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OPTICS OF THOUGHT: LOGIC AND VISION IN MÜLLER, HELMHOLTZ, AND FREGE

D. C. McCARTY

Abstract The historical antecedents of Frege's treatment of binocular vision in "The thought" were the physiological writings of Johannes Mueller, Hermann von Helmholtz, and Emil du Bois-Reymond. In their research on human vision, logic was assigned an unexpected role: it was to be the means by which knowledge of a world extended in three dimensions arises from stimuli that are at best two-dimensional. An examination of this literature yields a richer understanding of Frege's insistence that a proper epistemology requires us to recognize the existence and importance of nonsensible sources of knowledge.

1. Seeing in "The thought"

The physiology of the senses is a border land in which the two great divisions of human knowledge, natural and mental science, encroach on one another's domains; in which problems arise which are important for both, and which only the combined labor of both can solve. (Helmholtz [20], p. 61)

Frege's examples and metaphors are often visual or optical. In "On Sense and Reference" [8], for example, he envisioned a hydra-headed telescope with multiple eyepieces, so that a single objective image, and thereby a single celestial object, would be available for simultaneous observation by several viewers.[1] He intended the optical analogy to illustrate his trinitarian doctrine of senses, references, and ideas. In the opening pages of the same essay, Frege worried over the meanings of statements such as "Morgenstern = Abendstern," forms suitable for expressing the reidentification of a heavenly body from distinct visual appearances, ante- and postmeridian. In the preface to *Begriffsschrift* [10], Frege compared the advantages of his concept script over ordinary language with those of the microscope over the human eye. In this, he treated the eye as an instrument: "Considered as an optical instrument, to be sure, [the eye] exhibits many imperfections, which ordinarily remain unnoticed only on

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account of its intimate connection with our mental life. But, as soon as scientific goals demand great sharpness of resolution, the eye proves to be insufficient" ([10], p. 6). In "The thought," Frege imagined himself and a companion standing near a strawberry field. They both see the strawberries but, since the friend is colorblind, his visual impression of a strawberry is not markedly different from his impression of a green leaf. The words 'sense' in English and 'Sinn' in German can denote perception and comprehension.[2]

One goal of Frege's concept script was to make thought visible. According to his late philosophy, invisible thought gives us access to the visible world. In "The thought," he wrote,

> Certainly, the reception of sense impressions is necessary, but not sufficient, for seeing things. What must be added is nothing sensible. And it is this very thing that opens up the external world to us; for without this nonsensible, everyone would remain locked in his inner world. (Frege [9], p. 51)

At the close of that essay, Frege set out to demonstrate that his researches into logic, into the first principles of truth, do not wait upon psychology or any other science to build foundations beneath them. Rather, he held that precisely the reverse is true: perceptual psychology requires realistic logic, with thoughts and truth hypostatized, for its complete intelligibility. Frege maintained that the facts not only of reasoning but also of sensory perception cannot be as they are unless there exist thoughts, senses of sentences subsisting eternally unchanged in a realm apart from the subjective contents of human consciousness and from the phenomenal universe. For the assumption of something, indeed a plethora of somethings, existing unperceived, is required for the scientific study of perception:

> There is, for example, a sensory physiologist. As is appropriate for a researcher into natural science, he is, in the first place, far from taking the things he is convinced of seeing and touching to be his ideas. On the contrary, in sensory impressions, he believes himself to have the most certain evidence of things that subsist in total independence of his feeling, imagining and thinking, and that have no need of his consciousness. ([9], pp. 45–46)

Among the things the physiologist presupposes to exist are objects of perception—trees, clouds, and houses—as well as the media of the perceptual process: light rays strike the eye, pass through the aqueous humor, impinge on the retina, and then stimulate nerve cells and ganglia, all of which the scientist supposes to exist or obtain independently of any particular human. Frege warned his readership that this supposition on the scientist's part is limited in extent, but not in character, by the scientist's knowledge of sensory physiology:

> The stimulation of the optic nerve in no way requires light for it to occur. If lightning strikes in our vicinity, we think to see flames, even if we can't see the lightning itself. The optic nerve is then stimulated perhaps by an electrical current which arises in our bodies thanks to the lightning strike. If the optic nerve is thereby stimulated as it would be by rays of light emanating from flames, we think to see flames. It just depends on the stimulation of the optic nerve; how that stimulation arises is irrelevant. ([9], p. 46)

Even if the scientist recognizes, as a result of observation or experiment, that a manner of visual sensation can occur without its normal cause, that, for example, we can have visual images of flames when there are no flames in reality, the scientist still assumes that there are objects, perceived and unperceived, whether flames or lightning bolts or optic nerves, existing independently of our perceptions.

