...are not" and "Almost 100%...are not", respectively.)

Quantifier constructions which produce necessarily true or necessarily false propositions could, of course, be prohibited by placing an ad hoc restriction upon them, but in my opinion it is better simply to permit them. The consequences of such a policy are, I think, perfectly acceptable, provided we bear in mind a consistent distinction between validity and soundness. Thus it may be the case that certain syllogisms which employ these anomolous constructions will be valid, but unsound by virtue of having a necessarily false premise, or vacuous by virtue of having a necessarily true conclusion.

Given these primitive and defined quantifiers, it is possible to establish four types of quantification, each either affirmative or negative, yielding eight logically possible forms that a categorical proposition may take:

Affirmative	Negative
A: n% of *S* are *P*.	*E*: n% of *S* are not *P*.
P: Almost n% of *S* are *P*.	*B*: Almost n% of *S* are not *P*.
K: Many more than n% of *S* are *P*.	*G*: Many more than n% of *S* are not *P*.
I: More than n% of *S* are *P*.	*O*: More than n% of *S* are not *P*.

The following immediate inferences may be recognized as valid:

a. Conversion is valid only for *E* statements, where $n = 100$, and for *I* statements, where $n = 0$.
b. Contraposition is valid only for *A* statements, where $n = 100$, and for 0 statements, where $n = 0$.
c. Obversion is, as always, universally valid.

This system of categorical syllogisms includes both the complete Aristotelian system and the complete system of intermediate syllogisms developed by myself [5] and by Peterson and Carnes [3].[1] The following equivalencies show how.

Affirmative

A: "All *S* are *P*" is equivalent to "100% of *S* are *P*".
P: "Few *S* are not *P*" is equivalent to "Almost 100% of *S* are *P*".
T: "Most *S* are *P*" is equivalent to "More than 50% of *S* are *P*".
K: "Many *S* are *P*" is equivalent to "Many more than 0% of *S* are *P*".
I: "Some *S* are *P*" is equivalent to "More than 0% of *S* are *P*".

Negative

E: "No *S* are *P*" is equivalent to "100% of *S* are not *P*".
B: "Few *S* are *P*" is equivalent to "Almost 100% of *S* are not *P*".
D: "Most *S* are not *P*" is equivalent to "More than 50% of *S* are not *P*".
G: "Many *S* are not *P*" is equivalent to "Many more than 0% of *S* are not *P*".
O: "Some *S* are not *P*" is equivalent to "More than 0% of *S* are not *P*".

Besides showing that the older systems are contained in the present system, these equivalencies provide a rationale for assigning mnemonic letter names to the categorical statements recognized in this system. Except for *T* and *D* statements, the old letter names (explained in [5]) are simply carried over and extended to other statements of the same quality having the same quantifier form.

Thus, any affirmative statement with a quantifier of the form "n% of" may be referred to as an A statement. Any negative statement with a quantifier of the form "Almost n% of" may be referred to as a B statement. Hence, in the new system, T and D statements are merely I and O statements, respectively, where $n = 50$.

3 Distribution values In order to develop squares of opposition and rules of validity for this system it is necessary to have distribution values which can be assigned to the terms of each kind of categorical statement. In this system a distribution value, or distribution index,[2] will be a numerical value consisting of two parts:

1. the limiting value (or limit), which may be any real number, n, such that $0 \leq n \leq 100$. For all quantifiers receiving minimal interpretation, the 'n' of the limiting value will be just the 'n' of the percentile quantifier.

2. a modification indicator, written as a subscript to the limiting value. The function of the modification indicator is to measure the amount of slack or vagueness which the vague quantifiers have in a particular context. The modification indicator is expressed using two special variables, 'σ' and 'ι', which will require some explanation.

'σ' is the "significance level" of the context within which the statement occurs. Given a certain context, 'σ' is that value such that "Many more than n% of S are P" is *true* when the actual percentage of S's that are P's is $(n + \sigma)$ or greater. Because of the way in which "Many more than n% of" is defined, 'σ' is also that value such that "Almost n% of S are P" is *false* when the actual percentage of S's that are P's is $(n - \sigma)$ or less. 'σ' is defined arbitrarily, depending on the context of the statement but, if it is to function in accordance with its intended meaning, it cannot be less than or equal to 0 nor greater than 100, and in fact, like the significance level of statistical tests (with which it may be loosely identified), it will rarely be greater than 5.

