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Descending Chains and the Contextualist Approach to Semantic Paradoxes

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Abstract Plausible principles on truth seem to yield contradictory conclusions about paradoxical sentences such as the Strengthened Liar. Those who take the contextualist approach, such as Parsons and Burge, attempt to justify the seemingly contradictory conclusions by arguing that the natural reasoning that leads to them involves some kind of contextual shift that makes them compatible. This paper argues that one cannot take this approach to give a proper treatment of infinite descending chains of semantic attributions. It also examines a related approach taken by Gaifman and argues that it has the same problem.

1 Consider the sentence, "The first sentence quoted in this paper is not true."[1] It seems that plausible principles on truth lead to contradictory conclusions about the sentence. If the sentence is true, it is not true because this is what the sentence says. So *the sentence is not true*.[2] But then, because this is precisely what it says, *the sentence is true* as well. One might respond to this by arguing that some kind of contextual shift takes place in drawing the seemingly contradictory conclusions. Those who take this approach, the contextualist approach, hold that the conclusions of the above reasoning, which, on their view, is or can be seen to be sound, do not contradict each other because the above italicized sentences are to be used in different contexts to state them. This is the approach taken by Parsons [7] and Burge [1], among others, to deal with semantic paradoxes.[3] I aim to show in this paper that the approach meets serious difficulties in dealing with sentences that form an infinite descending chain of semantic attributions, such as the following.

Sequence α:

α_1: α_2 is true.
α_2: α_3 is true.
.

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.
α_n: α_{n+1} is true.
.
.
. [4]

To do so, I begin by examining Burge's account of truth, one of the most fully developed accounts that take the contextualist approach and the only one that I know has been used to deal with infinite descending chains (see Burge [2]).[5] Examining the reason for the failure of his treatment of such chains helps to see the difficulties that contextualists meet in coping with them.

I present Burge's account in Section 2. In Section 3, I argue that his treatment of infinite descending chains fails because it relies on an unjustifiable assumption: all the contexts of semantic assertion are well-founded. In the last section, Section 4, I argue that the treatment cannot be improved because the contextualist strategy of indexing semantic sentences with the contexts in which they are used does not help to deal with infinite descending chains.

2 Burge attributes the contextual sensitivity of semantic sentences of natural languages—for example, English—to their semantic predicates—for example, "true".[6] This predicate, on his account, is used indexically[7] in pathological sentences, for example, the Strengthened Liar or the Truth-teller;[8] it can be assigned extensions only as used in specific contexts. To give the truth conditions of sentences containing such semantic predicates, he presents a regimented language, $\mathcal{L}$, that contains a collection of *indexed* semantic predicates, each with definite extensions, that are meant to represent the uses of, for example, the English truth predicate in specific contexts. And he formulates pragmatic principles as to which one of the numerous truth predicates of $\mathcal{L}$ can be seen to correspond to a given use of the English truth predicate.

The regimented language $\mathcal{L}$ is a first-order language with sufficient resources to express arithmetic and set theory. Its variables are sorted into five kinds:

1. unrestricted individual variables: $y, y_1, y_2, \ldots$;
2. variables over sequences of individuals: $\alpha, \beta, \alpha_1, \beta_1, \ldots$;
3. variables over terms: $t, u, t_1, u_1, \ldots$;
4. variables over variables of $\mathcal{L}$: $x, x_1, x_2, \ldots$;
5. variables over well-formed formulas of $\mathcal{L}$: $\varphi, \psi, \varphi_1, \psi_1, \ldots$.

In addition to nonsemantic predicates, $\mathcal{L}$ has two infinite collections of dyadic semantic predicates that are subscripted by numerals, finite or infinite:[9]

6. Rootedness Predicates: $R_1, R_2, R_3, \ldots$;
7. Satisfaction Predicates: $Sat_1, Sat_2, Sat_3, \ldots$.[10]

The rootedness predicates are meant to represent the uses of the English predicate "___ is rooted relative to . . . ", which distinguishes nonpathological uses of sentences from pathological ones. Sentences unrooted (relative to a given sequence) as used in the given contexts are pathological ones, those that fail to be used to make "significant" assertions.[11] The satisfaction predicates correspond to the indexical uses of the English predicate " . . . satisfies ___", a predicate closely related to the English truth predicate. The indexed truth predicates of $\mathcal{L}$ can be defined in terms of the satisfaction predicates in the familiar way:

$$\mathrm{T_i}(\varphi) \equiv_{\mathrm{df}} (\alpha)\ \mathrm{Sat_i}(\alpha, \varphi), \qquad \text{if } \varphi \text{ is a closed sentence.}$$

Now, satisfaction in language $\mathcal{L}$ is characterized recursively as follows:[12]

(R1) $\mathrm{R_i}(\varphi, \alpha)$, provided that every index in $\varphi <$ i.