At a later phase of those same reflections, Frege hoped to reinforce his conclusions that thoughts inhabit a realm distinct from those of individual consciousness and everyday phenomenal objects and that, in thinking, we exercise a faculty of apprehension the business of which is to grasp those thoughts. For this reinforcing argument, Frege mentioned a problem of binocular vision that, he maintained, only his realistic theory of thoughts could solve.

> These [sense impressions] alone do not open up for us the outer world. Perhaps there is a creature that only has sense impressions, without seeing or touching things. The reception of sense impressions is not the seeing of things. How does it come to pass that I see the tree just there, where I do see it? Obviously it has to do with the sense impressions I have and on the special sort of impressions that arise because I see with two eyes. From the physical point of view, there arises on each of the two retinas a distinct image. Another person sees the tree in the same place. He also has two retinal images which differ from mine. We have to assume that these retinal images determine our impressions. Accordingly, we have not only different sense impressions, but impressions that differ markedly from one another. But even so we move around in the same outer world. ([9], p. 51)

Frege then explained the means by which we come into contact with beings from the realm of thoughts, arguing that only his account of thoughts and our relation of apprehension to them can wrest the unity of a shared, objective world from a multiplicity of diverse, subjective fantasies. The lines that appear next have already been quoted: “What must be added is nothing sensible. And it is this very thing that opens up the external world to us.” It would seem that “what must be added” is thought and a grasp of it.

For the background to his arguments, Frege relied heavily upon developments in psychology, physiology, and technology familiar to his educated readers, subscribers to the journal *Contributions to the Philosophy of German Idealism* in which “The thought” was first published. A history of those developments reveals an intellectual symbiosis between logic and physiology. For German intelligentsia of the time, sense impressions were the signs whose denotations, linked to them neither naturally nor conventionally, were the objects of sense—trees or strawberries—and the gap that seems, in Frege’s second argument, to yawn between those perceptual signs and their denotations was taken to coincide with a gap that logic and logical theory have always sought to fill: the space between premises and conclusions, between axioms and theorems. Even a brief rehearsal of that history suffices to refute claims (Kenny [12], pp. 190–94) that, at the close of the “The thought,” Frege was granting philosophical concessions to a skepticism that Wittgenstein was destined later to demolish. For, as we shall see, Frege was undertaking a demolition operation of his own against forms of skeptical idealism allied with the physiological theorizing of his era.

2. Laws of Nerve Energies: Johannes Müller and Emil du Bois-Reymond

The nineteenth century saw a renewal of intimacy between logic, on the one hand, and the psychology and physiology of vision on the other.[3] The word ‘renewal’ is appropriate since ancient Stoics and Pyrrhonists had once drawn these subjects into close alliance. Among the name indices of historical works on logic and those on nineteenth century perceptual physiology there is a good degree of overlap, with Kant, Hamilton, Mill, Lotze, Graßmann, Brentano, and Ladd-Franklin featuring in both. In those days, a liaison between thought and sight was recorded in the vocabulary that physiologists and psychologists adopted for visual processes and phenomena. Helmholtz, under the joint influence of Mill and Lotze, wrote of unconscious *inferences* from sensations as premises to perceptual *judgments* as conclusions. Such inferences were thought to ensure an understanding or *interpretation* of an external world. What got interpreted, in the first instance, were to be sensations treated as *signs* or *symbols* of their objects, a conception prefigured in Stoic logic. Lotze took perception to derive from stimuli which are "local signs," depending for their qualities on the regions at which they impinge on the body. Helmholtz also described the feelings of eye convergence and accommodation as affording *criteria* for judgment. Sensory physiologist Fechner sought to disentangle various *paradoxes* of vision. From midcentury, psychologists, philosophers, and physiologists, Mach among them, spoke of abstract *similarities* between the *serial relations* in stimuli and those in sensation, as well as of structure-preserving *isomorphisms* between patterns in sensation and regions of the brain, all expressible as indemonstrable *axioms* of psychophysical parallelism (see Mach [14]). Von Ehrenfels, an early Gestalt theorist, analyzed structured percepts in terms of *Fundamenta*, *Grundlage*, and *higher order* notions. A thorough account and assessment of this collaboration between logicians and physiologists would require a booklength work; we here restrict ourselves to some few contributions by those scientists, von Helmholtz and his mentor Müller, that are prerequisite to understanding Frege's arguments.

