

Defining "Good" and "Bad" in Terms of "Better"

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Abstract Monadic predicates for "good" and "bad" are inserted into structures already containing the dyadic predicate "better". A set of logical properties for "good" and "bad" is proposed, and a complete characterization is given of the pairs of monadic predicates that have these properties. It is argued that "good" and "bad" can indeed be defined in terms of "better", and a definition is given that is more generally applicable than those previously proposed.

1 Introduction Several authors have proposed definitions of the monadic value predicates "good" and "bad" in terms of the dyadic predicate "better". The best-known of these proposals is the definition of "good" as "better than its negation" and of "bad" as "worse than its negation". The first clear statement of this idea seems to be that by Brogan [4]. It has been accepted by, among others, Mitchell ([14], pp. 103–105), Halldén ([8], p. 109), von Wright ([19], p. 34, and [20], p. 162), and Åqvist [2]. Lenzen ([12], [13]) has investigated the formal properties of these and related definitions in a metricized structure, i.e., a structure in which a numerical value $u(x)$ is attached to each object x of value judgments, such that x is better than y if and only if $u(x) > u(y)$.

Chisholm and Sosa ([5], pp. 245–246) criticized Brogan's definitions, since they imply that the negation of a good state of affairs is always bad and that the negation of a bad state of affairs is always good. According to these authors, some existential statements are good or bad but have a negation that is neither good nor bad. Examples of this are "there are happy egrets" and "there are unhappy egrets".

Chisholm and Sosa proposed a different definition of "good" and "bad" in terms of "better". As a starting point, they defined the notion of an indifferent state of affairs. A state of affairs is indifferent, they say, if and only if it is nei-

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ther better nor worse than its negation. Then “a state of affairs is *good* provided it is better than some state of affairs that is indifferent, and . . . a state of affairs is *bad* provided some state of affairs that is indifferent is better than it”. Chisholm and Sosa’s approach may be seen as a special case of a more general format for defining “good” and “bad” delineated by van Dalen [6]. This format consists of the introduction of a set of neutral propositions. Then anything that is better than some neutral proposition is good, and anything that is worse than some neutral proposition is bad.

Danielsson ([7], p. 37) introduced another definition that falls within van Dalen’s general framework. According to Danielsson, a statement is good if and only if it is better than some tautology, and bad if and only if it is worse than some tautology. (This definition was, according to Danielsson, originally proposed by Bengt Hansson in an unpublished essay.) The use of contradictions, instead of tautologies, as indifference points, was hinted at by von Wright ([20], p. 164).

2 Preference structures This article is devoted to the investigation of the insertion of monadic predicates for “good” and “bad” into structures already containing a dyadic predicate for “better”. The predicates will be considered as referring to propositions that represent states of affairs. Throughout the article, standard first-order logic and the intersubstitutivity of logically equivalent propositions will be assumed to hold.

Definition D1 Let L be any language. Then a *preference structure* on L is a pair $\langle L, R \rangle$, where R is a reflexive relation on L .

In this paper, R represents the notion “better than or equal in value to”. The predicates P for “better than”, I for “equal in value to”, and C for “comparable to” will be defined in the usual way.

Definition D2

$$\begin{aligned} pPq &\leftrightarrow pRq \ \& \ \sim (qRp) \\ pIq &\leftrightarrow pRq \ \& \ qRp \\ pCq &\leftrightarrow pRq \ \vee \ qRp. \end{aligned}$$

Our definition of a preference structure is very weak. In particular, the conditions of transitivity ($(pRq \ \& \ qRr) \rightarrow pRr$) and completeness ($pRq \ \vee \ qRp$) are absent from the definition.

The transitivity of weak preference (R) implies transitivity of indifference (I). This is a highly controversial assumption that “has as an instance that if Henry is indifferent between cups of coffee with 1 grain of sugar and 2 grains, 2 grains and 3 grains, . . . , 499 grains and 500 grains, then Henry will be (and/or ought to be) indifferent between cups with 1 grain and 500 grains” ([15], p. 252). The transitivity of strict preference (P) has also been challenged by several authors (see [1], [10], [17]).

As I argue more in detail elsewhere (see [9]), completeness, i.e., full comparability between alternatives, does not always hold. I may be sure that I prefer winning \$5000 in a lottery to winning \$50 in that same lottery. I may also be sure

that I prefer my friend Bob being promoted on his job to him staying in the same position. However, I may still not know which I prefer, myself winning \$5000 or Bob being promoted.

In view of the problematic character of these two conditions, it should be seen as an advantage of the present formal system that it takes neither transitivity nor completeness for granted. However, a weaker comparability assumption will be needed for most of the results.

Definition D3 A preference structure $\langle L, R \rangle$ fulfills the *weak comparability assumption* (WCA) iff for all $p, q \in L$, if pCq and p and q are not logically equivalent, then $pC\sim p$.

The following defined dyadic operators will be needed.

Definition D4 pR^*q iff there are $r_1, \dots, r_n \in L$ such that $pRr_1 \& r_1Rr_2 \& \dots \& r_nRq$; pI^*q iff there are $r_1, \dots, r_n \in L$ such that $pIr_1 \& r_1Ir_2 \& \dots \& r_nIq$; pP^+q iff there are r_1 and $r_2 \in L$ such that $pR^*r_1 \& r_1Pr_2 \& r_2R^*q$.