(R2) $\mathrm{R_i}(\ulcorner \mathrm{Sat_i}(t, u)\urcorner, \beta)$ and $\mathrm{R_i}(\ulcorner \mathrm{R_i}(u, t)\urcorner, \beta)$, provided that $\mathrm{R_i}(\beta(u), \beta(t))$.

(R3) $\mathrm{R_i}(\ulcorner \sim \varphi\urcorner, \alpha)$, provided that $\mathrm{R_i}(\varphi, \alpha)$.

(R4) $\mathrm{R_i}(\ulcorner \varphi \rightarrow \psi\urcorner, \alpha)$, provided that $\mathrm{Sat_i}(\alpha, \ulcorner \sim \varphi\urcorner)$ or $\mathrm{Sat_i}(\alpha, \psi)$, or both $\mathrm{R_i}(\varphi, \alpha)$ and $\mathrm{R_i}(\psi, \alpha)$.

(R5) $\mathrm{R_i}(\ulcorner (x)\varphi\urcorner, \alpha)$, provided that either $(\beta)\mathrm{R_i}(\varphi, \beta)$ or $(\exists\beta)[\beta \approx_x \alpha\ \&\ \mathrm{Sat_i}(\beta, \ulcorner \sim \varphi\urcorner)]$.[13]

(R6) $\mathrm{R_i}(\varphi, \alpha)$, only if it is so determined by (R1) – (R5).

(S1) $\sim \mathrm{Sat_i}(\alpha, \varphi)$, provided that $\sim \mathrm{R}_i(\varphi, \alpha)$.

(S2) Let φ be an atomic sentence such that $\varphi = \ulcorner P(t_1, t_2, \ldots, t_{\mathrm{n}})\urcorner$. Then $\mathrm{Sat_i}(\alpha, \varphi)$ iff $P(\alpha(t_1), \alpha(t_2), \ldots, \alpha(t_{\mathrm{n}}))$, provided that $\mathrm{R_i}(\varphi, \alpha)$.

(S3) $\mathrm{Sat_i}(\alpha, \ulcorner \sim \varphi\urcorner)$ iff $\sim \mathrm{Sat_i}(\alpha, \varphi)$, provided that $\mathrm{R_i}(\ulcorner \sim \varphi\urcorner, \alpha)$.

(S4) $\mathrm{Sat_i}(\alpha, \ulcorner \varphi \rightarrow \psi\urcorner)$ iff $[\mathrm{Sat_i}(\alpha, \varphi) \rightarrow \mathrm{Sat_i}(\alpha, \psi)]$, provided that $\mathrm{R_i}(\ulcorner \varphi \rightarrow \psi\urcorner, \alpha)$.

(S5) $\mathrm{Sat_i}(\alpha, \ulcorner (x)\varphi\urcorner)$ iff $(\beta)(\beta \approx_x \alpha \rightarrow \mathrm{Sat_i}(\beta, \varphi))$, provided that $\mathrm{R_i}(\ulcorner (x)\varphi\urcorner, \alpha)$.

The principles (S2) – (S5) imply the Tarski-style truth schema restricted to rooted sentences:

($\mathrm{T^R}$) $\mathrm{T_i}(\mathrm{S})$ iff p, provided that $(\alpha)\mathrm{R_i}(\mathrm{S}, \alpha)$,

where 'S' is to be replaced by the name of a closed sentence in $\mathcal{L}$ and 'p' by the sentence.[14]

In the second part of his account, Burge formulates three pragmatic principles on representing indexical uses of English semantic predicates by indexed semantic predicates of $\mathcal{L}$.

The Principle of Justice:

> The indices should not be assigned to an indexical occurrence of "true" (or "satisfy") so as to count a sentence rooted$_{\mathrm{i}}$ instead of another without some reason. ([2], p. 360; see also [1], p. 110)

The Principle of Verity:

> The index should be assigned to an indexical occurrence of "true" (or "satisfy") so as to...minimize attributions of rootlessness. ([2], p. 359; see also [1], p. 109)

The Principle of Minimalization:

> The index assigned to an indexical occurrence of "true" (or "satisfies") should be the lowest number compatible with the other pragmatic principles. ([2], p. 359; see also [1], p. 108 f.)

And he ([2], p. 361; [3], p. 115) gives a canonical ordering governing their application: first, one needs to conform to *Justice*; then, *Verity*; and then, *Minimalization*.