To Lotze's theory of local signs, to Helmholtz's idea of unconscious inferences, and to the interplay of logic with sensory psychology generally, Müller's discoveries and doctrines, especially his laws of specific nerve energies, served as essential preliminaries. To say that Müller's laws, as set out in the second volume of his *Handbook of the Physiology of Humans* [15], captured the attention of scientists would be an understatement; Helmholtz likened the status of Müller's laws in physiology to that of Newton's law of gravitation in celestial mechanics. As we shall see, Müller's ideas of the 1830s were still familiar to scientific audiences in the 1870s; they gave a physiological foundation to Paul du Bois-Reymond's *General Function Theory* of 1882 ([6], p. 37).

Stoics and their skeptical critics had known that a familiar sensory appearance of a thing could be produced by a stimulus bearing no relation, either of apparent similarity or of structural congruence, to that thing in reality. Colored flashes can arise from pressure judiciously exerted on the eyeballs. A sharp blow to the head can make a person "see stars." Hot air rising over asphalt, when viewed from a distance and a sharp angle, creates an impression like that of a pool of water. They were equally aware that a single thing can appear differently to different senses: a painting that seems rough and pitted to sight can seem smooth to the touch. Müller, professor in Berlin and mentor to Ludwig, Brücke, Helmholtz, and du Bois-Reymond, took such phenomena not as a landing stage for skepticism but as multifarious data admitting a single and persuasive explanation: laws of specific nerve energies. With Müller's "energy," we should not associate an idea either of abstract physical quantity proportional, when kinetic, to mass times the square of velocity or of a tacit, perhaps unrealized, potentiality, but as closer kin to Aristotle's *energeia* in the sense of "effectiveness" or "proper activity." Müller took nerve energies to be qualities or states specific to individual nerves and indicative of their powers.

Those of Müller's laws that are relevant to present purposes can be handily summarized. First, we are not directly aware of objects in perception, but only of the qualities specific to particular nerves. Our sensations correspond in their features mainly to states in the nerves induced by stimulation and not to states of their distal causes. Second, the characters of our sensations are tied specifically to the nerves producing them and to the energies of those nerves. According to Müller, humans possess five kinds of nerves and, consequently, five kinds of sensory awareness. One nerve or one kind of nerve cannot replace another. Consequently, one and the same stimulus impinging upon different nerves gives rise to the different sensations appropriate to those nerves. In his famous *Ignorabimus* lecture of 1872 [4], du Bois-Reymond described these contributions of Müller with his trademark flair for verbal color. Du Bois-Reymond, following Müller, called the substrata of nerve qualities *Sinnsubstanzen*, 'sense substances':

> It is universally conceded that the sense-organs and the sense-nerves carry to their appropriate cerebral regions or, as Johannes Müller calls them, "sense-substances," a motion that is in all cases ultimately identical. As in the experiment suggested by Bidder and successfully made by Vulpian on the nerves of taste, and those of the muscles of the tongue, the sensory and motor nerves, on being cut across, so heal together that excitation of the one class of fibres is transmitted by the cicatrix to the other class: in like manner, were the experiment possible, fibres from different sets of nerves would blend perfectly together. With the nerves of vision and of hearing severed, and then crossed with each other, we should with the eye hear the lightning-flash as a thunder-clap, and with the ear we should see the thunder as a series of luminous impressions. ([4], p. 19)

