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Refining Temporal Reference in Event Structures

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Abstract This paper expands on the theory of event structures put forward in previous work by further investigating the subtle connections between time and events. Specifically, in the first part we generalize the notion of an event structure to that of a refinement structure, where various degrees of temporal granularity are accommodated. In the second part we investigate how these structures can account for the context-dependence of temporal structures in natural language semantics.

1 Introduction Reasoning and talking about time is to a great extent reasoning and talking about what actually happens or might happen at some time or another. This is perhaps not crucial if our concern is with abstract temporal reasoners or planners intended for specific applications, but it arguably matters for the prospects of knowledge representation and natural language semantics. The variety of the world is the variety of the things that happen, and we cannot deal with it without taking events at face value (just as we cannot deal with physical bodies or masses by confining ourselves to their spatial coordinates). This is the stance we took in [11], where we argued that the notion of an event structure can be given an autonomous characterization germane to both common sense and natural language. In [12] and [13] we also showed that the formal connection between the way events are perceived to be ordered and the underlying temporal dimension is essentially that of a construction of a linear ordering from the basic formal ontological properties of a domain of events—specifically, mereological and topological properties. The purpose of this paper is to expand on this by further investigating the subtle connections between time and events. After a brief review, in the first part we shall generalize the notion of an event structure to that of a refinement structure, where various degrees of temporal granularity are accommodated. In the second part we shall then investigate how these structures can account for the context-dependence of temporal structures in natural language semantics.

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2 *Refining event structures*

2.1 Preliminaries The basic notion of an event structure is presented in detail in [13]. The underlying mereotopological machinery is developed within a first-order language with identity and descriptions. To allow for the possibility of improper descriptive terms, we use a free logic supplemented with Lambert's axiom [8]:

(1) $\forall y(y = \iota x\varphi x \leftrightarrow (\varphi y \wedge \forall x(\varphi x \rightarrow x = y)).$

(This is not crucial; alternative logical systems—e.g., based on Russell's theory of descriptions—may also be adequate for most purposes.)

The primitive mereological and topological relations are "part of" and "boundary of," symbolized by 'P' and 'B' respectively (cf. Varzi [17]). Additional derived notions can be defined as usual:

(2)	$x = y$	$=_{def}$	$\mathsf{P}(x,y) \wedge \mathsf{P}(y,x)$	identity
(3)	$\mathsf{O}(x,y)$	$=_{def}$	$\exists z(\mathsf{P}(z,x) \wedge \mathsf{P}(z,y))$	overlap
(4)	$\mathsf{X}(x,y)$	$=_{def}$	$\mathsf{O}(x,y) \wedge \neg\mathsf{P}(x,y)$	crossing
(5)	$\mathsf{PP}(x,y)$	$=_{def}$	$\mathsf{P}(x,y) \wedge \neg\mathsf{P}(y,x)$	proper part
(6)	$\mathsf{BP}(x,y)$	$=_{def}$	$\mathsf{P}(x,y) \wedge \mathsf{B}(x,y)$	boundary part
(7)	$\sigma x\varphi x$	$=_{def}$	$\iota x\forall y(\mathsf{O}(y,x) \leftrightarrow \exists z(\varphi z \wedge \mathsf{O}(z,y)))$	sum
(8)	$\pi x\varphi x$	$=_{def}$	$\sigma x\forall z(\varphi z \rightarrow \mathsf{P}(x,z))$	product
(9)	$x + y$	$=_{def}$	$\sigma z(\mathsf{P}(z,x) \vee \mathsf{P}(z,y))$	join
(10)	$x \times y$	$=_{def}$	$\sigma z(\mathsf{P}(z,x) \wedge \mathsf{P}(z,y))$	meet
(11)	$x \sim y$	$=_{def}$	$\sigma z(\mathsf{P}(z,x) \wedge \neg\mathsf{O}(z,y))$	difference
(12)	$\sim x$	$=_{def}$	$\sigma z(\neg\mathsf{O}(z,x))$	complement
(13)	$\mathsf{c}(x)$	$=_{def}$	$x + \sigma z(\mathsf{B}(z,x))$	closure
(14)	$\mathsf{C}(x,y)$	$=_{def}$	$\mathsf{O}(\mathsf{c}(x),y) \vee \mathsf{O}(\mathsf{c}(y),x)$	connection
(15)	$\mathsf{Cn}(x)$	$=_{def}$	$\forall y\forall z(x = y + z \rightarrow \mathsf{C}(y,z))$	self-connectedness.