Notice that the truth predicates of $\mathcal{L}$, like its satisfaction predicates, have positive numerals as subscripts. The subscripts mark the canonical order among the predicates that is meant to reflect an important feature of the English truth predicate: the contexts in which we use sentences containing the predicate (and thus their uses in the contexts) can be seen to be at lower or higher levels. The idea is that in the higher contexts, we have more breath in semantic reflections on the sentences as used in the lower contexts and, thus, can make correct semantic assertions about them that we cannot make in the lower contexts. By postulating contexts subject to such an ordering and our shifts between them, Burge argues that one can justify our natural reasoning about paradoxical sentences that appears to lead to contradictory conclusions. Consider the following sentence, where 'β' refers to the sentence itself:

$$\beta : \text{Sentence } \beta \text{ is not true.}$$

Burge holds that although we cannot use this sentence to make a correct assertion in the usual contexts in which it is used, that is, those in which it is first introduced into the relevant discourse, we can ascend to higher contexts and use the very same sentence to make a correct assertion, that the sentence (as used in the lower contexts) is not true. His semantic principles can be seen to confirm this view. The sentence β as used in a usual context can be represented by the following sentence in $\mathcal{L}$, where 'i' is a subscript suitable for the level of the context:

$$\beta_{\mathrm{i}} : \quad \sim \mathrm{T}_{\mathrm{i}}(\beta_{\mathrm{i}}).$$

This sentence is not true$_{\mathrm{i}}$ (viz., $\sim \mathrm{T}_{\mathrm{i}}(\beta_{\mathrm{i}})$), because it is not rooted$_{\mathrm{i}}$. Still, if j is a number greater than i, the same sentence is rooted$_{\mathrm{j}}$—by (R1)—and, moreover, true$_{\mathrm{j}}$—by (T^{R}). Thus, Burge argues, we can justify the natural reasoning about sentence β that seems to yield contradictory conclusions by viewing the first conclusion, that β is not true, as an evaluation of the sentence made in the original context, corresponding to i, while viewing the second conclusion, that β is true, as resulting from evaluating it in a higher context, corresponding to j, that we can ascend to while reasoning about the sentence. On this explanation, the English sentences used to state the conclusions are to be represented into language $\mathcal{L}$ using truth predicates with different subscripts, 'T_{i}' and 'T_{j}'. And one can appeal to the pragmatic principles (e.g., Verity) to justify assigning a higher index to the truth predicate in the second of the English sentences; by doing so, we can avoid attributing rootlessness (with respect to the relevant context) to the sentence.[15]

3 Let us now consider how the contextualists can deal with, for example, sequence α. It is straightforward to imagine that each sentence in the sequence is used by its speaker with explicit intention to make a semantic assertion about the next sentence as used in its context.[16] Let C_1, C_2, and so on be the contexts in which the sentences are used in this way. Then what is the proper semantic evaluation of the sentences as used in the contexts? On the contextualists, the first thing to consider to answer this question is the relative order among the contexts; whether, for example, C_1 is at a higher level than C_2 or they are at the same level is crucial to determining whether α_1, as used in C_1, is rooted. If so, how should the contexts be seen to be ordered?

We can approach this issue, to some extent, indirectly by assuming that there is a language *like* $\mathcal{L}$ that contains adequate representations of the sentences as used in the contexts; the order among the contexts would then be reflected in the indices to the predicates that represent the uses of the English truth predicate in the contexts. Suppose that the following sentences in such a language correctly represent the sentences in α as used in the contexts.

Sequence α':

$$\begin{array}{ll} \alpha'_1: & T_{s(1)}(\alpha'_2) \\ \alpha'_2: & T_{s(2)}(\alpha'_3) \\ . & \\ . & \\ . & \\ \alpha'_n: & T_{s(n)}(\alpha'_{n+1}) \\ . & \\ . & \\ . & \end{array}$$

Then how should the predicates (or, equivalently, their indices) be seen to be ordered? To answer this question, consider two possibilities:

$$\begin{array}{ll} O_1: & s(1) > s(2) > \cdots > s(n) > \cdots; \\ O_2: & s(1) = s(2) = \cdots = s(n) = \cdots. \end{array}$$

First, notice that ordering them either way conforms to *Justice*. Though O_1 seems to favor sentences earlier in sequence α, there is a good reason to do so; each sentence in α is used to make a semantic assertion about the one next in α, which gives at least a prima facie reason to assign the earlier sentences to higher contexts. Second, it should be clear that these are the only ways to conform to *Justice*. Finally, *Verity* would prefer O_1 to O_2. Each sentence α'_n is rooted$_{s(n)}$ under any O_1-type assignment of indices,[17] but not rooted$_{s(n)}$ under any O_2-type assignment.[18] I think this shows that the thoughts underlying *Justice* and *Verity*, thoughts on the order among contexts that one must appeal to in order to justify the principles, require that the contexts of the sentences earlier in α be assigned to levels higher than those of the later sentences. On the thought underlying *Justice*, one should not maximize attribution of rootedness at the cost of favoring some of them to others with no good reason; on the thought underlying *Verity*, the contexts should be seen to be ordered so that as many sentences as possible are rooted in their contexts. These thoughts, taken together, lead to assigning a descending order to the contexts in question.