In his *Handbook*, Müller reminded his readers that sensations need not have external causes at all, but can be caused internally by direct stimulation of a nerve or, as Frege mentioned, by an electrical impulse. Müller was familiar with Tourtual's observation that, in the instant when a patient's eyeball is removed by slicing the optic nerve, he or she may report seeing a great light. Müller held that suitable electrical impulses can produce sensations in any of the familiar five modalities and that light flashes appear to the eye when one terminal of a battery is attached to the eyelid and the other to the upper palate. Here is his brief report on electrical experiments conducted by Volta:

> In the hearing organ, electrical stimulation excites the sensation of sound. Volta experienced, after he placed his ears between the poles of a battery composed of forty pairs of plates and closed the circuit, a hissing and pulsating sound that continued throughout the time that the current was flowing. ([15], vol. 2, p. 253)

Physiologists and philosophers took Müller's laws to ring the death knell for a copybook account of perception according to which the perceptual process is a delivery service to the mind of loads of tiny pictures of external objects, the correctness of those pictures consisting in a likeness to their originals. Müller was deemed to have shown that the sensory nerves feed us symbols or signs of their causes. These symbols were supposed to be properties of sensory nerves that somehow track their ultimate causes, but certainly need not resemble them pictorially. No musical tone could be a literal picture of the corresponding sonic vibration. Further, if our perceptual signs of the world outside our heads are to afford us real knowledge of that world, they will have to be interpreted in some reliable fashion. 'Interpretation [*Auslegung*]' was the term Müller chose in expounding his laws. In the first paragraph of "From the senses" he wrote,

> That which comes to consciousness through the senses is, in the first place, only properties and states of our nerves. However, imagination [*die Vorstellung*] and judgment [*das Urtheil*] are ready to interpret [*auslegen*] the processes produced in our nerves by external causes as properties and alterations of bodies themselves outside us. ([15], vol. 2, p. 249)

Müller also took sensory ideas to be, in a literal sense, signs of sensations. In his chapter, "On the life of the soul," he claimed,

> Consequently, ideas relate themselves to sensations much more like a sign for a thing, . . . Idea and sensation doubtless relate to each other as a word to a thing, a melody written in musical notes to the melody itself. ([15], vol. 2, pp. 526–27)

Though Müller's physiology may have been outstanding, his logic was far from impeccable. That I can, by pushing, pinching, or shocking the eyeball electrically, produce in myself sensations of red similar to those I experience while looking at the red costume of a department store Santa, hardly proves that the color of his trousers is a feature of my own optic nerve and not of his outfit. Similarly, the fact that I can make a portion of my hand feel hot by sprinkling a tiny amount of acid on it cannot show that a sandy beach isn't hot on a summer day and that my nerves are either the principal bearers of heat or internal telegraphs transmitting codes that I may wish to interpret as heat. Frege spotted and successfully avoided these troubled waters in "The thought" when he objected to the notion that a green field of which I am aware is reducible to my idea of a green field. As Frege reminded us, my ideas are one and all invisible; a fortiori they cannot be green ([9], pp. 44–45).

Those German philosophers and scientists who drew explicitly idealistic conclusions from Müller's results became targets of Frege's criticism. Du Bois-Reymond was one of them. In the same lecture that hoisted the banner of *Ignorabimus*, he announced,

> That there are in reality no [sensory] qualities follows from the analysis of our sensory perceptions. . . . It is the sense-substances that first translate the stimuli that are similar in all nerves into sensation and, as the real bearers of Johannes Müller's 'specific energies,' produce the various qualities according to their natures. The Mosaic [assertion], "And there was light," is false physiologically. There was light only when the first red eyespot of an infusorium distinguished light from dark for the first time. Without the visual and auditory substances, the world that glows with color and resounds with noise would be dark and silent. And silent and dark in itself, that is, without [sensory] qualities, is the world according to the mechanical outlook, won through objective consideration. (du Bois-Reymond [5], pp. 109–10)

Frege's response to this brand of idealism was perfectly effective:

> As is appropriate for a researcher into natural science, he is, in the first place, far from taking the things he is convinced of seeing and touching to be his ideas. On the contrary, in sensory impressions, he believes himself to have the most certain evidence of things that subsist in total independence of his feeling, imagining, and thinking, and that have no need of his consciousness. ([9], pp. 45–46)