As specific axioms we assume at least those of classical extensional mereology (see Simons [15] for an overview) supplemented with the analogues of the basic topological axioms for closure systems (cf. Smith [16]):

(16) $\mathsf{P}(x,y) \leftrightarrow \forall z(\mathsf{O}(z,x) \rightarrow \mathsf{O}(z,y))$

(17) $\exists x\varphi x \rightarrow \exists x(x = \sigma x\varphi x)$

(18) $\mathsf{c}(\mathsf{c}(x)) = \mathsf{c}(x)$

(19) $\mathsf{c}(x+y) = \mathsf{c}(x) + \mathsf{c}(y).$

This yields a minimal theory which proves fit for some basic patterns of mereotopological reasoning. Further principles (concerning, e.g., the dependent nature of boundaries) can be added as required.

2.2 Event structures An *event structure* is an ordered pair $\langle E, \delta\rangle$, where E is a mereotopologically self-connected domain:

(20) $\forall z(\mathsf{O}(z,x) \vee \mathsf{O}(z,y)) \rightarrow \mathsf{C}(x,y),$

and δ picks out a maximal class of "divisors" closed under the basic operations of join, meet, and difference (within specific limits):

(21) $\delta(x) \rightarrow \neg\mathsf{Cn}(\sim x)$

(22) $\delta(x) \wedge \delta(y) \rightarrow (\delta(x+y) \leftrightarrow \mathsf{C}(x,y))$

(23) $\delta(x) \wedge \delta(y) \rightarrow (\delta(x \times y) \leftrightarrow \mathsf{O}(x,y))$

(24) $\delta(x) \wedge \delta(y) \rightarrow (\delta(x-y) \leftrightarrow \mathsf{X}(x,y))$.

Intuitively, the δs are somewhat distinguished items that separate their complement into two disconnected parts. Taking E as a set of events, the idea is to think of these divisors as comprising all that happens during certain "periods," counting the events on one side as past events, and those on the other side as future events. (The closure conditions (22)–(24) are easily motivated: if every (bounded) event must be included in some divisor, then any two connected divisors must make up a (thicker) divisor; and if divisors are to divide the entire domain into *two* parts, past and future, then they must not themselves consist of disconnected divisors. Moreover, divisors must have a uniform orientation, hence the common part of any two overlapping divisors and the difference between any two crossing divisors must themselves be divisors.)

We regard these as minimal conditions. Further constraints on E and/or δ can of course be added to select specific structures.

2.3 Oriented structures Event structures can be used to provide a characterization of the intuitive notion of an event (or a family of events) separating past from future events. This is so because the divisors of any given structure form a closure system in which every (bounded) event can be associated with the smallest divisor containing it:

(25) $\mathsf{d}(x) =_{def} \pi z(\delta(z) \wedge \mathsf{P}(x,z))$.

Event structures say nothing, however, about whether a given event actually lies in the past or in the future of another event (divisor). That is, event structures are not temporally oriented.

Oriented structures can be obtained as follows. Define:

(26) $\mathsf{d}^\star(x) =_{def} x + \sigma z \exists y(\mathsf{P}(y,x) \wedge z = \mathsf{d}(y))$

(27) $\mathsf{F}(z_1, x, z_2) =_{def} \neg\mathsf{O}(z_1 + z_2, \mathsf{d}(x)) \wedge \neg\mathsf{C}(\mathsf{d}^\star(z_1), \mathsf{d}^\star(z_2))$

(28) $\mathsf{S}(z_1, x, z_2) =_{def} \mathsf{F}(z_1, x, z_2) \wedge z_1 + z_2 = \ \sim \mathsf{d}(x)$.

(Intuitively, $\mathsf{d}^\star$ extends d to unbounded events; F is a relation of two events, z_1 and z_2, flanking (i.e., lying on two opposite sides of) a third one, x; and S is the relation of one event, x, separating its complement into two parts, z_1 and z_2.) Then a triple $\langle E, \delta, e\rangle$ is an *oriented event structure* if and only if $\langle E, \delta\rangle$ is an event structure and e is a distinguished element of E such that

(29) $\exists x \exists y(\mathsf{S}(e,x,y))$.

That is, an oriented structure is obtained by singling out an "anchor" element e relative to which every other event can be positioned on the assumption that e covers one of the two sides (intuitively, either the past or the future) of some event x. The positioning is obtained via the following:

(30) $f(x) =_{def} \iota z \exists y(\mathsf{S}(z,x,y) \wedge (\mathsf{O}(x,e) \rightarrow \mathsf{P}(z,e)) \wedge (\neg\mathsf{O}(x,e) \rightarrow \mathsf{P}(e,z)))$

(31) $f'(x) =_{def} \sigma z(\mathsf{P}(x, f(z)))$.