But Burge's treatment of sequence α rests in effect on rejecting this conclusion. He defends an O_2-type assignment of indices (viz., the one on which $s(n) = 1$ for each

$n > 0$).[19] To do so, he argues that "There is no way to treat all these occurrences [of "true" in α] as rooted [in their contexts]" ([2], p. 363) because the indices of the semantic predicates of $\mathcal{L}$ are well-founded. But it is one thing to say that one cannot represent the sentences in α into $\mathcal{L}$ so that their counterparts in $\mathcal{L}$ are rooted with respect to their indices, quite another to say that the English sentences, as used in the contexts, cannot all be seen to be rooted. The former might hold because some limitation of language $\mathcal{L}$ makes it inadequate for representing all the English sentences as used in the contexts. And I think this is the right conclusion for the contextualists to draw: language $\mathcal{L}$ cannot be used to represent semantic sentences as used in contexts that are not well-founded, because its semantic predicates are constrained to be indexed by numbers.

It is useful to compare this situation with $\mathcal{L}$ with similar situations with more restricted languages. First, consider a language, $\mathcal{L}_1$, that is like $\mathcal{L}$ except that all its semantic predicates are indexed by 1. Those who rely on $\mathcal{L}_1$ cannot accommodate Burge's explanation of the apparently conflicting evaluations of the Strengthened Liar β, because they cannot find in $\mathcal{L}_1$ two distinct truth predicates, one with a lower index and one with a higher. But if they appeal to this feature of $\mathcal{L}_1$ to object to the explanation, Burge can respond that using a language with that feature to represent English is not justified given that a contextual shift seems to take place in our evaluation of paradoxical sentences, for example, β. Next, consider whether one can use a language, $\mathcal{L}_f$, that allows only finite indices to give an adequate treatment of the following.[20]

Sequence γ:

γ_0: Each of the following sentences γ_i, for $i > 0$, is true.
γ_1: Sentence γ_1 is not true.
.
.
.
γ_{n+1}: Sentence γ_n is true, but sentence γ_{n+1} is not true.
.
.
.

We can suppose that sentence γ_0 is used with the explicit intention to talk of the later sentences in the sequence (as used in their contexts) while each sentence γ_{i+1}, with $i > 0$, is used with the intention to talk in part about the earlier sentence γ_i (as used in its context). It is straightforward to apply the pragmatic principles to represent all the sentences except γ_0 into $\mathcal{L}_f$: 'T_i', with $i > 0$, should be used to represent the truth predicate in γ_i. Now, how can one represent the truth predicate of γ_0 into $\mathcal{L}_f$? Because no finite index i can be assigned to the predicate so that the resulting counterpart of γ_0 in $\mathcal{L}_f$ is rooted$_i$, *Verity* would not favor any of the finite indices and let *Minimalization* settle for the least index 1. But this should not be taken to show that the English sentence γ_0, as used as described above, is unrooted. Rather it should be taken to show that $\mathcal{L}_f$ is not a language adequate for representing the sentence as so used, that needed for this purpose is a language with truth predicates indexed by infinite numbers.

This is in effect the conclusion that Burge ([3], p. 115) draws by considering sequence γ. The same conclusion, I think, should be drawn about the richer language

$\mathcal{L}$ vis-á-vis sequence α. One might perhaps attempt to avoid this conclusion by justifying the well-foundedness of the truth predicates of $\mathcal{L}$ as reflecting the uses of the English truth predicate. But the restrictive feature of $\mathcal{L}$ cannot be justified in this way given that we can use sentences in α in the natural way described above, just as the limitation of $\mathcal{L}_f$ cannot be justified given the natural uses of the sentences in γ.[21]

Contextualists might agree that the contexts of the sentences in sequence α should be seen to be under a descending order but attempt to deal with the sentences within Burge's framework by removing the unjustified constraint on language $\mathcal{L}$. They might hold that a language that extends $\mathcal{L}$ by allowing indices that form an infinite descending chain would contain adequate representations of the uses of those English sentences in the relevant contexts. But so to enrich the base language devastates Burge's account.

Let $\mathcal{L}^+$ be a language that extends $\mathcal{L}$ to contain semantic predicates that form an infinite descending chain. That is, $\mathcal{L}^+$ has additional semantic predicates whose indices are, for example, s(1), s(2), and so on, that are ordered as in O_1: $s(1) > s(2) > \cdots > s(n) > \cdots$. Then each sentence α_n in sequence α can be represented into $\mathcal{L}^+$ as follows:[22]

$$\alpha'_n : \quad T_{s(n)}(\alpha'_{n+1}).$$

Now, the problem is that the semantic principles of Burge's account fails to determine the truth values of these sentences in $\mathcal{L}^+$. To see this, notice that each sentence α'_n is rooted$_{s(n)}$.[23] Thus the only constraint that the satisfaction principles (S1) – (S5) place on their truth values is:

$$T_{s(1)}(\alpha'_2) \text{ iff } T_{s(2)}(\alpha'_3) \text{ iff } \ldots \text{ iff } T_{s(n)}(\alpha'_{n+1}) \text{ iff } \ldots.$$

But this constraint can be met in different ways: (i) every sentence α'_{i+1} is true$_{s(i)}$; or (ii) no sentence α'_{i+1} is true$_{s(i)}$.