'ι' is a special variable denoting "infinitesimal positive magnitude". The principles of an arithmetic using infinitesimal numbers, as originally envisioned by Leibniz, have been worked out in Robinson [4]. For our purposes only two of the properties of infinitesimals need to be noted:

a. $(n + \iota) > n$
b. if $m < n$, then $m < (n - (x \cdot \iota))$

where 'ι' is some positive infinitesimal, and 'm', 'n', and 'x' are any finite real numbers whatsoever.[3] Being greater than 0, 'ι' is a value such that "More than n% of S are P" is *true* when the actual percentage of S's that are P's is $(n + \iota)$ or greater. Consequently, 'ι' is also a value such that "Almost n% of S are P" is *true* when the actual percentage of S's that are P's is greater than, or equal to, $(n - (\sigma - \iota))$, or, what is the same thing, $(n + (\iota - \sigma))$.[4]

The following rules of distribution explain how a distribution index may be assigned to the terms of a categorical proposition:

1. Distribution by Quality
   a. Any affirmative statement gives its predicate a distribution index of 0_ι.
   b. Any negative statement gives its predicate a distribution index of 100_0.
2. Distribution by Quantity
   a. Any categorical statement with a quantifier of the form "n% of" gives its subject a distribution index of n_0.
   b. Any categorical statement with a quantifier of the form "Almost n% of" gives its subject a distribution index of $n_{(\iota-\sigma)}$.
   c. Any categorical statement with a quantifier of the form "More than n% of" gives its subject a distribution index of n_ι.
   d. Any categorical statement with a quantifier of the form "Many more than n% of" gives its subject a distribution index of n_σ.
   e. Any categorical statement with a quantifier of the form "Less than n% of" gives its subject a distribution index of $(100 - \mathrm{n})_\iota$.

The distribution indices of two terms may be added to or subtracted from each other as follows.

For addition: $m_x + n_y = (m + n)_{(x+y)}$.
For subtraction: $m_x - n_y = (m - n)_{(x-y)}$.

In general, comparisons between two values must consider both the limiting value and the modification indicator. As long as these two factors agree, there is no problem. Thus it is *true* that '$83_0 < 100_\iota$' since both '$83 < 100$' and '$0 < \iota$' are true. In cases where the two factors disagree, however, it is necessary to make some calculations in which the value of 'σ' may need to be considered. We may state the formula for comparison of distribution values as,

$$(m_x > n_y) \text{ iff } (m - n) > (y - x) \ .$$

This makes it possible to state the general principle of implication for categorical statements. Let *Sp* be the distribution index of the subject of some categorical proposition, *p*, and let *Sq* be the distribution index of the subject of some proposition, *q*, having the same terms as *p*. We may say that *p* implies *q* iff

1. *p* and *q* have the same quality, and
2. $Sp \geq Sq$.

This allows us to account for such intuitively valid immediate inferences as,

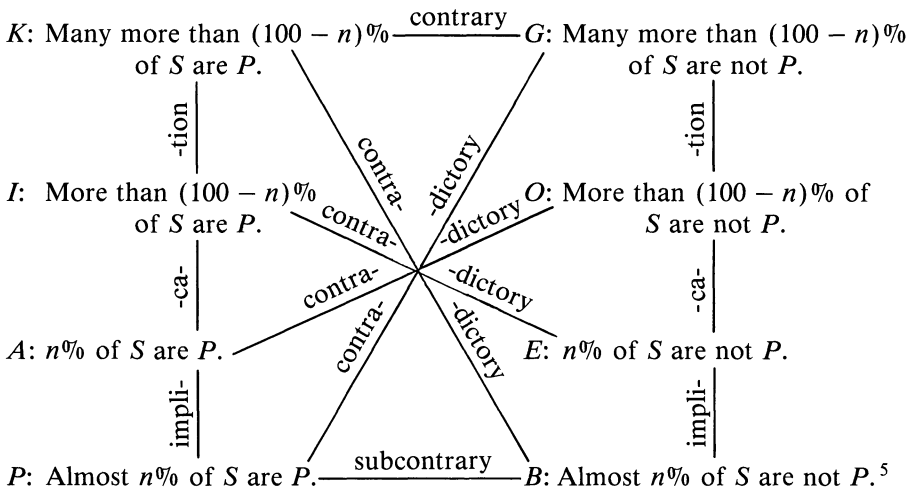
$$\frac{47\% \text{ of } S \text{ are } P.}{\therefore \text{Almost } 50\% \text{ of } S \text{ are } P.}$$