Frege was certainly right to point out that no conclusion that the phenomenal world does not really exist or is lacking in the familiar sensory qualities can be legitimately drawn from Müller's laws of specific nerve energies, not to mention the copious evidence Müller cited in illustration and support of them. Indeed, any such conclusions are flatly inconsistent with the *ipsissima verba* of laws that explicitly require familiar objects to be causes of sensation and that treat of nerves and nerve fibers, brains, light rays, and electric currents as causal agents or lines of transmission existing prior to and, hence, independently of any perceptions of them. These are all aspects of an extramental world that can exist unperceived and are, with the possible exception of electric currents, identified by means of the properties they commonly reveal to sensory inspection. Du Bois-Reymond was therefore guilty of fallacy in inferring from Müller's work that there are no sensory qualities in reality and, hence, no light and no everyday phenomenal world.

A nagging question remains to be answered. If the phenomenal world does exist but our sensory impressions are merely irregular and unreliable signs of its features, how is that world made available, or—to use Frege's apt expression—opened up to us? This is a question von Helmholtz, among others, sought to answer by locating those principles by which we learn, from the signs bequeathed to us via stimulation, about the properties of that objective world that does the stimulating.[4]

3. Helmholtz on Analysis, Logic, and Binocular Vision

I see with two eyes a regularly patterned and spatially extended visual world of reasonably persistent, readily reidentifiable objects spread out in three dimensions rather than either a random welter of sense impressions or a rotating pinwheel of double images because I, in command of my eyes and nerves, am, if tacitly, a mathematician and logician. This, according to von Helmholtz, is the short answer to the question, "How is the objective world opened up for us by means of binocular vision?" Müller had held that binocular vision produces a single image not thanks to logic but due to anatomy. He hypothesized a physical coincidence between paired receptors on the two retinas of a single person, one receptor of each pair per retina. The proper action of the pairs is to combine their identical but doubled efforts to produce a single image from the separate and, as Frege also knew, disparate retinal illuminations falling on them. Müller supposed that the corresponding receptors on the two retinas are the paired roots of a single nerve fiber, split at the optic chiasma. Because of the hypothesized identity in function between corresponding receptors, Müller's idea was known as an "identity theory of binocular vision."

Experimental results, many collected in the 1850s through ingenious and persistent uses of new devices such as the opthalmoscope and stereoscope, encouraged vision theorists to think that the single image in relief produced by binocular vision is a result of psychological, learned processes, "an act of the soul," and not due to any anatomically guaranteed identity between retinal points. Wheatstone invented the stereoscope in 1838; shortly thereafter stereoscopic experiments of Dove showed that binocular vision creates, from a black image presented to one eye and a congruent white image to the other, not a single black, white, or grey image, as the identity theory would lead one to expect, but an image of what seems to be a metallic three-dimensional object that shines. This is the phenomenon of stereoscopic luster. In 1859, physiologist Volkmann, reflecting on this new evidence, launched a full-scale attack on Müller's identity theory. Volkmann wrote,

> The perception of images falling on noncorresponding retinal points as a single image is of psychological origin. . . . The unification of images falling on non-corresponding retinal points is an act of the soul which presupposes experience of the real unity of the object seen, and which we can only acquire through the training of the senses. (Volkmann [18], pp. 86–87, quoted in Lenoir [13], p. 131)

By the 1860s, von Helmholtz, at one time a surgeon in the Prussian army, was well established in the community of German physiologists, having invented the opthalmoscope. He had already touched off a heated factional dispute in academic circles when, at age 26, he read his "On the conservation of force" before the Berlin Physical Society. The address was a trenchant criticism of the vitalism defended by members of the older generation, Müller among them, but abhorred by the young Turks, including Helmholtz and du Bois-Reymond. In the eleven years from 1855 to 1866, Helmholtz produced the three volumes of his *Treatise on Physiological Optics* [19], the last of which contained his logico-mathematical theory of binocular vision. There, Helmholtz envisioned the eye as a complex optical instrument for taking measurements—he placed the title "The Eye as an Optical Instrument" over the first section of his contemporaneous lectures "The recent progress of the theory of vision" ([20], p. 61). Further, he imagined the mind, at least those powers of it devoted to vision, to be a scientist who carefully targets and adjusts his living optical instrument to take required measurements and who conducts crucial experiments by exploiting a tacit but sophisticated knowledge of mathematics and logic.