R^* will be called “iterated weak preference”, I^* “iterated indifference”, and P^+ “iterated strict preference”. Conjunctions with the symbols R, P, I, C, R^*, I^* , and P^+ will be contracted, writing $pRqIr$ for $pRq \& qIr$, $pR^*qI^*sI\sim s$ for $pR^*q \& qI^*s \& sI\sim s$, etc.

3 The logical properties of “good” and “bad” “Good” is a positive value predicate, just like “best”, “not worst”, “very good”, “excellent”, “not very bad”, etc. Each of these predicates has the property of being applicable to everything that is better or equal in value to something that it is applicable to. This property will be called “positivity”.

Similarly, “bad” is one of the predicates with the converse property, that it is applicable to whatever is worse than or equal in value to something that the predicate is applicable to.

Definition D5 Let H be a monadic predicate on the preference structure $\langle L, R \rangle$. Then H fulfills *positivity* iff for all $p, q \in L$, $Hp \& qRp \rightarrow Hq$. H fulfills *negativity* iff for all $p, q \in L$, $Hp \& pRq \rightarrow Hq$.

A pair $\langle G, B \rangle$ of monadic predicates on $\langle L, R \rangle$ fulfills the combined property of *positivity/negativity* (PN) iff G fulfills positivity and B fulfills negativity.

PN is fulfilled not only by “good” and “bad”, but also by many other pairs of monadic value predicates, such as “very good” and “very bad”, or “not worst” and “not best”. Other intuitions will have to be invoked, therefore, to characterize the notions “good” and “bad”.

One such intuition is mutual exclusiveness. It does not seem reasonable to say that something is (from the same point of view) both good and bad.

Definition D6 Let $\langle G, B \rangle$ be a pair of monadic predicates on $\langle L, R \rangle$. Then $\langle G, B \rangle$ fulfills *mutual exclusiveness* (ME) iff for all $p \in L$, $\sim(Gp \& Bp)$.

A related intuition is that a state of affairs and its negation are not both good or both bad. If I say that it is good that p , but also that it would be good

if $\sim p$, then I will be taken to refer to different standards in the two uses of the word “good”. This property will be called “nonduplicity”.

Definition D7 Let $\langle G, B \rangle$ be a pair of monadic predicates on $\langle L, R \rangle$. Then $\langle G, B \rangle$ fulfills *nonduplicity* (ND) iff for all $p \in L$, $\sim(Gp \ \& \ G\sim p)$ and $\sim(Bp \ \& \ B\sim p)$.

Furthermore, “good” and “bad” are a pair of operators that come very close to each other, so that they only have “neutral” values between them. One way to express this is “that things which are neither good nor bad are not among themselves better or worse” ([20], p. 161). Another is that “if two things are of unequal value, then at least one of them must be good or at least one of them bad” (ibid.). These two formulations can be shown to be equivalent for pairs of operators that fulfill PN.

Theorem T1 Let $\langle L, R \rangle$ be a preference structure and $\langle G, B \rangle$ a pair of monadic predicates over $\langle L, R \rangle$ that fulfills PN. Then each of the following two conditions on $\langle G, B \rangle$ holds iff the other holds:

- (a) $\sim Gp \ \& \ \sim Bp \ \& \ \sim Gq \ \& \ \sim Bq \rightarrow \sim(pPq) \ \& \ \sim(qPp)$
- (b) $pPq \rightarrow Gp \vee Bq$.

(For proofs of the theorems, see the Appendix.)

The shorter of the two formulas, viz. (b), will be used in the definition of closeness.

Definition D8 Let $\langle G, B \rangle$ be a pair of monadic predicates on the preference structure $\langle L, R \rangle$. Then $\langle G, B \rangle$ fulfills *closeness* (CL) iff for all $p, q \in L$, $pPq \rightarrow Gp \vee Bq$.

It will not be assumed that all states of affairs are comparable to their negations. However, the following weaker comparability condition will be needed in several of the results of the following sections.

Definition D9 Let $\langle G, B \rangle$ be a pair of monadic predicates on the preference structure $\langle L, R \rangle$. Then $\langle G, B \rangle$ fulfills *comparability with negation* (CN) iff for all $p \in L$, $Gp \vee Bp \rightarrow pC\sim p$.

The search for pairs corresponding to “good” and “bad” will take the form of a search for pairs that have the properties positivity/negativity (PN), mutual exclusiveness (ME), nonduplicity (ND), closeness (CL), and comparability with negation (CN). However, mutual exclusiveness (ME) will not be explicitly referred to, since it follows from the other properties.

Theorem T2 Let $\langle G, B \rangle$ be a pair of monadic predicates on the preference structure $\langle L, R \rangle$. Then if $\langle G, B \rangle$ fulfills PN, ND, and CN, it also fulfills ME.

4 A complete characterization of predicates fulfilling the properties In this section, some new predicates for “good” and “bad” will be introduced, including “canonical good” and “canonical bad” (Definition D11). Furthermore, some formal results for these predicates will be given, including a complete characterization of pairs of predicates that fulfill PN, CL, ND, and CN (Theorem T6).