The value of *Sp* is 47_0, while the value of *Sq* is $50_{(\iota-\sigma)}$. Let us assume $\sigma = 5$. Then,

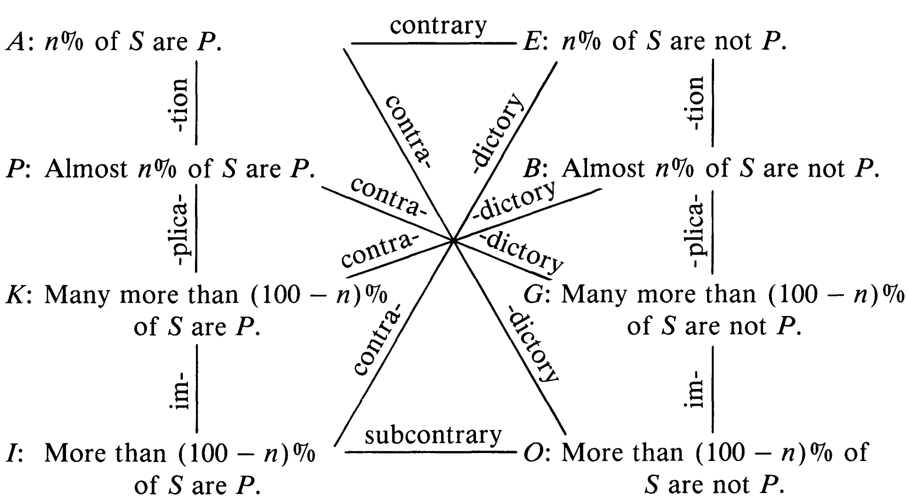
$$(47 - 50) = -3 > -(5 + \iota) = (\iota - 5) - 0 \ .$$

Hence the inference is valid, in settings where $\sigma = 5$, or indeed in any settings where $\sigma > 3$.

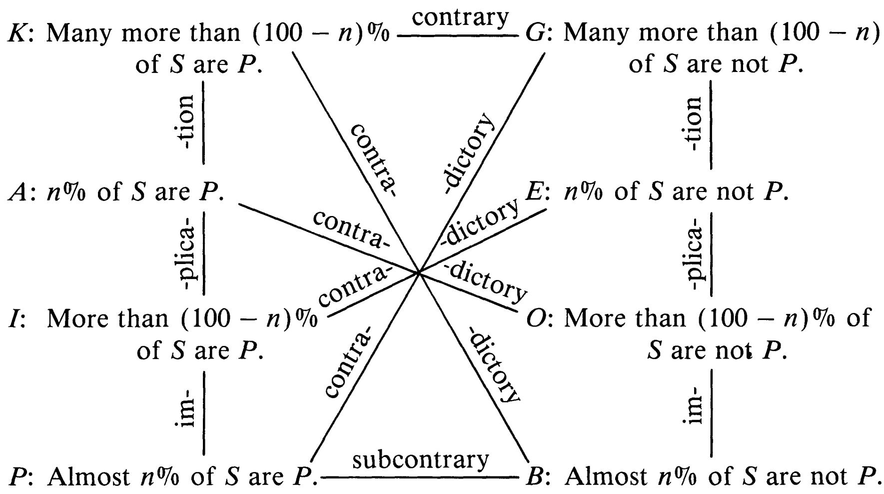
4 Square of opposition We have now defined all of the relations which make up the Square (or rather Squares) of Opposition for this system. In addition to the implication relation just stated, the relation of contradiction follows from the definitions of the derived quantifiers. Contrary and subcontrary relations are derivable from the implication and contradiction relations, so do not need to be independently defined. Thus, for any number, n, it is possible to construct a complete square of opposition. There are, however, three distinct patterns which such a square may have. For $n \leq 50$, the following pattern occurs:



For $n > 50$, the pattern depends upon the value of 'σ'. Normally 'σ' will be less than the difference between n and 50, and when this is so, the Square of Opposition looks like this:



Sometimes, however, it is possible for 'σ' to be *greater* than $n - 50$ (for example, when $\sigma = 5$, and $n = 51$). In this unusual circumstance, the following pattern occurs[6]:



*5 **Rules of validity*** The ability to add, subtract, and compare distribution values makes it possible to use mathematical formulas to express the rules of validity for this system of categorical syllogisms. Let *M1* and *Pp* be the distribution indices of the terms of the major premise (the middle term and the major term, respectively) Let *M2* and *Sp* be the distribution indices of the terms of the minor premise (the middle term and minor term, respectively). Let *Sc* and *Pc* be the distribution indices of the terms of the conclusion (the minor term and major term, respectively). Finally, let *PM* be the distribution index of the predicate of the major premise, and let *Pm* be the distribution index of the predicate of the minor premise. The maximum distribution which any single occurrence of a term can receive is 100_0, so a term with a distribution index of 100_0 may be said to be fully, or maximally distributed. The rules of validity may be stated as follows:

Rule 1. The middle term must be more than maximally distributed by the premises, i.e., $M1 + M2 > 100_0$.

Rule 2. The minor term in the premises must be distributed to at least the same degree as the minor term in the conclusion, i.e., $Sp \geq Sc$.

Rule 3. The major term in the premises must be distributed to at least the same degree as the major term in the conclusion, i.e., $Pp \geq Pc$.

Rule 4. The number of negative premises must equal the number of negative conclusions, i.e., $PM + Pm = Pc + 0_\iota$.

Some examples will show how this system functions. To take the most obvious type of example, the following syllogism is valid.

All *M* are *P*.	$M1 = 100_0$	$Pp = 0_\iota$
<u>37.2% of *S* are *M*.</u>	$Sp = 37.2_0$	$M2 = 0_\iota$
∴37.2% of *S* are *M*.	$Sc = 37.2_0$	$Pc = 0_\iota$

The syllogism meets Rule 1, since $(M1 + M2) = (100_0 + 0_\iota) = (100 + 0)_\iota = 100_\iota$; and it is true that $100_\iota > 100_0$, since $(100 - 100) = 0 > -\iota = (0 - \iota)$. The syllogism meets Rule 2, since $37.2_0 \geq 37.2_0$. The syllogism meets Rule 3, since $0_\iota \geq 0_\iota$.

Rule 4 captures the principle that a valid syllogism may have a negative conclusion if and only if one (and only one) of the premises is negative. If all three propositions are affirmative (as in this case), then $PM + Pm = 0_\iota + 0_\iota = Pc + 0_\iota$. If one of the premises were negative, then $PM + Pm$ would be equal to $100_0 + 0_\iota$. In that case the conclusion would also need to be negative so that $Pc + 0_\iota$ would equal $100_0 + 0_\iota$ as well.

Rules 2, 3, and 4 are relatively straightforward. Most of the complications in the system are to be encountered in the application of Rule 1 to syllogisms in the third figure. For example, the following syllogism is valid.

Almost 27% of M are not P.	$M1 = 27_{(\iota-\sigma)}$	$Pp = 100_0$
Many more than 73% of M are S.	$M2 = 73_\sigma$	$Sp = 0_\iota$
∴ Some S are not P.	$Sc = 0_\iota$	$Pc = 100_0$

Again the syllogism meets Rule 1, since $(M1 + M2) = (27_{(\iota-\sigma)} + 73_\sigma) = (27 + 73)_{((\iota-\sigma)+\sigma)} = 100_\iota$. The other rules are, of course, also met. To understand why this syllogism is valid it is important to remember that "almost" and "many more than" are defined as having complementary meanings, like "few" and "many" in the intermediate syllogisms (see [2] and [5] for discussion). Thus, granting that we do not know precisely how much short of 27% we may be in the major premise, we know that we are at least that much over 73% in the minor premise. That is, we know that there must be at least some overlap among the M's spoken of in the first premise and those spoken of in the second premise.

The following syllogism is also valid.

Less than 25% of M are P.	$M1 = 75_\iota$	$Pp = 100_0$
25% of M are S.	$M2 = 25_0$	$Sp = 0_\iota$
∴ Some S are not P.	$Sc = 0_\iota$	$Pc = 100_0$

$M1$ has the value 75_ι since the "Less than n% of" quantifier construction gives its subject term a value of $(100 - n)_\iota$, which in this case is $(100 - 25)_\iota = 75_\iota$. Pp has the value 100_0, since the "Less than n% of" quantifier receives maximal interpretation, so that the major premise in this case is a negative proposition. Rule 1 is met since $(75_\iota + 25_0) = 100_\iota > 100_0$. Again there is no particular problem with the other rules, all of which are met.