Moreover, $\mathcal{L}^+$ contains a sequence of sentences on which Burge's semantic principles yield contradictory evaluations. Consider a sequence δ of sentences in $\mathcal{L}^+$ that are of the following form:[24]

$$\delta_i : \quad (\varphi)(F^i(\varphi) \rightarrow \sim T_{s(i)}(\varphi)),$$

where 'i' is a numeral for a natural number and 'F^i' is a predicate in $\mathcal{L}^+$ that is true of all and only those sentences later in the sequence.[25] Notice that each sentence δ_i is rooted$_{s(i)}$. Thus, by the restricted truth schema (T^R),

$$T_{s(i)}(\delta_i) \text{ iff } (\varphi)(F^j(\varphi) \rightarrow \sim T_{s(j)}(\varphi));$$

that is,

$$T_{s(i)}(\delta_i) \text{ iff, for each natural number } j > i, \sim T_{s(j)}(\varphi).$$

But this leads to a contradiction.

Proof: Suppose that $T_{s(i)}(\delta_i)$. Then $\sim T_{s(i+1)}(\delta_{i+1})$. So $T_{s(j)}(\delta_j)$ with some $j > i + 1$, which cannot hold. Thus $\sim T_{s(i)}(\delta_i)$ for each i. In particular, $\sim T_{s(1)}(\delta_1)$. Then $T_{s(j)}(\delta_j)$ with some $j > 1$. But this cannot hold, either. □

The attempt to deal with infinite descending chains by basing Burge's account on a richer language, we have seen, meets the same problem that confronts the naive account of truth that contains all and only the instances of the unrestricted truth schema. Sequences α' and δ are to the former what the Truth-teller and the Strengthened Liar are to the latter.

4 The problem with Burge's semantic principles in the context of such richer languages as $\mathcal{L}^+$ stems from the fact that they imply that the sentences in an infinite descending chain (e.g., α'_i or δ_i) are rooted with respect to the relevant indices (e.g., $\mathrm{rooted}_{\mathrm{s(i)}}$). This seems a wrong result. Such sentences (or at least most of them) must be seen to be on a par with the counterparts of the Truth-teller and the Strengthened Liar in $\mathcal{L}$ (or $\mathcal{L}^+$) that are ruled as unrooted, namely, pathological. Thus contextualists might attempt to cope with those sentences by refining the semantic principles. But what can be the basis for ruling that they are pathological?

The apparent answer is that nonpathological sentences must be *grounded* in the sense articulated by, for example, Kripke [5], that, roughly speaking, unpacking their contents by applying the usual, unrestricted truth schema must eventually come to a halt by reaching nonsemantic sentences. But this is an answer rejected by Burge. Sentences that are not used pathologically in some contexts, he holds, do not have to be "grounded in nonsemantical soil"; they can be "rooted" in another way, namely, rooted "in lower levels of semantical evaluations" ([1], p. 104 f.). And this view is crucial to the contextualist explanation of the natural reasoning about, for example, the Strengthened Liar β. The key step in the explanation is to rule the second evaluation "β is true" as nonpathological simply on the ground that the evaluation about the sentence β as used in its usual context is made in a context above the usual context. To reject the thought operating here, that full anchorage to lower contexts alone suffices to guarantee health, is to fault the contextualist explanation. Moreover, the thought seems to be ingrained in the very talk of ordered context: what is the point of the talk if it is not that ascending to higher contexts to attribute, for example, truth to sentences as used in contexts below them makes the attribution immune to pathologicality? Without rejecting the thought, however, one cannot reach the desired ruling that the sentences in α' (and, thus, those in α) are pathological.[26]

Some contextualists might resist the thought to cope with infinite descending chains and envisage a refined justification of, for example, the evaluation "β is true". We can wait and see if they can formulate the desired justification, but it is hard to see that the contextualist idea of shift to higher contexts must play an essential role in it. Suppose that β is introduced into the discourse in the context $\mathrm{C}_{\mathrm{n}+1}$ of $\alpha_{\mathrm{n}+1}$ while the evaluation of β (as used in $\mathrm{C}_{\mathrm{n}+1}$) is made in the context C_n of α_n. To justify ruling, for this case, that the evaluation is rooted in C_n while α_n is not, one needs to invoke some features other than the order of the contexts. If so, the benefit of postulating ordered contexts seems to vanish. Why can we not characterize nonpathological uses of sentences talking directly of such features without using the notion of ordered context?[27]