An image of the eye's mind as mathematician had already featured in Müller's *Handbook*: he compared the mind's interpretation of perceptual signs to its grasp of mathematical equations. In Müller's Volume Two, we read, "It is not necessary that the idea [*Vorstellung*] of spatial objects is itself extended in space. Rather, the idea can be related to the sensible object as the expression of a figure in an algebraic equation to the figure itself or as the infinitely small differential to the integral in analysis" ([15], vol. 2, p. 527). For Helmholtz, the instrumental eye is moved by the mathematical mind in accord with a psychophysical principle: the law of easiest orientation. In this, Helmholtz seems to have been inspired by Wundt's hypothesis of least exertion, that "the eye always adopts that position in which the opposition of its muscles is least" ([22], p. 92, quoted in [13], p. 134), an hypothesis Wundt tested on his new ophthalmotrope, a spring-driven model of the eyeball and attendant muscles. Wundt discovered that a good quantitative expression for a principle of least exertion is an equation quadratic in the measure by which individual eyeball muscles lengthen or shorten. Wundt noted that his equation was identical in form with the Gaussian probabilistic law of least squares governing observational error in scientific measurement. Hence, wrote Wundt, "[W]henever we move the visual axis into a new position, the eye proceeds just like a mathematician when he compensates for errors according to the rules of the probability calculus" ([23], pp. 58–59, quoted in [13],

p. 137). In guiding the eyes, the mind calculates, just as a human mathematician, the errors among multiple observations in obedience to a principle of least squares.

Helmholtz adopted the idea that the movement of the two eyes in directing themselves toward a single point in the visual field is the practical expression of the mind's tacit knowledge of a law and its ability to calculate accordingly. Therefore, to perceive a single persistent object on the basis of a series of sensations, the visual mind would act as an experimental scientist, performing experiments with the instrument which is the eye. In effect, the mind poses an hypothesis, for example, "these sensations are signs of a single motionless object," which it attempts to confirm by willing the eye to move, and so focusing attention on one or another location in the sensational series. The hypothesis is deemed confirmed, and any apparent motion is due solely to the motion of the eyeball rather than to relative motion among members of the series, if the deviation or error in eyeball motion submits to the law of least squares.

But if our minds, in perceiving relatively permanent objects, confirm or disconfirm laws concerning the constancy of those objects, and if, in that, they act like scientists setting up instruments, then there must be forms of reasoning in perception that intervene between stimulus and percept. Helmholtz, once more in step with his colleague Wundt, embraced this consequence with the notion of unconscious inference [*unbewußter Schluß*]. Here, the mind of the eye is thought to be a logician, drawing tacit conclusions from premises tacitly understood. Helmholtz imagined his inner logician desiring to draw a conclusion concerning the location of a presented object in three-dimensional space. He pictured the logician as working from two premises. The relevant major premise may be a psychophysical generalization of the form "Whenever the muscles of the eye are innervated in fashion xyz, then the eyes focuses on an object at position *abc* in visual space." The minor premise is the unconscious claim that the muscles of the eye are being innervated in fashion xyz. The unconscious conclusion is that the eyes are focusing on an object at position abc.[5]

One can detect a loose tripartite analogy linking Helmholtz's unconscious inferences to the "nothing sensible" that Frege believed added to sense impressions so that the external world is opened up. First, both function as supplements to immediate sensation not encompassed in the strict content of that sensation. Müller had already distinguished within perception between the presentation [*Vorstellung*] and the sensation [*Empfindung*], only the second of which is literally given ([15], vol. 2, p. 526). Historian of psychology Boring's depiction of Helmholtz's doctrine of unconscious inference confirms that Helmholtz adopted it to anatomize those aspects of perception that do not reduce to sense impressions:

> Helmholtz was arguing, in the first place, that perception may contain many experiential data that are not immediately represented in the stimulus, a view which ought to have the support of every psychologist who has ever studied an illusion. He was arguing, in the second place, that these aspects of the perception that do not immediately represent the stimulus are, in a sense, additions which accrue to the perception in accordance with its development in past experience. (Boring [1], p. 308)