The following syllogism may or may not be valid, depending on the value of σ in the setting in which the syllogism occurs.

Almost 50% of M are P.	$M1 = 50_{(\iota-\sigma)}$	$Pp = 0_\iota$
Almost 55% of M are S.	$M2 = 55_{(\iota-\sigma)}$	$Sp = 0_\iota$
∴ Some S are P.	$Sc = 0_\iota$	$Pc = 0_\iota$

The calculation for Rule 1 gives us $(M1 + M2) = (50_{(\iota-\sigma)} + 55_{(\iota-\sigma)}) = 105_{2(\iota-\sigma)}$. The question is whether $105_{2(\iota-\sigma)} > 100_0$. The formula for comparison recasts this as $(105 - 100) = 5 > (2\sigma - 2\iota) = 0 - 2(\iota - \sigma)$. Let us suppose

$\sigma = 5$. In that case the syllogism is invalid, since it is false that $5 > (10 - 2\iota)$. Since 'ι' is an infinitesimal number, the quantity $(10 - 2\iota)$ is *infinitely close* to 10; and any number infinitely close to 10 will naturally be greater than 5. Now let us suppose that $\sigma = 1$. In that case the syllogism is valid, since it is true that $5 > (2 - 2\iota)$. Thus we may say that this syllogism is ambiguously valid, since, without knowing the context of the argument we cannot know whether σ is small enough to make the conclusion follow from the premises or not.

Of the 105 valid classical and intermediate syllogisms (vide Peterson and Carnes [3]) several may be regarded, technically speaking, as ambiguously valid. In all of these cases, however, the syllogisms could only be regarded as invalid for $\sigma > 50$. For example, the *BPO-3* syllogism looks like this:

Few *M* are *P*.	$M1 = 100_{(\iota-\sigma)}$	$Pp = 100_0$
Few *M* are not *S*.	$M2 = 100_{(\iota-\sigma)}$	$Sp = 0_\iota$
∴Some *S* are not *P*.	$Sc = 0_\iota$	$Pc = 100_0$

The calculation for Rule 1 goes as follows. $(M1 + M2) = (100_{(\iota-\sigma)} + 100_{(\iota-\sigma)} = 200_{2(\iota-\sigma)}$. The comparison $200_{2(\iota-\sigma)} > 100_0$ becomes $(200 - 100) = 100 > (2\sigma - 2\iota)$. The syllogism is therefore valid for any settings in which $\sigma \leq 50$. It is difficult to imagine any occasions, even in ordinary unscientific arguments, where the value of σ would go much over 5. σ greater than 50 is virtually inconceivable, so it is probably safe to say that all of the classical and intermediate syllogisms are unambiguously valid, speaking at least in practical terms.

There are also a couple of situations in which first figure syllogisms may be ambiguously valid. The more mundane type of situation involves the application of Rule 2. For example,

All *M* are *P*.	$M1 = 100_0$	$Pp = 0_\iota$
83% of *S* are *M*.	$Sp = 83_0$	$M2 = 0_\iota$
∴Many more than 80% of *S* are *P*.	$Sc = 80_\sigma$	$Pc = 0_\iota$

Whether Rule 2 is met in this case depends upon the value of σ. Let us suppose $\sigma = 1$. In that case $83_0 \geq 80_\sigma$ is true, since $(83 - 80) = 3 > \sigma = (\sigma - 0)$. That is, the syllogism is valid since $3 > 1$ is true. But if $\sigma = 5$, then the syllogism is invalid, because $3 > 5$ is false. Ambiguities of this type are in fact related to ambiguities on Rule 1. Using the reverse of Aristotle's method of reducing to first figure, we may replace the major premise of the above syllogism with the denial of the conclusion, and replace the conclusion with the denial of the major premise, to obtain a new syllogism.

Almost 20% of *S* are not *P*.	$M1 = 20_{(\iota-\sigma)}$	$Pp = 100_0$
83% of *S* are *M*.	$M2 = 83_0$	$Sp = 0_\iota$
∴Some *M* are not *P*.	$Sc = 0_\iota$	$Pc = 100_0$

This syllogism, like the previous one, is valid for $\sigma = 1$, but invalid for $\sigma = 5$. $(M1 + M2) = (20_{(\iota-\sigma)} + 83_0) = 103_{(\iota-\sigma)}$. Given $\sigma = 1$, then $103_{(\iota-\sigma)} > 100_0$, since $(103 - 100) = 3 > (1 - \iota) = 0 - (\iota - 1)$. Given $\sigma = 5$, then $103_{(\iota-\sigma)} > 100_0$ is false, since $(103 - 100) = 3 \not> (5 - \iota) = 0 - (\iota - 5)$.