The following definition and theorem provide a necessary and sufficient condition for a preference structure having a pair $\langle G, B \rangle$ that fulfills the four conditions PN, CL, and ND.

Definition D10 A preference structure $\langle L, R \rangle$ fulfills the *weak structure condition* (WSC) iff there are no $p, q \in L$ such that $\sim pRpP^+qR\sim q$.

Theorem T3 Let $\langle L, R \rangle$ be a preference structure fulfilling WCA. Then if and only if it fulfills WSC is there a pair $\langle G, B \rangle$ of monadic operators on $\langle L, R \rangle$ that fulfills PN, CL, and ND.

Next, a pair of monadic operators, $\langle G_C, B_C \rangle$, will be defined. It can be shown to fulfill PN, CL, ND, and CN, in all preference structures fulfilling WCA and WSC. The index “C” of G_C and B_C stands for “canonical”, for reasons to be explained in Section 6.

Definition D11 Let $\langle L, R \rangle$ be a preference structure. Then the predicates G_C for *canonical good* and B_C for *canonical bad* are defined as follows:

$$\begin{aligned} G_C p &\leftrightarrow pP\sim p \ \& \ \sim(\exists q)(\sim qRqR^*p) \\ B_C p &\leftrightarrow \sim pPp \ \& \ \sim(\exists q)(pR^*qR\sim q). \end{aligned}$$

Theorem T4 Let $\langle L, R \rangle$ be a preference structure fulfilling WCA and WSC. Then $\langle G_C, B_C \rangle$, as defined on $\langle L, R \rangle$, fulfills PN, CL, ND, and CN.

The pair $\langle G_C, B_C \rangle$ can be shown to be “maximal” in the sense that if another pair $\langle G, B \rangle$ fulfills the conditions for being a plausible candidate for “good” and “bad”, then Gp implies $G_C p$ and Bp implies $B_C p$.

Theorem T5 Let $\langle L, R \rangle$ be a preference structure, $\langle G_C, B_C \rangle$ the canonical pair on $\langle L, R \rangle$, and $\langle G, B \rangle$ any pair of predicates on $\langle L, R \rangle$ that fulfills PN, ND, and CN. Then $Gp \rightarrow G_C p$ and $Bp \rightarrow B_C p$.

The following definitions and theorem provide a complete characterization of all the pairs of monadic predicates that, given a preference structure $\langle L, R \rangle$, fulfill the conditions PN, CL, ND, and CN in this structure.

Definition D12 An I^* -class S is a subset of L such that, for all $p, q \in L$, $p \in S \rightarrow (q \in S \leftrightarrow pI^*q)$.

It should be obvious that I^* -classes are equivalence classes.

Definition D13 A *marginally good set* is an I^* -class S such that (1) if $p \in S$, then $G_C p$, and (2) if $p \in S$, then for all $q \in L$, $pP^+q \rightarrow \sim qPq$.

A *marginally bad set* is an I^* -class S such that (1) if $p \in S$, then $B_C p$, and (2) if $p \in S$, then for all $q \in L$, $qP^+p \rightarrow qP\sim q$.

M^+ is the set of all marginally good sets. M^- is the set of all marginally bad sets.

Theorem T6 Let $\langle L, R \rangle$ be a preference structure fulfilling WCA and WSC. Furthermore let $\langle G, B \rangle$ be a pair of monadic predicates on $\langle L, R \rangle$. Then $\langle G, B \rangle$ fulfills PN, CL, ND, and CN iff there is a subset K^+ of M^+ and a subset K^- of M^- such that:

(1) If $p \in UK^+$ and $q \in UK^-$, then $\sim(pCq)$

- (2) $Gp \leftrightarrow G_C p \ \& \ p \notin UK^+$
 (3) $Bp \leftrightarrow B_C p \ \& \ p \notin UK^-$.

Corollary C1 *Let $\langle L, R \rangle$ be a preference structure fulfilling WCA and WSC. Furthermore, let $\langle G, B \rangle$ and $\langle G', B' \rangle$ be two pairs of monadic predicates on L , such that both fulfill PN, CL, ND, and CN. Then for no $p \in L$ is it the case that $Gp \ \& \ B'p$.*

$\langle G_C, B_C \rangle$ is the only pair of monadic operators that can fulfill PN, CL, and ND in a preference structure with full comparability.

Theorem T7 *Let $\langle L, R \rangle$ be a preference structure such that full comparability holds (i.e., pCq holds for all $p, q \in L$). Furthermore, let $\langle G, B \rangle$ be a pair of monadic predicates on $\langle L, R \rangle$. Then $\langle G, B \rangle$ fulfills PN, CL, and ND iff $\langle L, R \rangle$ fulfills WSC, and $\langle G, B \rangle$ is identical to $\langle G_C, B_C \rangle$.*

It also follows from Theorem T6 that predicates for “good” and “bad” can be defined that are minimal in the same sense that G_C and B_C are maximal.