This effectively amounts to defining f and f' as a pair of Galois connections so that

(32) $\mathsf{S}(f(x), x, f'(x))$

(32′) $\mathsf{S}(f'(x), x, f(x))$

always hold. We then just stipulate that e represents the past. That is, we treat f as a function of temporal orientation associating each event in the domain with the totality of events that precede it; and correspondingly, we treat f' as a function associating each event with the events that follow it. This is a conventional choice (the alternative stipulation would do as well), but we can show that it is coherent throughout. For instance, the following are all consequences of (29) (given (21)–(24)):

(33) $f(x) = f(\mathsf{d}(x))$

(33′) $f'(x) = f'(\mathsf{d}(x))$

(34) $\mathsf{P}(x, y) \to \mathsf{P}(f(y), f(x))$

(34′) $\mathsf{P}(x, y) \to \mathsf{P}(f'(y), f'(x))$

(35) $\mathsf{P}(x, f(y)) \to \mathsf{P}(y, f'(x))$

(35′) $\mathsf{P}(x, f'(y)) \to \mathsf{P}(y, f(x))$

(36) $\mathsf{P}(x, f(y)) \to \mathsf{P}(f(x), f(y)))$

(36′) $\mathsf{P}(x, f'(y)) \to \mathsf{P}(f'(x), f'(y)))$.

In fact, it can be shown that if $\langle E, \delta, e\rangle$ is an oriented event structure with orientation functions f and f', the temporal dimension can be fully retrieved. For instance, define temporal precedence and overlap:

(37) $\mathsf{TP}(x, y) =_{def} \mathsf{P}(x, f(y)))$

(38) $\mathsf{TO}(x, y) =_{def} \mathsf{O}(\mathsf{d}^\star(x), \mathsf{d}^\star(y))$.

Then we can prove the mereological counterparts of Kamp's [6] axioms for strict linear orders (see Pianesi and Varzi [13] for details):

(39) $\mathsf{TO}(x, x)$

(40) $\mathsf{TO}(x, y) \to \mathsf{TO}(y, x)$

(41) $\mathsf{TP}(x, y) \to \neg\mathsf{TO}(x, y)$

(42) $\mathsf{TP}(x, y) \to \neg\mathsf{TP}(y, x)$

(43) $\mathsf{TP}(x, y) \wedge \mathsf{TP}(y, z) \to \mathsf{TP}(x, z)$

(44) $\mathsf{TP}(x, y) \wedge \mathsf{TO}(y, z) \wedge \mathsf{TP}(z, t) \to \mathsf{TP}(x, t)$

(45) $\mathsf{TP}(x, y) \vee \mathsf{TP}(y, x) \vee \mathsf{TO}(x, y)$.

2.4 Refinement structures The kind of temporal structure that emerges from oriented event structures strictly depends on the choice of the relevant divisor condition δ. Thus, for instance, dense orders can be derived by imposing suitable conditions on δ, much as is the case of discrete orders. We now consider more complex structures involving not just one dividing condition δ, but entire collections of such conditions. This will provide a suitable framework to account for shifting temporal perspectives.

A *refinement event structure* is a triple $\langle E, \{\delta_i : i \in I\}, e\rangle$ such that (i) for each $i \in I$, $\langle E, \delta_i, e\rangle$ is an oriented event structure, and (ii) the family of divisors $\{\delta_i : i \in I\}$ is closed under meet, i.e., for all $x, y \in E$ and all $i, j \in I$ there exists some $k \in I$ satisfying the following:

(46) $\delta_i(x) \wedge \delta_j(y) \to \delta_k(x \times y)$.

(This has the effect of securing coherence among the various constituent event structures. Equivalently, we could define a refinement structure as a class $\{\mathcal{S}_i : i \in I\}$ of oriented event structures $\mathcal{S}_i = \langle E, \delta_i, e\rangle$ closed under meet.) Note that we implicitly require that every oriented structure involved in a refinement have the same anchor e. This has a natural motivation, considering that oriented structures whose anchor elements are related by a parthood relation induce the *same* ordering. That is, if $\langle E, \delta, e_1\rangle$ and $\langle E, \delta, e_2\rangle$ are two oriented event structures, and f_1, f_1', f_2, and f_2' their respective orientation functions, we have:

(47) $\mathsf{P}(e_1, e_2) \vee \mathsf{P}(e_2, e_1) \rightarrow f_1 = f_2 \wedge f_1' = f_2'.$

Thus there are only two ways of orienting an event structure $\langle E, \delta\rangle$, and these can be obtained by picking out any pair of oriented structures whose anchor elements do not overlap. It is then easy to verify that such structures would reverse the order, i.e.,

(48) $f_1 = f_2' \wedge f_1' = f_2.$

On the other hand, it is clear from (47) that the above-mentioned implicit condition could be weakened to the requirement that oriented event structures may enter into a refinement provided that of any two of them, the anchor of one is part of the anchor of the other. That is, we could consider structures $\langle E, \{\delta_i : i \in I\}, \{e_i : i \in I\}\rangle$ with the property that, for all $j \in I$,

(49) $\mathsf{P}(e_i, e_j) \vee \mathsf{P}(e_j, e_i).$

However, since this generality yields no significant gain, in the following we shall confine ourselves to refinements in which the anchor element is kept fixed.

