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Book Review

Malinowski, Grzegorz. *Many-Valued Logics*. Oxford University Press, Oxford, 1993. 131 pages.

The latest addition to the *Oxford Logic Guides* is a short but informative monograph on many-valued logics. The author naturally gives pride of place to the achievements of the Polish school, but many other authors and topics are touched upon. After a short survey of classical logic, Malinowski begins with the work of Łukasiewicz and the Lvov-Warsaw school, focusing mostly on issues of philosophy and interpretation. Next there are two chapters on the topic of logical algebras and matrices followed by chapters on Łukasiewicz logics (both finite- and infinite-valued), on the logics of Post, and on the systems of three-valued logic introduced by Kleene and Bochvar. The remaining chapters cover an assortment of topics, including probability and manyvaluedness, quantifiers in many-valued logic, fuzzy logic, and applications of manyvalued systems. There is also a somewhat incongruous chapter on intuitionism and the modal logics of Lewis; this is included because the corresponding sentential logics can be characterized by infinite-valued matrices, but the topic seems somewhat distant from the main stream of research in many-valued systems.

The author provides an efficient survey of the large and rather diffuse literature on many-valued logics. The major results and techniques are covered, though proofs of theorems are mostly omitted. In places where some details are provided (such as the completeness theorems of Rosser and Turquette), the proof given is a sketch rather than a complete account. The book, though, gives a very good overview for those wishing to orient themselves in the field, and provides a substantial list of references for anyone wishing to follow up particular topics. It is the best introductory monograph in the field, and can be confidently recommended to students.

Many-valued logic has had a rather checkered career, eliciting remarkably varied opinions as to its significance and value. On the one hand, Jordan, in his booklet of 1945, *The Development of Mathematical Logic and of Logical Positivism in Poland between the Two Wars*, described Łukasiewicz's discovery of many-valued systems as "a discovery of the first order, eclipsing everything done in the field of logical research in Poland." This remark is the more surprising if we remember that the results eclipsed include the fundamental work of Tarski, Lindenbaum and others.

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Tarski himself took a different view. In Reid's recent biography of E. T. Bell, Feferman reports on Tarski's attitude in the 1940s to many-valued truth:

I think he found it of technical interest, but I don't think he ever took it as seriously as Bell did. As it happens, many-valued truth has had no impact on mainstream logic. It just doesn't correspond to any natural form of deductive reasoning. If we don't know that something is certainly true or certainly false, would we assign a truth value of 1/2 to it? (*The Search for E. T. Bell*, p. 265).

Feferman's last remark points out the major interpretive difficulty with the logics of Łukasiewicz. The assignment of intermediate values seems somewhat arbitrary. A further interpretive difficulty, first posed by Gonseth in 1938, is discussed by Malinowski in his second chapter. For a fixed undetermined proposition α , its negation $\neg \alpha$ is also undetermined in the three-valued logic of Łukasiewicz, hence so is $\alpha \land \neg \alpha$, contrary to our intuitions. Similar difficulties arise in areas such as fuzzy logic. It is presumably such difficulties that have restricted many-valued logic to a rather marginal position in contemporary studies.

It is to Malinowski's credit that he does not shirk these difficulties of interpretation but discusses in an interesting and fair manner the problems of interpreting manyvalued systems in various contexts. His verdict on fuzzy logic is cautiously favorable; he does not mention the huge wave of commercial hype surrounding the recent rash of Japanese patents for fuzzy auto transmissions, anti-skid braking systems, shower systems and so forth. According to *Time* magazine (September 25, 1989), "the only barrier remaining to wider use of fuzzy logic is the philosophical resistance of the West." Well—maybe so. In the meantime, those of us in the West who wish for a more sober guide to many-valued systems will find Malinowski's little book an excellent read.

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