Definition D14 Let $\langle L, R \rangle$ be a preference structure. Then the predicates G_{\min} for *minimally good* and B_{\min} for *minimally bad* are defined as follows:

$$G_{\min}p \leftrightarrow G_C p \ \& \ p \notin UM^+ \\
 B_{\min}p \leftrightarrow B_C p \ \& \ p \notin UM^-.$$

Theorem T8 *Let $\langle L, R \rangle$ be a preference structure fulfilling WCA and WSC, and let $\langle G, B \rangle$ be any pair of predicates on $\langle L, R \rangle$ that fulfills PN, CL, ND, and CN. Then $G_{\min}p \rightarrow Gp$ and $B_{\min}p \rightarrow Bp$.*

As was seen in Theorem T4, the maximally good and the maximally bad, i.e., G_C and B_C , combine into a pair $\langle G_C, B_C \rangle$ that fulfills PN, CL, and ND in all preference structures that are capable of containing any pair with this property. The same does not apply to the minimally good (G_{\min}) and the minimally bad (B_{\min}). This can be seen from the following preference structure: $L = \{p, \sim p, q, \sim q\}$, where R and P are transitive, and such that $\sim qPpPqP\sim p$. In this structure $\langle G_C, B_C \rangle$, $\langle G_C, B_{\min} \rangle$, and $\langle G_{\min}, B_C \rangle$ all fulfill PN, CL, ND, and CN, but $\langle G_{\min}, B_{\min} \rangle$ does not fulfill CL, since $pPq \ \& \ \sim G_{\min}p \ \& \ \sim B_{\min}q$. A more general result, including a sufficient condition for $\langle G_{\min}, B_{\min} \rangle$ to fulfill PN, CL, ND, and CN, is given in the following definition and theorem.

Definition D15 A preference structure $\langle L, R \rangle$ fulfills the property of *infinite divisibility* (DIV) iff for all $p, q \in L$, $pPq \rightarrow (\exists r)(pPrPq)$.

Theorem T9 *Let $\langle L, R \rangle$ be a preference structure fulfilling WCA and WSC. Then:*

- (1) $\langle G_C, B_{\min} \rangle$ and $\langle G_{\min}, B_C \rangle$ both fulfill PN, CL, ND, and CN, and
 (2) If $\langle L, R \rangle$ fulfills DIV, then $\langle G_{\min}, B_{\min} \rangle$ fulfills PN, CL, ND, and CN.

5 Negation-related and indifference-related “good” and “bad” In this section, two of the definitions of “good” and “bad” mentioned in Section 1 will be introduced into the formal system. The first of these is the definition of “good” as “better than its negation”, and of “bad” as “worse than its negation”.

Definition D16 Let $\langle L, R \rangle$ be a preference structure. Then the predicates G_N for *negation-related good* and B_N for *negation-related bad* are defined as follows:

$$G_N p \leftrightarrow pP \sim p$$

$$B_N p \leftrightarrow \sim pP p.$$

The following definition and theorem provide a condition for $\langle G_N, B_N \rangle$ to be, essentially, a plausible rendition of “good” and “bad”.

Definition D17 Let $\langle L, R \rangle$ be a preference structure. It fulfills the *strong structure condition* (SSC) iff for all $p, q \in L$, $\sim pR pR^* qR \sim q \rightarrow \sim pI pI^* qI \sim q$.

The intuitive understanding of the two conditions WSC and SSC may perhaps be furthered by a consideration of preference structures in which R, I, and P are transitive. In such structures, R and R* coincide, as do I and I* and P and P+. Then WSC excludes the following four structures (V denotes “better than” and \approx denotes “equal in value to”):

$$\begin{array}{cccc} \sim p & \sim p & & \\ \vee & \vee & & \\ p & p & p \approx \sim p & p \approx \sim p \\ \vee & \vee & \vee & \vee \\ q & q \approx \sim q & q & q \approx \sim q \\ \vee & & \vee & \\ \sim q & & \sim q & \end{array}$$

SSC excludes these same four structures, and in addition the following three:

$$\begin{array}{ccc} \sim p & \sim p & \\ \vee & \vee & \\ p \approx q & p \approx q \approx \sim q & \sim p \approx p \approx q \\ \vee & & \vee \\ \sim q & & \sim q \end{array}$$

Theorem T10 Let $\langle L, R \rangle$ be a preference structure that fulfills WCA. Then:
 (1) If $\langle G_N, B_N \rangle$ fulfills PN and CL then $\langle L, R \rangle$ fulfills SSC
 (2) If $\langle L, R \rangle$ fulfills SSC, then for all $p \in L$, $G_C p \leftrightarrow G_N p$ and $B_C p \leftrightarrow B_N p$
 (3) If $\langle L, R \rangle$ fulfills WSC and SSC, then $\langle G_N, B_N \rangle$ fulfills PN, CL, ND, and CN.

Theorem T11 Let $\langle L, R \rangle$ be a preference structure fulfilling WCA and SSC. Further, let $\langle G, B \rangle$ be any pair of predicates on $\langle L, R \rangle$ that fulfills PN, CL, ND, and CN. Then:

- (1) $G_N p \leftrightarrow G p \vee B \sim p$
- (2) $B_N p \leftrightarrow B p \vee G \sim p$.