If $\langle E, \{\delta_i : i \in I\}, e\rangle$ is such a refinement structure, we can then define a refinement relation $\succcurlyeq$ among its constitutive divisors as follows:

(50) $\delta_i \succcurlyeq \delta_j =_{def} \forall x(\delta_i(x) \rightarrow \exists y(\delta_j(y) \wedge \mathsf{P}(y, x))).$

Thus, intuitively, δ_j is a refinement of δ_i iff the former draws at least the same temporal distinctions as the latter (and perhaps more). It is immediately verified that this relation is reflexive, transitive, and asymmetric. Furthermore, $\succcurlyeq$ is monotonic with respect to the ordering conditions f_i, f_i' (induced in the obvious way):

(51) $\delta_i \succcurlyeq \delta_j \rightarrow \forall x \forall y(\mathsf{P}(x, f_i(y)) \rightarrow \mathsf{P}(x, f_j(y))$

(52) $\delta_i \succcurlyeq \delta_j \rightarrow \forall x \forall y(\mathsf{P}(x, f_i'(y)) \rightarrow \mathsf{P}(x, f_j'(y)).$

This means that $\succcurlyeq$ behaves as a homomorphism with respect to f and f' and, ultimately, with respect to the ordering relations. (Note that this depends crucially on the above requirement on anchors). Thus, whenever an event x precedes another event y in a given oriented structure, the same obtains in every event structure whose divisor condition is a refinement of the given one:

(53) $\mathsf{TP}_i(x, y) \wedge \delta_i \succcurlyeq \delta_j \rightarrow \mathsf{TP}_j(x, y)$

(where TP_i and TP_j are the relations of temporal precedence induced by δ_i and δ_j, respectively).

3 *Refining temporal reference*

3.1 Density Refinement structures seem particularly suited to account for the effect of context on the choice of temporal structures. Landman [9] observes that if language exhibits the possibility of indefinitely refining temporal relations among events—as seems to be the case with natural language—the underlying model of time must be dense. Thus, for instance, we can imagine a process of gradual refinement:

(54) John and Mary met last year. More exactly, they met during summer vacation. To be precise, it was the 15th of August. If fact, they met while having brunch. John was just having his first sip of coffee…

Even if there is a point beyond which refinement is no longer practically feasible, it seems that this is not enough to posit discreteness as linguistically relevant.

In the present framework, density can be obtained by adding the mereotopological counterpart of the usual axiom for dense linear orders on closed (or, equivalently, open) intervals:

(55) $\mathsf{TP}(\mathsf{c}(x), \mathsf{c}(y)) \rightarrow \exists z(\mathsf{TP}(\mathsf{c}(x), \mathsf{c}(z)) \wedge \mathsf{TP}(\mathsf{c}(z), \mathsf{c}(y)))$.

More generally, in the context of a refinement structure $\langle E, \{\delta_i : i \in I\}, e\rangle$ this corresponds to assuming the following to hold for relevant $i \in I$:

(56) $\mathsf{P}(\mathsf{c}(x),\ f_i(\mathsf{c}(y))) \rightarrow \exists z(\mathsf{P}(\mathsf{c}(x),\ f_i(\mathsf{c}(z))) \wedge \mathsf{P}(\mathsf{c}(z),\ f_i(\mathsf{c}(y))))$.

However this does not fully capture the idea behind (54). The interesting question is what kind of divisors are presupposed by the underlying unlimited refining process. Clearly they must be infinite in number (which in turn presupposes that the domain E must have infinite cardinality). But, more importantly, they cannot include a minimal element (with respect to the ordering $\succcurlyeq$). This amounts to the following requirement:

(57) For every $i \in I$ there exists $j \in I$ such that $\delta_i \succcurlyeq \delta_j$ but not $\delta_j \succcurlyeq \delta_i$.

This entails that divisors must themselves be infinitely divisible, i.e., in the terminology to be developed in the next subsection, there can be no absolute punctual events.

From a cognitive perspective, the kind of event domain required by (55)–(57) may seem somewhat too rich: does our common sense notion of an event support the idea of a really *dense* course of events? (The issue does not arise within merely temporal models, since we are more confident about the idea that the time line is infinite, without end points, and dense.) It seems that natural language gives us the possibility of refining temporal relations without any limitation. But capturing the properties of natural language and describing the common-sense world are two distinct matters and should be kept apart. If so, this would be an argument in favor of the view that natural language is an autonomous cognitive system—i.e., in the case at hand, that the interpretive properties of natural language cannot be derived directly from the structure of the common-sense world. A different perspective would be to assimilate the discrepancy between language and cognitive ontology to the difference between properties-in-intension and properties-in-extension, as Habel [3] seems to suggest. Thus, the possibility of indefinitely refining temporal relations would not (*contra* Landman [9]) require an underlying infinite, dense ontology; rather, it would be a property of *language as a process*. Representations can be broken up and made finer.