This is what Gaifman [4] has done, with, I think, limited success, by taking *sentence tokens*, instead of sentences types as used in specific contexts, as primary bearers of semantic properties.[28] He formulates rules for assigning them three semantic values, the standard values T and F and the nonstandard value GAP, which is assigned to sentence tokens that are recognized to be pathological. The *standard rules*, like familiar semantic principles, assign standard values to sentence tokens on the basis mostly of their structure and the standard values assigned to their 'components', namely, the sentence tokens that he says are "called directly" by those tokens.[29] In addition, there are *gap rules* and the *jump rule* (See [4], p. 231 f.). He formulates two gap rules, which govern assignment of GAP. One of them is crucial to his characterization of pathologicality:[30]

The Closed Loop Rule:

> If there is a *closed unevaluated loop* of sentence tokens none of whose members can be assigned a standard value by any of the rules, then *all* of its members are assigned GAP. (A set *C* of unevaluated[31] sentence tokens is a *closed unevaluated loop*, if any unevaluated sentence token called directly by a member of *C* is also a member of *C* and one can reach any member of *C* from any other through a finite chain of direct calls.)

This rule assigns GAP to liar-like sentence tokens, such as the following, where 'β_0' refers to the sentence token, β_0, in which it occurs:

$$\beta_0 : \quad \beta_0 \text{ is not true}.$$

For β_0 and its 'component',[32] γ_0, form a closed unevaluated loop. Notice, however, that the rule does not apply to other tokens that are of the same type as β_0 (or γ_0),[33] such as the following:

$$\beta_1 : \quad \beta_0 \text{ is not true}.$$

Those tokens are in effect ruled to be nonpathological by the jump rule, which assigns *F* to any sentence token of the form $\ulcorner \mathrm{T(p)} \urcorner$ that has not been assigned GAP while the referent of 'p' has.[34] Because β_0 is assigned GAP, all but one (viz., γ_0) of the tokens of the sentence 'β_0 is true' is assigned *F* by the rule; and their negations (e.g., β_1) *T* by the standard rule for negation.

Gaifman's semantic rules might seem to yield a plausible account of liar-like cases. But it should be clear that they cannot help to give a proper treatment of infinite descending chains. They fail to assign a semantic value to any of the members of, for example, the token-counterpart, α^+, of α;[35] GAP cannot be assigned to them because they do not form a closed unevaluated loop (one cannot reach, for example, the first member of α^+ from, for example, the second via a chain of direct calls).

Gaifman seems to think it is straightforward to deal with such cases by adding other gap rules. He discusses adding the *give-up rule*: if there remain unevaluated sentence tokens to which no other rules apply, then all of them are assigned GAP ([4], p. 231, note 5). This rule assigns GAP not only to the members of, for example, α^+ but also to any tokens of sentences that can be used to attribute truth (or other semantic properties) to them. But this seems to go too far. Some people might deliberate on the members of α^+, judge them all to be pathological, and, thus, conclude "None of them is true." Though others might find some fault in the reasoning and object to the conclusion, it seems hardly plausible to hold that the conclusion is pathological; those who hold this view leave no room for even disagreeing with the conclusion. Certainly, it is not a view that can be maintained by those who, like Gaifman, hold that one can sometimes make a correct statement by saying, for example "β_0 is not true." Defenders of Gaifman might attempt to avoid the invidious consequence of the give-up rule by replacing it with, for example, the rule that assigns GAP to all the members of an infinite descending chain of unevaluated sentence tokens. But this rule fails to decide the truth-values of the members of α^+. They can be assigned different values depending on how one applies the rule, because, for example, the sequence obtained from α^+ by removing its first member, α_1^+, is also an infinite descending chain; α_1^+ is assigned F if we apply the rule to this sequence, GAP if we apply it to α.[36]

We can see that Gaifman's difficulty in coping with infinite descending chains stems from a crucial difference between their members and the liar-like cases. In dealing with liar-like cases, there seems to be substantial advantage in assigning semantic values not to sentences but to their tokens. There is a clear disparity, of apparent semantic significance, among tokens of the same liar-like sentences; among tokens of 'β_0 is not true', only β_0 can be seen to be, for example, self-referential in an intuitive sense (or lead to a loop). The disparity allows Gaifman to ascribe pathologicality to some, but not all, of them (for example, β_0); the others can then be taken to make nonpathological assertions about them. In dealing with infinite descending chains, however, one does not gain much with Gaifman's strategy. Turn as you may an infinite descending chain of sentences into the corresponding chain of tokens, you are still left with the same perplexity—bottomless descent that in no way turns around. In particular, the strategy does not help to explain how semantic assertions (for example, 'α_1^+ is true') on members of a pathological descending chain can avoid pathologicality; adding such an assertion on top of the chain, of tokens or types may it consist, yields another abyss.