As was also mentioned in Section 1, Chisholm and Sosa [5] propose another definition of “good” and “bad” in terms of “better”. They suggest that a state of affairs is good if and only if it is better than some other state of affairs that is in its turn equal in value to its negation. Correspondingly, a state of

affairs is bad if and only if it is worse than some other state of affairs that is in its turn equal in value to its negation. This version of “good” and “bad” can be introduced into the formal system as follows:

Definition D18 Let $\langle L, R \rangle$ be a preference structure. Then the predicates G_I for *indifference-related good* and B_I for *indifference-related bad* are defined as follows:

$$\begin{aligned} G_I p &\leftrightarrow (\exists q)(pPqI\sim q) \\ B_I p &\leftrightarrow (\exists q)(\sim qIqPp). \end{aligned}$$

Theorem T12 Let $\langle L, R \rangle$ be a preference structure that fulfills WCA and WSC. Then for all $p, q \in L$:

- (1) $pCqI\sim q \rightarrow (G_I p \leftrightarrow G_C p)$
- (2) $pCqI\sim q \rightarrow (B_I p \leftrightarrow B_C p)$.

Corollary C2 Let $\langle L, R \rangle$ be a preference structure that fulfills WCA and WSC, and such that $(\forall p)(\exists q)(pCqI\sim q)$. Then for all $p \in L$, $G_I p \leftrightarrow G_C p$ and $B_I p \leftrightarrow B_C p$.

Corollary C3 Let $\langle L, R \rangle$ be a preference structure that fulfills WCA, WSC, and SSC, and such that $(\forall p)(\exists q)(pCqI\sim q)$. Then for all $p \in L$, $G_I p \leftrightarrow G_N p$ and $B_I p \leftrightarrow B_N p$.

Corollary C2 shows that $\langle G_I, B_I \rangle$ coincides with $\langle G_C, B_C \rangle$ in those preference structures for which the definition of $\langle G_I, B_I \rangle$ seems to be intended. Corollary C3 shows that, in all preference structures in which both $\langle G_I, B_I \rangle$ and $\langle G_N, B_N \rangle$ are plausible renditions of “good” and “bad”, the two coincide.

6 Conclusions Of the different candidates for “good” and “bad” G_C and B_C have properties that give them a special standing. Provided that the weak comparability assumption (WCA) holds, the pair $\langle G_C, B_C \rangle$ fulfills the conditions of positivity/negativity (PN), closeness (CL), and nonduplicity (ND) in all preference structures in which any pair of predicates can fulfill these conditions (Theorems T3 and T4).

If full comparability holds, then $\langle G_C, B_C \rangle$ is the only pair that fulfills these conditions (Theorem T7). If full comparability does not hold, there may also be other pairs that fulfill the conditions. They may be seen as weakened forms of $\langle G_C, B_C \rangle$, assigning a neutral value to some (marginal) states of affairs that are assigned “good” or “bad” by $\langle G_C, B_C \rangle$ (Theorem T6).

Two other definitions of “good” and “bad” proposed by previous authors have a strong intuitive plausibility. They are negation-related “good” and “bad” as proposed by Brogan, and indifference-related “good” and “bad” as proposed by Chisholm and Sosa. They are both closely related to $\langle G_C, B_C \rangle$, and may be seen as special cases of the latter, though applicable in a smaller range of preference structures.

Negation-related “good” and “bad” ($\langle G_N, B_N \rangle$) fulfill plausible conditions for “good” and “bad” only in preference structures fulfilling the strong structure condition (SSC) of Definition D17. In such structures, G_N and G_C coincide, as do B_N and B_C (Theorem T10).

Indifference-related “good” and “bad” ($\langle G_I, B_I \rangle$) are plausible only in structures where each proposition is comparable to a proposition that is equal in value to its negation. In such structures, G_I and G_C coincide, as do B_I and B_C (Corollary C2 of Theorem T12).

In preference structures where both negation-related ($\langle G_N, B_N \rangle$) and indifference-related ($\langle G_I, B_I \rangle$) “good” and “bad” are plausible (i.e., where it is both the case that SSC holds and that each proposition is comparable to a proposition that is equal in value to its negation), G_N and G_C coincide, as do B_N and B_C (Corollary C3 of Theorem T12). This result indicates that the difference between these two definitions of “good” and “bad” should not be seen as concerning how to define these two monadic value-terms, given a preference structure. Rather, it should be seen as resulting from different views on which preference structures are plausible.

The above formal results lend support to the conclusion that “good” and “bad” are definable in terms of “better”. $\langle G_C, B_C \rangle$ is the most general definition of them, whereas $\langle G_N, B_N \rangle$ and $\langle G_I, B_I \rangle$ are special cases, applicable only in a smaller range of preference structures than is $\langle G_C, B_C \rangle$. This is the reason for the suggested names “canonical good” and “canonical bad” for G_C and B_C .

As was pointed out to me by Thorild Dahlquist, however, although $\langle G_C, B_C \rangle$ is in this sense the most general pair of predicates for “good” and “bad”, the definition format proposed by van Dalen [6] is in another sense more general. As was mentioned in Section 1, this format consists in the introduction of a set of neutral propositions, such that anything is good (bad) that is better (worse) than some neutral proposition. Since this format does not refer to negations, it may be applied to objects of valuation other than propositions, such as physical objects *per se*. The same style of definition may also be used for other triads of adjectives that are analogous to good-better-bad, such as big-bigger-small and happy-happier-unhappy. (On the logic of the positive, comparative, and opposite of adjectives see [3].)