We shall leave the issue open. But we shall observe that the two theses could be reconciled if the density-in-intension property is what marks a difference (among others) between language and the other cognitive systems.

3.2 Punctuality Just as natural language appears to allow us to indefinitely refine temporal relations as illustrated in (54), it also permits us to discretize time at will:

(58) That's how they met: *at a certain point*, John asked the waiter to invite her at his table; *the next moment* she was sitting in front of him.

This is another fundamental manifestation of the inherent context-dependency of time granularity: what counts as a moment in one context may be structurally analyzed in another, and vice versa. Plain event structures do not allow one to account for this variability. For although they capture the intuition that the segmentation of time is not absolute (it depends on the divisor condition δ), they supply no means for making this explicit (within every oriented structure, the divisor condition δ is fixed). Refinement structures provide a natural way to overcome this limitation: the variety of possible choices is reflected in the variety of available δs.

Intuitively, punctual events are instantaneous, i.e., do not extend over any time *interval*: they are *located in* time but do not *take up* time. These include for instance boundary events traditionally classified as "culminations" or "achievements." Within the present setting, this does not amount to a requirement of mereological atomicity: what counts as instantaneous, as opposed to extended in time, depends entirely on the relevant δ. For divisors not only provide the basis for temporal orientation but, in a sense, also for temporal measurement. Punctuality is a relative notion.

This is not to deny that punctuality rests on some sort of minimality: punctual events cannot accommodate more structured ones. However, contrary to a rather standard practice, we need not in this regard consider the distinction between instants and intervals—or more generally any distinction based on such absolute notions as size or duration—as the relevant parameters. We also need not impose any specific axioms for characterizing punctuality. Rather, the distinguishing properties of punctual events and instant algebras can be derived from more basic aspects of event structures.

To see this, define the notion of a *minimal divisor* relative to an oriented event structure $\langle E, \delta, e \rangle$:

(59) $\mathsf{M}\delta(x) =_{def} \delta(x) \wedge \forall y(\mathsf{P}(y, x) \to \neg\delta(y))$.

Thus, a divisor x is minimal if and only if it does not contain other divisors (relative to the same δ). As a consequence, every event that is part of such an x has x as its divisor:

(60) $\mathsf{M}\delta(x) \wedge \mathsf{P}(y, x) \to \mathsf{d}(y) = x$.

This is a welcome consequence, since (60) entails that "temporal" differences are neglected inside a minimal divisor. In fact, we can show that any events that are parts of such a divisor are simultaneous, i.e., are temporally overlapped by the same events:

(61) $\mathsf{M}\delta(x) \wedge \mathsf{P}(y, x) \to \forall z(\mathsf{TO}(z, x) \leftrightarrow \mathsf{TO}(z, y))$.

More generally, we have:

(62) $\mathsf{M}\delta(x) \wedge \mathsf{TO}(y, x) \wedge \mathsf{TO}(z, x) \rightarrow \mathsf{TO}(y, z)$

(63) $\mathsf{M}\delta(x) \wedge \mathsf{P}(w, x) \wedge \mathsf{TO}(y, w) \wedge \mathsf{TO}(z, w) \rightarrow \mathsf{TO}(y, z)$.

Thus, if two events temporally overlap a minimal divisor (or a part thereof), then they temporally overlap each other. Vice versa, we have that divisors that can be temporally overlapped only by temporally overlapping events are minimal:

(64) $\delta(x) \wedge \forall y \forall z(\mathsf{TO}(y, x) \wedge \mathsf{TO}(z, x) \rightarrow \mathsf{TO}(y, z)) \rightarrow \mathsf{M}\delta(x)$.

Putting (62) and (64) together, the fundamental properties characterizing punctual events according to Kamp [6] can be shown to hold of minimal—and only minimal—divisors. We can then propose the following definition for punctual events:

(65) $\mathsf{PE}(x) =_{def} \mathsf{M}\delta(\mathsf{d}(x))$.

Thus, punctual events are not merely—and not necessarily—atomic events, i.e., events with no proper parts (although of course every atomic event is punctual, regardless of δ). Rather, they are events whose internal structure is irrelevant for the purpose of temporal distinctions.