I think the situation is essentially the same with the contextualists. Those who take semantic attributions to be properly made only of sentences-as-used-in-contexts and thus couple, for example, the Strengthened Liar β with the contexts of its use for semantic evaluation can see a significant disparity among its uses in different contexts. Those who use the sentence to withhold truth to the same sentence as used in a context can be seen to use it, for example, self-referentially only if they do so in the same context. This allows the contextualists to plausibly rule the sentence as used pathologically in the usual contexts while leaving room for nonpathological uses of it. By contrast, one cannot find much disparity among various infinite descending chains by taking contexts into account. Couple the sentences in any infinite descending chain with their contexts (and their names occurring in the subsequent sentences with the same contexts), and you get an infinite descending chain of sentences-cum-contexts—descending, no matter how the contexts are related, in the clear sense that each one of them makes a semantic assertion about the next one. Thus the strategy of contextualizing, too, fails to elucidate a basis for telling infinite descending chains with some healthy members from those with only pathological members.

Contextualists might respond that whether or not sentences-cum-contexts that form an infinite descending chain are all pathological depends on the order among the contexts: If the contexts are all at the same level, all the sentences are used pathologically; otherwise, the sentences used in contexts at higher levels than those of the subsequent sentences are used nonpathologically. But this response, like Burge's account, rests on the assumption that all the contexts of semantic assertions are well-founded; for one cannot plausibly hold that all the sentences in, for example, α (or the English counterpart of δ) are used nonpathologically. This crucial assumption cannot be maintained, as we have seen, given the asymmetry in the linear pattern of semantic attribution in the natural uses of the sentences in, for example, α; there is nothing, it seems, to counteract the support that the asymmetry lends to assigning a descending order to their contexts. Some contextualists might respond to this by denying that the fact that, for example, α_1 is used in context C_1 to make a semantic assertion about α_2 as used in context C_2 gives a prima facie reason for taking C_1 to be at a higher level than C_2. To do so, they might hold that there is no reason at all for taking C_1 to be at a higher level than C_2 unless the assertion of α_1 in C_1 results from substantial deliberation on the use of α_2 in C_2. I think this response comes from tackling an important question that the contextualists have yet to address to articulate their approach: "What in the semantic attributions gives rise to ascent to higher contexts?" But the response does not help to defend the assumption of well-founding of contexts. An immortal, for example, Bob, might resolve to say, for example, "What I will say tomorrow is not true" (and nothing else) each day and carry it out henceforth. Suppose that he says so each day after serious reasoning that leads to the conclusion that what he will say the following day is not true. Then the contexts of his assertions must be taken to be under a descending order even on the present proposal.[37]

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NOTES

1. A sentence of this form is called the *Strengthened Liar*. See also sentence β in Section 2.

2. Moreover, it seems that we can use the same sentence to say, correctly, that it is not true, because it is the first sentence quoted in this paper.

3. See also Burge [2] & [3], and Parsons [7]. Gaifman [4] takes a related approach by attributing truth to sentence tokens instead of sentence types as used in specific contexts. See also Skyrms [9], who attempts to resolve the contradiction by viewing "true" as an intensional predicate.

4. To see how such a chain of sentences might be used, imagine that Bob, an immortal, says "What I will say tomorrow is true", and nothing else, each day.

5. Gaifman [4], who takes a closely related approach, discusses an infinite chain of sentences to argue that certain assumptions of semantic evaluation wrongly makes its first member, a Strengthened Liar, a "black hole", but the chain is not a descending chain. His approach, too, fails to deal with descending chains (see Section 4).

6. Parsons [7] in effect agrees. He would translate the predicate, if predicated of sentences, as "expresses a true proposition," because he attributes truth primarily to propositions; but this complex predicate is context-sensitive, on his view, because "expresses" or "a" is so.

7. On his view, the predicate is sometimes used schematically (see e.g., [2], p. 353). I do not discuss his account of its schematic use, because it is only indirectly relevant to his treatment of pathological sentences. Thus, when I talk of uses of English semantic predicates (or sentences containing them), I mean their indexical uses (or, more precisely, what Burge would count as such uses).

8. The Truth-teller is a sentence that attributes truth to itself.

9. In Burge [1], for example, p. 100, it seems implicit that all the subscripts are finite numerals; but in Burge [2], p. 356, and Burge [3], p. 115, it is made explicit that infinite numerals are allowed.

10. The rootedness predicates indicate relations between sentences and sequences of individuals; the satisfaction predicates, those between sequences and sentences.

11. On Burge's account, both the pathological sentences and their negations are false.

12. Burge [1] presents three different schemes of characterizing it. Presented above is his "Construction C3", the most permissive among them. (It is the only one that he presents in [2].) But my discussion below is neutral with respect to the three schemes.