The definability of “good” and “bad” in terms of “better” lends some plausibility to the opinion that “better” is “the fundamental value universal” [4], “the value-fundamental” [18], or “the basic notion of normative logic” ([11], p. 197).

However, it does not follow that “[v]alue judgements . . . have the form ‘A is better than B’ or [that] they can be reduced to this form” ([14], p. 114; cf. [16]). Although two such important monadic value predicates as “good” and “bad” can be defined in terms of “better” (as can, of course, “best” and “worst”), other monadic value predicates cannot be so defined. Informal discourse on values would be sadly incomplete without such predicates as “very bad”, “acceptable”, and “excellent”.

Appendix: Outline of proofs An outline of proofs will be given for Theorems T4, T6, T9, T10, and T12. The proofs of the rest of the theorems are straightforward enough to be left to the reader.

Proof of Theorem T4 (and the “if” part of Theorem T3): To show that CL is fulfilled, suppose it is not. Then there are $p, q \in L$ such that $pPq \ \& \ \sim G_C p \ \& \ \sim B_C q$. From pPq it follows, by WCA, that $pP\sim p \vee \sim pRq$ and $qR\sim q \vee \sim qPq$.

Case 1: $\sim pRp$ and $qR\sim q$. Then $\sim pRpPqR\sim q$, contrary to WSA.

Case 2: $\sim pRp$ and $\sim qPq$. From $\sim B_Cq$ and $\sim qPq$ follows $(\exists s)(qR^*sR\sim s)$. Then $\sim pRpPqR^*sR\sim s$, so that $\sim pRpP^+sR\sim s$, contrary to WSA.

Case 3: $pP\sim p$ and $qR\sim q$. This case can be proved in the same way as Case 2.

Case 4: $pP\sim p$ and $\sim qPq$. From $pP\sim p$ and $\sim G_Cp$ follows $(\exists r)(\sim rRrR^*p)$. From $\sim B_Cq$ and $\sim qPq$ follows $(\exists s)(qR^*sR\sim s)$. Thus $\sim rRrR^*pPqR^*sR\sim s$, so that $\sim rRrP^+sR\sim s$, contrary to WSA.

Thus, in all four cases, a contradiction can be derived. This completes the proof that CL holds. The proofs that PN, ND, and CN hold are all straightforward.

Proof of Theorem T6: For the proof from right to left: Assume that K^+ , K^- , G, and B are as stated in the theorem. It has to be shown that $\langle G, B \rangle$ fulfills PN, CL, ND, and CN.

To see that G fulfills positivity, suppose that Gp and qRp . Then since by (2) Gp implies G_Cp , we have G_Cp and qRp . By the positivity of G_C (Theorem T4) G_Cq follows.

From qRp it follows that either qIp or qPp . First suppose qIp . From Gp it follows, by (2), that $p \notin UK^+$. From qIp and $p \notin UK^+$ it follows, since UK^+ is a set of I*-classes, that $q \notin UK^+$.

Next, suppose qPp . In this case, suppose $q \in UK^+$. Then it follows from qPp , $q \in UK^+$, and clause (2) of the definition of a marginally good set (Definition D13) that $\sim pPp$, contrary to G_Cp . Thus $q \notin UK^+$.

Thus in both cases G_Cq & $q \notin UK^+$, i.e., Gq . This proves the positivity of G. The negativity of B can be proved in the same way, so that $\langle G, B \rangle$ fulfills PN.

To see that $\langle G, B \rangle$ fulfills CL, suppose pPq . By CL for $\langle G_C, B_C \rangle$ (Theorem T4) $G_Cp \vee B_Cq$ follows.

Suppose G_Cp & $\sim Gp$. Then by (2) $p \in UK^+$. From $p \in UK^+$ and pPq it follows, by clause (2) of the definition of a marginally good set (Definition D13), that $\sim qPq$.

Next, suppose there is a t such that $qR^*tR\sim t$. Then $pPqR^*tR\sim t$, so that $pP^+tR\sim t$, and hence, by clause (2) of the definition of a marginally good set (Definition D13), $p \notin UK^+$, contrary to the condition. Thus we have $\sim qPq$ & $\sim (\exists t)(qR^*tR\sim t)$, i.e., B_Cq . Since this follows from G_Cp & $\sim Gp$, we have G_Cp & $\sim Gp \rightarrow B_Cq$.

Similarly it can be shown that B_Cq & $\sim Bq \rightarrow G_Cp$. From $G_Cp \vee B_Cq$, G_Cp & $\sim Gp \rightarrow B_Cq$, and B_Cq & $\sim Bq \rightarrow G_Cp$ it follows that $\sim Gp$ & $\sim Bq \rightarrow G_Cp$ & B_Cq .