Helmholtz was also willing to allow, like Frege, that the nonsensible in perception might manifest itself independently of strict sensory content. To quote Boring on Helmholtz,

> If the sense-impressions are entirely lacking and we have only the imaginal equivalent of the *Anschauung*, then the experience may be called a *Vorstellung*, a use of this German word more like the English *idea* than is usually the case. . . . *Vorstellungen* lie outside the universe of discourse. ([1], p. 312)

Second, Helmholtz's unconscious inferences and Frege's "nothing sensible" feature in instances of a species of mental act that would be, when explicit, associated with a higher, ratiocinative consciousness, projecting a completed world in three dimensions from the thin and uneven data given to the senses. Third, both can be considered semantic in nature:

> Now this connection between names and objects, which demonstrably must be *learnt*, becomes just as firm and indestructible as that between sensations and the objects which produce them. We cannot help thinking of the usual signification of a word, even when it is used exceptionally in some other sense; . . . just in the same way as we cannot get rid of the normal signification of the sensations produced by any illusion of the senses, even when we know that they are not real. ([20], p. 134)

As Boring's comments suggest, Helmholtz endorsed a thoroughgoing empiricism about our knowledge of logic and mathematics, in their conscious and unconscious manifestations, that would have been anathema to Frege. It is therefore ironic that the fruits of Frege's rationalism and logicism, staunchly defended by his assaults on psychologism and empiricism, seem to us now a nearly perfect realization of Helmholtz's dreams. Were it consistent, the system of Frege's *Grundgesetze* would have provided an attractive, unitary treatment of mathematical analysis and logical inference within a detailed formalism that could be mechanical and calculatory in operation. With Frege's discoveries in hand (and plenty of hindsight), Helmholtz might have reduced the heavy theoretical overhead of the mathematician-cum-logician in the visual mind to a mechanism that makes automated deductions from symbolized premises. Frege's mature viewpoint, set out in "On sense and reference" and "The thought," allows us (in the twenty-first century) to realize the extent to which Helmholtz required, for the cogency of his theory, some such reduction. For if, as Frege believed, a nonmechanical inference from premises to conclusion requires a prior grasp, by the reasoner, of the content of premises and conclusion, then Helmholtz was caught in a vicious circle. Frege's treatment of identity statements and his analogy of the telescope encourage us to ask whether Helmholtz's problem of breaking out from our varied and private sensations to a relatively unvarying and shared object in the phenomenal world that they jointly represent is, at bottom, the very same as that of breaking out from our varied and private ideas to a grasp of the unvarying and public meanings involved in making a judgment. In effect, Frege's viewpoint presented Helmholtz with a charge of circularity: if the logician who works the eye is to make contentful inferences, he or she must grasp propositions, even if tacitly. Then he or she must already be equipped with what is, in effect, a further eye of a semantic sort and a further mind that calculates objective senses from subjective ideas. For Helmholtz himself claimed that the problem of tying a linguistic sign to its meaning is the very same as that of tying a sensation to its object. Hence, an explanation of binocular vision in terms of an understanding of premises and conclusions for inference will not do, since it presupposes that we already possess a semantic brand of binocular vision that allows us to grasp the sense of 'even numbers are just those divisible by two' and to "see" that the words ‘even number’ and ‘number divisible by two’ present the same meaning twice over. It is this further eye and its operation that Helmholtz has failed to explain.

4. Conclusion: from Helmholtz to Frege

I close with three remarks on Frege and the legacy of Helmholtz and Müller, thereby introducing three themes each of which is large enough to call for more extended treatment.

First, even before Helmholtz’s death in 1894, there was reaction, some of it antiempiricistic, against his logical theory of visual, and especially binocular, perception. The physiological theories of Hering, which Helmholtz thought to have refuted conclusively in the final volume of the *Treatise*, gained new recognition and adherents. Like Müller, Hering sought to explain binocular vision more on the basis of explicit anatomy than of unconscious psychology. He offered a vigorous defense of a version of the identity theory, and, like Frege in his final days, took geometry, rather than analysis or logic, to supply the only stable foundation for his efforts. Hering worked out a conception of an innate coordinate system of “spatial feelings” and applied to it techniques from projective geometry introduced into mathematics by Steiner. (The account of sensory illusions that Frege sketched in his unpublished essay of 1924/25, “Sources of knowledge of mathematics and the mathematical natural sciences,” employed geometrical reasoning exclusively ([11], p. 268). For Hering, and perhaps for the later Frege, the capacity for binocular vision was far more an innate endowment than a learned facility.