Punctuality is thus relativized to the particular event structure at hand—hence, ultimately, to the particular divisor condition δ. By changing δ, events previously treated as punctual may become nonpunctual, in that their internal temporal structure is made available, and vice versa. This notion of "change," as we said, is purely metalinguistic if we focus on plain structures. However, refinement structures are endowed with families of divisor conditions and may therefore accommodate this variability directly, by drawing connections between the available δs. (There is a clear modal flavor to this, which is reminiscent of the way Kripke structures can be used to account for intensional notions such as necessity and possibility.) This can be made more precise as follows.

3.3 Putting everything into semantics First of all, here is how some key semantic notions can be recovered within the basic framework. Let $\mathcal{E} = \langle E, \delta, e\rangle$ be an ordered event structure. For every $K \subseteq E$ we can introduce the following restricted relations:

(66) $\sqsubseteq_{\mathcal{E},K} = \{\langle x, y\rangle \in K \times K : \mathsf{P}(x, y)\}$

(67) $<_{\mathcal{E},K} = \{\langle x, y\rangle \in K \times K : \mathsf{TP}(x, y)\}$

(68) $\mathsf{o}_{\mathcal{E},K} = \{\langle x, y\rangle \in K \times K : \mathsf{TO}(x, y)\}$.

Now we can define a *temporal structure* induced by $\mathcal{E}$ to be any tuple $T_{\mathcal{E}} = \langle K, \sqsubseteq_{\mathcal{E},K}, <_{\mathcal{E},K}, \mathsf{o}_{\mathcal{E},K}\rangle$ with $K \subseteq E$. In particular, $T_{\mathcal{E}}$ qualifies as the *period structure* induced by $\mathcal{E}$ if $K = \{x \in E : \delta(x)\}$, and it qualifies as the *instant structure* if $K = \{x \in E : \mathsf{M}\delta(x)\}$. Since $<_{\mathcal{E},K}$ behaves as a relation of temporal precedence in view of (39)–(45), these two notions correspond to the standard notions of period and instant structures (divisors and minimal divisors acting as counterparts of intervals and instants, respectively). Standard temporal (instant or interval) semantics for a tensed language $\mathcal{L}$ can then be obtained by defining a *model* for $\mathcal{L}$ to be any structure $\mathcal{M} = \langle T_{\mathcal{E}}, h\rangle$ where $T_{\mathcal{E}} = \langle K, \sqsubseteq_{\mathcal{E},K}, <_{\mathcal{E},K}, \mathsf{o}_{\mathcal{E},K}\rangle$ is a temporal structure induced by some oriented event structure $\mathcal{E} = \langle E, \delta, e\rangle$ and h is an interpretation function determining a truth-value assignment for every atomic sentence/formula of $\mathcal{L}$ relative to arbitrary elements of K.

To illustrate, if $\mathcal{L}$ is some language supplied with the tense operators 'P' ("it has been the case that") and 'F' ("it will be the case that"), we obtain a classical Priorean semantics (as in [14]) for $\mathcal{L}$ by requiring the satisfaction relation $\models$ to meet the following conditions for all instant models $\mathcal{M}$ and all relevant "instants" t (we write '$>$' for the inverse of '$<$', omitting subscripts):

(69) $\mathcal{M} \models_t P\varphi$ if and only if $\mathcal{M} \models_{t'} \varphi$ for some $t' < t$

(70) $\mathcal{M} \models_t F\varphi$ if and only if $\mathcal{M} \models_{t'} \varphi$ for some $t' > t$.

(The variety of resulting logics would depend on the properties of $<$, hence ultimately on the specific mereotopology of E and δ.) The semantics of other tense operators can then be defined as usual. For instance, the following define Kamp's [5] operators 'S' ("since") and 'U' ("until"):

(71) $\mathcal{M} \models_t S\varphi\psi$ if and only if $\mathcal{M} \models_{t'} \varphi$ and $\mathcal{M} \models_{t''} \psi$ for some $t' < t$ and every $t'' > t'$ such that $t > t''$

(72) $\mathcal{M} \models_t U\varphi\psi$ if and only if $\mathcal{M} \models_{t'} \varphi$ and $\mathcal{M} \models_{t''} \psi$ for some $t' > t$ and every $t'' < t'$ such that $t < t''$.

Likewise, we can obtain a classical interval semantics as in Humberstone [4] by referring to interval models instead. The conditions for 'P', 'F', etc. remain the same, and we can in addition specify the semantics for the downward and upward "holds" operators 'H_d' and 'H_u' (again, we write '$\sqsupseteq$' for the inverse of '$\sqsubseteq$', omitting subscripts):

(73) $\mathcal{M} \models_t H_d\varphi$ if and only if $\mathcal{M} \models_{t'} \varphi$ for every $t' \sqsubseteq t$

(74) $\mathcal{M} \models_t H_u\varphi$ if and only if $\mathcal{M} \models_{t'} \varphi$ for every $t' \sqsupseteq t$.