It follows by (2) that $\sim Gp$ & $G_Cp \rightarrow p \in UK^+$ and by (3) that $\sim Bq$ & $B_Cq \rightarrow q \in UK^-$. From these two results and $\sim Gp$ & $\sim Bq \rightarrow G_Cp$ & B_Cq it follows that $\sim Gp$ & $\sim Bq \rightarrow p \in UK^+$ & $q \in UK^-$. By pPq and clause (1) of the theorem it follows that $p \in UK^+$ & $q \in UK^-$ does not hold. Thus $\sim Gp$ & $\sim Bq$ does not hold, i.e., $Gp \vee Bq$ has been derived from pPq , so that CL holds.

To see that ND holds for $\langle G, B \rangle$, suppose it does not. Then there is a p such that either $Gp \ \& \ G\sim p$ or $Bp \ \& \ B\sim p$. In the first case it follows that $G_C p \ \& \ G_C \sim p$, and in the second that $B_C p \ \& \ B_C \sim p$. In both cases, it follows by Definition D11 that $pP\sim p \ \& \ \sim pPp$, in contradiction to Definition D2.

That CN holds for $\langle G, B \rangle$ follows from $Gp \rightarrow pP\sim p$ and $Bp \rightarrow \sim pPp$.

For the proof from left to right, let $\langle G, B \rangle$ be a pair of predicates that fulfills PN, CL, ND, and CN. Furthermore, let K^+ be the set of I^* -classes of elements p such that $G_C p \ \& \ \sim Gp$, and let K^- be the set of I^* -classes of elements p such that $B_C p \ \& \ \sim Bp$. Then it has to be shown:

- (1) that $K^+ \subseteq M^+$
- (2) that $K^- \subseteq M^-$
- (3) that if $p \in UK^+$ and $q \in UK^-$, then $\sim(pCq)$
- (4) that $Gp \leftrightarrow G_C p \ \& \ p \notin UK^+$
- (5) that $Bp \leftrightarrow B_C p \ \& \ p \notin UK^-$.

(1): Let p be an element of L , such that $G_C p \ \& \ \sim Gp$, and let S be the I^* -class of p . We proceed to show that S is an element of M^+ , i.e., that it is a marginally good set. By Definition D13, to do this we must prove that: (i) $q \in S \rightarrow G_C q$, and (ii) $q \in S \ \& \ qP^+r \rightarrow \sim rPr$.

For (i), suppose $q \in S$. Then pI^*q . From this and the positivity of G_C (Theorem T4) follows $G_C q$.

For (ii), suppose $q \in S$ and qP^+r . Since qP^+r there are s and t such that qR^*sPtR^*r . Since CL is assumed to hold for $\langle G, B \rangle$, $Gs \vee Bt$. Suppose Gs . Then, by qR^*s and the positivity of G , Gq . Furthermore, by pI^*q and the positivity of G , Gp , contrary to the conditions. Thus $\sim Gs$. Then, since $Gs \vee Bt$, we have Bt .

By the negativity of B , Br can be derived from Bt and tR^*r . From Br and CN, $\sim rPr \vee rR\sim r$ follows. Suppose $rR\sim r$. Then, by the negativity of B , $B\sim r$, contrary to ND. Thus $\sim rPr$.

(2): The proof is similar to the proof of (1).

(3): Suppose this does not hold. Then there are $q \in UK^+$ and $r \in UK^-$ such that qCr . From qCr , $rRq \vee qPr$ follows.

First suppose rRq . From $q \in UK^+$ it follows that there is a p such that $G_C p$ and pI^*q . Then, by the positivity of G_C (Theorem T4), $G_C q$. By $G_C q$, rRq , and the positivity of G_C , $G_C r$ follows. Furthermore, from $r \in UK^-$ it follows that there is an s such that $B_C s$ and sI^*r . Then, by the negativity of B_C , $B_C r$. Thus both $G_C r$ and $B_C r$. Since $G_C r$ implies $pP\sim p$ and $B_C r$ implies $\sim pPp$, this is a contradiction. Thus $\sim(rRq)$.

Next suppose qPr . Then, since CL has been assumed to hold for $\langle G, B \rangle$, $Gq \vee Br$. Suppose Gq . Since $q \in UK^+$, there is a p such that pI^*q and $G_C p \ \& \ \sim Gp$. From Gq and pI^*q it follows, by the positivity of G , that Gp . Thus Gq leads to a contradiction. Suppose instead Br . Since $r \in UK^-$, there is an s such that sI^*r and $B_C s \ \& \ \sim Bs$. From Br and sI^*r it follows, by the negativity of B , that Bs . Thus Br , too, leads to a contradiction. Thus $\sim(qPr)$.

In summary we have $rRq \vee qPr$, $\sim(rRq)$ and $\sim(qPr)$. This contradiction concludes this part of the proof.

(4): The proof of $Gq \leftrightarrow G_Cq \ \& \ q \notin UK^+$ will be divided into two parts.

(i) To prove $Gq \rightarrow G_Cq \ \& \ q \notin UK^+$, let q be an element of L such that Gq . Then G_Cq follows by Theorem T5. Suppose $q \in UK^+$. Then, by the above definition of K^+ , there is an r such that rI^*q and $\sim Gr$. By Gq and the positivity of G , Gr follows. It follows from this contradiction that $q \notin UK^+$.