Finally there are some ambiguously valid first figure syllogisms that run afoul of Rule 1. For example,

Many more than 97% of M are P.	$M1 = 97_\sigma$	$Pp = 0_\iota$
42% of S are M.	$Sp = 42_0$	$M2 = 0_\iota$
∴ 42% of S are P	$Sc = 42_0$	$Pc = 0_\iota$

The situation in this case is complicated. Under normal circumstances a person would not use a construction like "Many more than 97% of" unless he was not prepared to say simply "all". Thus we would normally suppose that, in such an instance, σ would be less than $(100 - n)$. If this is so, then the syllogism is invalid. Let us suppose $\sigma = 1$. In that case, $(M1 + M2) = (97_\sigma + 0_\iota) = 97_{(\sigma+\iota)}$. It is, however, false that $97_{(\sigma+\iota)} > 100_0$, since $(97 - 100) = -3 \not> -(1 + \iota) = 0 - (\sigma + \iota)$. If, on the other hand, we suppose that σ is greater than $(100 - n)$, then the syllogism is valid, but cannot possibly be sound. Let us suppose, for example, that $\sigma = 5$. In that case the syllogism would be valid since $(97 - 100) = -3 > -(5 + \iota) = -(\sigma + \iota) = 0 - (\sigma + \iota)$. But then the major premise couldn't possibly be true, since the actual percentage of M's that are P's would have to be at least 102% or greater, which is absurd. Thus the syllogism would be valid but not sound. In only one kind of case could such a syllogism be both valid and sound, namely the situation in which σ is precisely equal to $(100 - n)$.

6 Conclusion In my opinion, the main virtue of this system rests in its ability to reflect our ordinary intuitions about arguments. In particular, by permitting ambiguities I think that this system captures our ordinary intuitions better than would a system with no ambiguities. Moreover, this system is able to express arguments that cannot be captured by the lower predicate calculus. Despite recent work by Brown [1], the predicate calculus still cannot recognize as valid any arguments which employ quantifiers other than the existential and the universal, and even then it cannot recognize as valid any arguments in which a particular conclusion is drawn from universal premises. The system of syllogisms presented in this paper and the predicate calculus each recognize infinitely many argument forms, but their area of overlap is a mere fifteen forms: the classical Boolean syllogisms. In light of this, it will be interesting to see whether a system of predicate logic can be developed which would include the capacities of both systems.

NOTES

1. The system developed in [5] fails to recognize twelve valid syllogisms in the third figure. For example the TTI-3 syllogism,

   Most M are P.
   Most M are S.
   Some S are P.

   should have been recognized as valid. The system developed by Peterson and Carnes in [3] corrects this defect in my original system.

2. The term "distribution index" was coined by Peterson and Carnes [3].

3. Robinson defines a number, a, as infinitesimal if $|a| < m$ for all positive finite values of m. As Robinson points out, this definition makes 0 an infinitesimal. Infinitesimal values not equal to 0 may be defined in this manner:

   a. A number, r, is said to be infinite if it is not the case that there exists a finite real number, m, such that $|r| \leq m$.
   b. A number a, is said to be infinitesimal if there exists an infinite number, r, such that $a = r^{-1}$.

4. The referee has pointed out that if 'ι' is conceived as a specific value, then given that "More than n% of S are P" is true when $(n + \iota)$% of S's are P's, then it will be possible to postulate some smaller infinitesimal number, ι', so that we may imagine a situation in which "$(n + \iota')$% of S are P" is true, while "$(n + \iota)$% of S are P" is false. Under such conditions we would wish to say that "More than n% of S are P" is true, while in fact it fails to meet the specified truth conditions. I think this problem can be side-stepped by treating 'ι' not as a specific infinitesimal value, but as a variable which may be satisfied by any infinitesimal positive magnitude whatsoever.

5. Not every relation is shown on this, and on the following squares. The relations not shown are, however, fairly easy to discover.

6. In the still more unusual case where σ is exactly equal to $(n - 50)$, the square collapses from eight statements to four. The truth conditions of the A statement coincide with those of the K statement, those for I coincide with those for P, those for E with those for G, and those for B with those for O.

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