As a further example, Dowty's [2] operator 'B' ("comes to be the case") can be characterized by the following condition:

(75) $\mathcal{M} \models_t B\varphi$ if and only if (i) $\mathcal{M} \models_{t_1} \neg\varphi$ for some $t_1 \mathsf{O} t$ for which there exists no $t' \sqsubseteq t$ such that $t' < t_1$, and (ii) $\mathcal{M} \models_{t_2} \varphi$ for some $t_2 \mathsf{O} t$ for which there exists no $t' \sqsubseteq t$ such that $t' > t_2$.

Of course we can also extend these semantics by relativizing the satisfaction relation to all sorts of events (not just divisors), so as to read $\mathcal{M} \models_x \varphi$ simply as "sentence φ holds in model $\mathcal{M}$ throughout event x." This means using temporal structures $T_{\mathcal{E}} = \langle K, \sqsubseteq_{\mathcal{E},K}, <_{\mathcal{E},K}, \mathsf{O}_{\mathcal{E},K}\rangle$ where K is a proper superset of the sets $\{x \in E : \delta(x)\}$ and $\{x \in E : \mathsf{M}\delta(x)\}$. This may be useful, for instance, to account for a logic of change in the spirit of Kamp [7]. Moreover, it is understood that if $\mathcal{L}$ is, say, a first order language, then the event domain K will also serve as a domain of quantification for event-based semantics in the spirit of Davidson [1]. For instance, on Parsons's tensed formulation in [10], a sentence φ such as "John met Mary in the dining room" would have the following truth condition:

(76) $\mathcal{M} \models_t \varphi$ if and only if there exists some $x < t$ such that x is an event of John's meeting Mary and x takes place in the dining room.

(The full-blown picture would of course have to consider many-sorted models in which the domain includes other entities as well.) These developments are obvious, and we shall not consider specific applications.

Rather, let us now consider how the picture can be fruitfully extended by construing models out of refinement event structures. If $\mathcal{R} = \langle E, \{\delta_i : i \in I\}, e\rangle$ is such a structure, we can define a corresponding *refinement temporal structure* to be a family $T_{\mathcal{R}} = \{T_{\mathcal{E}_i} : i \in I\}$ of temporal structures $T_{\mathcal{E}_i} = \langle K_i, \sqsubseteq_{\mathcal{E}_i,K_i}, <_{\mathcal{E}_i,K_i}, \mathsf{O}_{\mathcal{E}_i,K_i}\rangle$, one for each $i \in I$. (We do not require that these be all of the same sort, for instance, that they be all interval structures. On the contrary, as we saw above, the point of introducing refinement is precisely to be able to switch naturally from one (kind of) temporal structure to another.) Note that since e is fixed, the temporal orderings will be coherent throughout, i.e., the following will hold for all $x_i, y_i \in K_i$ and all $x_j, y_j \in K_j$ $(i, j \in I)$:

(77) $\mathsf{TP}(x_i, x_j) \wedge \mathsf{TP}(y_j, y_i) \rightarrow (x_j <_{\mathcal{E}_j,K_j} y_j \rightarrow x_i <_{\mathcal{E}_i,K_i} y_i)$.

A *refinement model* will then be a pair $\mathcal{M} = \langle T_{\mathcal{R}}, h\rangle$ where h is a family of interpretation functions $\{h_i : i \in I\}$ each of which determines a truth-value assignment for every atomic sentence/formula of the language relative to arbitrary elements of the corresponding domain K_i.

With respect to such structures, the customary semantic conditions for tensed languages present no significant difficulty, and we can proceed as before. However, the relation of satisfaction will now have to be relativized with respect to divisors as well, i.e., with respect to arbitrary elements of arbitrary domains K_i, the latter being determined by the corresponding divisors of the underlying refinement event structure. For instance, (69)–(70) will have to be formulated along the following lines:

(78) For all $i \in I$ and all $t \in K_i : \mathcal{M} \models_{t,i} P\varphi$ if and only if $\mathcal{M} \models_{t',i} \varphi$ for some $t' \in K_i$ such that $t' < t$

(79) For all $i \in I$ and all $t \in K_i : \mathcal{M} \models_{t,i} F\varphi$ if and only if $\mathcal{M} \models_{t',i} \varphi$ for some $t' \in K_i$ such that $t' > t$.