(ii) For the proof in the other direction, suppose $G_Cq \ \& \ \sim Gq$. Then, by the definition of K^+ , $q \in UK^+$. We therefore have $G_Cq \ \& \ \sim Gq \rightarrow q \in UK^+$, or, equivalently, $G_Cq \ \& \ q \notin UK^+ \rightarrow Gq$.

(5): The proof is similar to the proof of (4).

Proof of Theorem T9: (1) follows directly from Theorem T6. As can also be seen from Theorem T6, to prove (2) it is sufficient to prove that if $\langle L, R \rangle$ fulfills WCA, WSC, and DIV, then for all $p, q \in L$: If $p \in UM^+$ and $q \in UM^-$, then $\sim(pCq)$.

Suppose that DIV holds and that $p \in UM^+$, $q \in UM^-$, and pCq . Then either qRp or pPq .

First suppose qRp . By $p \in UM^+$ it follows that G_Cp and by $q \in UM^-$ B_Cq follows. By B_Cq , $\sim qPq$ follows. It therefore follows that $\sim qPqRp$. By G_Cp , however, $\sim(\exists q)(\sim qRqR^*p)$ follows. Therefore, qRp does not hold.

Next, suppose pPq . By DIV it follows that there is an r such that $pPrPq$. By $p \in M^+$ and pPr it follows, according to Definition D13, that $\sim rPr$. By $q \in M^-$ and rPq it follows, according to the same definition, that $rP\sim r$. By this contradiction, pPq does not hold either.

We then have $qRp \vee pPq$, $\sim(qRp)$, and $\sim(pPq)$. This contradiction completes the proof.

Proof of Theorem T10: For (1), let $\langle L, R \rangle$ be a preference structure that does not fulfill SSC. Then there are p and q such that either $\sim pPpR^*qR\sim q$, $\sim pRpP^+qR\sim q$, or $\sim pRpR^*qP\sim q$. Suppose PN and CL hold for G_N and B_N .

Case 1: $\sim pPpR^*qR\sim q$. Since B_N fulfills negativity, by $\sim pPp$, i.e., B_Np , and pR^*q , B_Nq follows, i.e., $\sim qPq$, contrary to the assumption.

Case 2: $\sim pRpP^+qR\sim q$. Then there are r and s such that $\sim pRpR^*rPsR^*qR\sim q$. Since $\langle G_N, B_N \rangle$ fulfills CL, rPs yields $G_Nr \vee B_Ns$. Since G_N fulfills positivity, it follows from pR^*r and G_Nr that G_Np . Since B_N fulfills negativity, it follows from sR^*q and B_Ns that B_Nq . Thus it follows that $G_Np \vee B_Nq$, i.e., $pP\sim p \vee \sim qPq$, contrary to the assumptions.

Case 3: $\sim pRpR^*qP\sim q$. Since G_N fulfills positivity, by $qP\sim q$, i.e., G_Nq , and pR^*q , G_Np follows, i.e., $pP\sim p$, contrary to the assumption.

Thus in all three cases a contradiction has been derived. This completes the proof of (1).

The proofs of parts (2) and (3) are straightforward.

Proof of Theorem T12: Only part (1) will be proved here; part (2) can be proved in the same way.

From $pCqI\sim q$ it follows that either $pPqI\sim q$, $pIqI\sim q$, or $\sim qIqPp$. The proof proceeds by these three cases.

Case 1: $pPqI\sim q$. By $pPqI\sim q$, $G_I p$ follows. By pPq and WCA it follows that either $\sim pRp$ or $pP\sim p$. Suppose $\sim pRp$. Then $\sim pRpPqI\sim q$, contrary to WSC. Thus $pP\sim p$. Suppose $(\exists r)(\sim rRrR^*p)$. Then $\sim rRrR^*pPqI\sim q$, so that $\sim rRrR^*pI\sim q$, contrary to WSC. Thus $\sim(\exists r)(\sim rRrR^*p)$. From $pP\sim p$ and $\sim(\exists r)(\sim rRrR^*p)$ it follows that $G_C p$. Thus $G_C p \& G_I p$, thus $G_C p \leftrightarrow G_I p$.

Case 2: $pIqI\sim q$. Then $\sim qRqR^*p$, from which $\sim G_C p$ follows. Suppose $G_I p$. Then $(\exists s)(pPsI\sim s)$, thus $\sim qIqI pPsI\sim s$, so that $\sim qIqP^+sI\sim s$, contrary to WSC. Thus $\sim G_I p$. We then have $\sim G_C p \& \sim G_I p$, and thus $G_C p \leftrightarrow G_I p$.

Case 3: $\sim qIqPp$. Suppose $pP\sim p$. Then $\sim qIqPpP\sim p$, contrary to WSC. Thus not $pP\sim p$, thus $\sim G_C p$. Next, suppose $G_I p$. Then $(\exists s)(pPsI\sim s)$ and $\sim qIqPpPsI\sim s$, so that $\sim qIqP^+sI\sim s$, contrary to WSC. Thus $\sim G_I p$. We then have $\sim G_C p \& \sim G_I p$, and thus $G_C p \leftrightarrow G_I p$. This concludes the proof of the theorem.

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