Second, the “nonsensibles” on which Frege called to open up the phenomenal world to us *may* have been for him propositional and inferential in character rather than ideate, imagic or conceptual, but nonpropositional. In “Sources of knowledge of mathematics and the mathematical natural sciences,” he took nonempirical geometrical and inferential knowledge to safeguard sensory knowledge from illusion, arguing that explanatory circularity upsets any attempt (like that of Helmholtz) to explain empirical knowledge along strictly empirical lines:

> Of course, if there were no laws governing events, or if the laws governing events in the physical world were unknowable for us, we would lack the means for recognizing illusions for what they are and thus for rendering them harmless. . . . In order to know the laws of nature we need perceptions that are free from illusion. And so, on its own, sense perception can be of little use to us, since to know the laws of nature we also need the other sources of knowledge: the logical and the geometrical. ([11], p. 268)

Frege’s logical source issues in correct inferences, his geometrical source in axioms for Euclidean geometry. If this interpretation of Frege is adequate and extends to “The thought,” we can place Frege’s views on nonsensibles in perception in close company with those of Helmholtz, for whom the nonsensibles were also both propositional and inferential in nature, but apart from those of Mach, the psychologists of the Würzburg school, and the early Gestalt theorists, for whom the nonsensible components in perception were often nonpropositional forms or structures of presented objects.

Third, although the solutions Müller and Helmholtz offered to shared problems of vision and thought were later questioned, even discarded, the problems themselves and the attitudes of these great scientists to those problems were not. Helmholtz took the problem of binocular vision to have signal import for epistemology generally, as did Frege, who seems to have conceived of it as an analogue to a philosophical problem of the objective world, namely, the problem of constructing not a single object from different retinal images but a common world, not reducible to any individual's stream of consciousness, from completely different mindsets. Were Helmholtz to have succeeded with binocular vision, then an answer to the latter, more philosophical, question would be suggested: our common access to a single logic and a single mathematics underwrites our success in living within a single, unitary, spatially-extended world containing all physical objects, their motions, and all our behaviors, despite the gross disparities among the ways that world is presented to us in sense. One such mathematics, be it classical analysis or Euclidean geometry, would have to be true and binding on the thought of all if there is to be a firm guarantee that we all construct the same external world in the same space. Frege, I think, took on this problem and a good part of Helmholtz's answer, but insisted that Helmholtz's empiricism was insufficient to the task. Some nonsensible source of knowledge would be necessary.[6]

Notes

1. The heliometer, employed to measure the sun's diameter, was a telescope with a split objective lens. Helmholtz's design for the ophthalmometer, which he used for measuring the dimensions of the human eye, was based upon that of the heliometer ([13], p. 148).

2. Ernst Abbe, supervisor of Frege's *Habilitationsschrift* [7], achieved fame in mathematical optics; his innovative ideas inspired new lens designs. The mathematics faculty at the University of Jena, of which Frege was a member, received funds from the nearby Zeiss optical works. Zeiss supported Frege's own research with a grant.

3. In the twentieth century, the liaison between logic and the study of vision was reflected in those notebooks of Wittgenstein [21] where attention was paid to the logical properties of the visual field.

4. A fuller examination of "The thought" in the light of Müller's *Handbook* would take into consideration Frege's observations on the ego in relation to ideas of portions of the perceiver's own body ([9], pp. 46–47). On the basis of similar observations, Müller criticized the idealists of his day for failing to delimit within the external world a "double outer world," that is, for failing to distinguish the subject's own body from other living bodies and inanimate objects ([15], vol. 2, p. 269).

5. Notions of unconscious inferences clearly played a role in the thinking of German logicists. See Dedekind's "Preface to the first edition" of *Was Sind und Was Sollen die Zahlen?* [2].

6. Unless otherwise indicated, translations from the German are by the author.

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Department of Philosophy
Indiana University
Sycamore Hall 026
Bloomington IN 47405-7005
dmccarty@indiana.edu