These conditions will not be affected by the possibility of varying the second contextual feature (the index i). In addition, however, we can now specify the semantics of operators which do depend on the variable granularity of the divisors. Consider for instance the operators '$M_{\sqsubseteq}$', '$N_{\sqsubseteq}$', 'M', and 'N' defined by the following clauses (where '$<$' denotes the union of the relevant $<_{\mathcal{E}_i,K_i}$ relations, and '$>$' the corresponding inverse relation):

(80) For all $i \in I$ and all $t \in K_i : \mathcal{M} \models_{t,i} M_{\sqsubseteq}\varphi$ if and only if $\mathcal{M} \models_{t',i'} \varphi$ for some $i' \in I$ and some $t' \in K_{i'}$ such that $t' \sqsubseteq t$

(81) For all $i \in I$ and all $t \in K_i : \mathcal{M} \models_{t,i} N_{\sqsubseteq}\varphi$ if and only if $\mathcal{M} \models_{t',i'} \varphi$ for every $i' \in I$ and every $t' \in K_{i'}$ such that $t' \sqsubseteq t$

(82) For all $i \in I$ and all $t \in K_i : \mathcal{M} \models_{t,i} M\varphi$ if and only if $\mathcal{M} \models_{t',i'} \varphi$ for some $i' \in I$ and some $t' \in K_{i'}$ such that $\delta_i \succcurlyeq \delta_{i'}$ and $t' \sqsubseteq t$

(83) For all $i \in I$ and all $t \in K_i : \mathcal{M} \models_{t,i} N\varphi$ if and only if $\mathcal{M} \models_{t',i'} \varphi$ for every $i' \in I$ and every $t' \in K_{i'}$ such that $\delta_i \succcurlyeq \delta_{i'}$ and $t' \sqsubseteq t$.

These are only a few among a large variety of possible operators that can be distinguished (just permute or change the quantifiers 'some' and 'all' or the mereotemporal relations $\sqsubseteq$ and $<$ to get a first extra stock), but they serve the purpose of illustration. Consider for instance the operator '$M_{\sqsubseteq}$' (80), and suppose for simplicity that $\mathcal{M} = \langle T_{\mathcal{R}}, h\rangle$ is based on a family $T_{\mathcal{R}}$ of instant structures. Then we can think of this operator as specifying that the argument sentence φ is true at a certain instant t with granularity i (i.e., true throughout an event treated as punctual under the i-th way of drawing divisors, δ_i) if and only if there is some way of changing temporal granularity (relative to the range of possibilities admitted by the underlying refinement structure $\mathcal{R} = \langle E, \{\delta_i : i \in I\}, e\rangle$) so as to make φ true at some sub-instant of t. What this means is that '$M_\sqsubseteq$' behaves essentially as a "precisification" operator: if you count time in moon cycles, you might not be able to make certain relevant distinctions (you might not be able to establish the truth of (58), "John asked the waiter to invite Mary at his table; *the next moment* she was sitting in front of him"); but if you count time in minutes, then things may change. In other words, the relevant sentence, φ, may be false *not* because things went differently (e.g., because Mary refused to accept the invitation), but because the relevant temporal granularity is too coarse for φ to be recognized as true. If you can get down to a sufficiently refined temporal structure, this may become apparent and φ may be recognized as true. Thus, we can think of '$M_\sqsubseteq$' as an operator allowing one to double check the possibility for a sentence to come out true under suitable temporal refinements. (Within certain obvious limits, this would correspond to the English "more precisely.") Likewise, '$N_\sqsubseteq$' is essentially a "no matter how" operator: no matter how you change granularity (within the limits set by the underlying refinement structure), if φ is true, it remains true, and if it is false, you can find some sub-instant where it is false.

The operators defined by (82) and (83) are similar, but somewhat more illustrative of the intensional flavor of refinement processes. In the foregoing example we have implicitly assumed that changing granularity is a very regular process: you may count time in moon cycles, weeks, days, or minutes; but once you choose one grain, you apply it throughout (until you change grain). That is, if δ_i is your moon-cycle divisor, it divides the whole of history into moon-cycles: it does not vary from one "epoch" to another. This is intuitive, but there is nothing of course in our notion of a(n instant-based) refinement model that guarantees it. And perhaps there are good reasons to consider models where this is not the case after all. If so, then the operators '$M_\sqsubseteq$' and '$N_\sqsubseteq$' are not quite the appropriate counterparts of the intuitive operations discussed above, and reference to 'M' and 'N' becomes necessary. Unlike the former, the semantics of these latter operators makes explicit reference to the sort of granularity to be considered in the refinement process. In other words, these operators do not force you to consider every possible alternative granularity, but only those alternatives that correspond to an actual *refinement* of the initial δ_i.

The semantic mechanism operating here is reminiscent of an idea familiar from modal logics: modal operators do not range over all possible worlds, but only over those worlds that are "accessible" from the given one. If the analogy is acceptable, then the richness of the basic framework need hardly be emphasized. The variety of interesting accessibility relations among refinements is very large indeed, and appears to be a rewarding subject for further exploration.

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