13. '$\beta \approx_x \alpha$' abbreviates '$(t)(\sim t = x \rightarrow \beta(t) = \alpha(t))$'.

14. But they do not imply the unrestricted schema: $\mathrm{T_i(S)}$ iff p.

15. For $\ulcorner \mathrm{T_i}(\beta_\mathrm{i})\urcorner$ is not rooted$_\mathrm{i}$, but $\ulcorner \mathrm{T_j}(\beta_\mathrm{i})\urcorner$ is rooted$_\mathrm{j}$.

16. For example, Bob might mean to attribute truth to what he will say the following day when he says "What I will say tomorrow is true."

17. By (R1), each sentence $\alpha'_{\mathrm{n+1}}$ is rooted$_{\mathrm{s(n)}}$ under the assignment; so, by (R2) and (R5), α'_n (viz., $\ulcorner \mathrm{T_{s(n)}}(\alpha'_{\mathrm{n+1}})\urcorner$) is also rooted$_{\mathrm{s(n)}}$.

18. And the result of applying *Minimalization* cannot override the result of applying *Justice* and *Verity*.

19. Thus he concludes that they are all unrooted and so, false.

20. This is a minor variation of the example that Parsons uses in his [8], p. 259 f., to argue for the need to allow infinite indices.

21. Burge ([1], p. 106) attempts to justify the other key feature, linearity, of the ordering of the semantic predicates of $\mathcal{L}$, but makes no attempt to justify their well-foundedness. He does not even explicitly mention this feature, which is presupposed in his justification of their comparability (ibid.) and formulation of *Minimalization*, as well as his treatment of infinite descending chains.

22. Thus the resulting sequence of sentences in $\mathcal{L}^+$ is α'.

23. See note 17.

24. δ is the representation in $\mathcal{L}^+$ of the sequence that Yablo [10] shows to be paradoxical.

25. That is, $F^i(y)$ iff there is a natural number $j > 1$ such that $y = \ulcorner(\varphi)(F^j(\varphi) \rightarrow \sim T_{s(j)}(\varphi))\urcorner$.

26. The semantic principle (R1), which implies that β_i is rooted$_j$ simply because $j > i$, can be seen to encapsulate the thought. It implies, for the same reason, that α'_{j+1} is rooted$_{s(j)}$. Given this, one cannot rule that $\ulcorner T_j(\beta_i)\urcorner$ is rooted$_j$ without also ruling that α'_j (viz., $\ulcorner T_{s(j)}(\alpha'_{j+1})\urcorner$) is rooted$_{s(j)}$.

27. The contextualists might argue that we cannot do so because a sentence used to attribute truth in some context to a sentence as used in its context cannot be rooted unless the former context is above the latter. But this view is hard to defend, because the first evaluation 'β is not true' that is made in the natural reasoning about β is to be seen to be correct (and so nonpathological) but made in the original context.

28. Gaifman prefers "pointer" to "sentence token" partly because, for example, there can be a token of a sentence whose negation does not have a token. I ignore this problem, which cannot be avoided by using the unfamiliar word, and state his view using the familiar phrase.

29. See Gaifman [4], p. 228. The negation of a sentence token is said to *directly call* the token; a disjunction, its disjuncts; a sentence token of the form $\ulcorner T(p)\urcorner$, the sentence token that 'p' refers to (he treats quantified sentences as infinite disjunctions or conjunctions). I say "mostly" because (i) a disjunction whose disjunct is assigned T is assigned T no matter what, if any, value the other disjunct is assigned, and (ii) the rules for assigning truth values to *basic* sentence tokens, tokens of predicational sentences with nonsemantic predicates, appeal to other kinds of semantic values assigned to their components.

30. The other is the *simple gap rule*: a sentence token is assigned GAP, if all tokens it calls directly have been assigned values but the sentence token cannot be assigned a value by the other rules. This rule assigns GAP to the negation of a sentence token that is assigned GAP and, for example, a disjunction whose disjuncts are both assigned GAP.

31. A sentence token is *unevaluated* (at a stage of evaluation) if no truth value has been assigned to it.

32. That is, the token of the sentence 'β_0 is true' that is called directly by β_0.

33. For $\{\beta_1, \gamma_1\}$ is not a closed unevaluated loop—β_1 is not reachable from γ_1.

34. Standard rules assign T (or F) to $\ulcorner T(p)\urcorner$ if the referent of 'p' is assigned T (or F).

35. The sequence α^+ is like α except that it consists of sentence tokens (thus, the tokens have 'terms' that refer to the tokens that are the next members of the sequence). Incidentally, Gaifman would hold that α^+ gives a better representation of the sequence of Bob's assertions than α does (see note 4).

36. Tokens of "α_1^+ is true", too, get assigned different values for the same reason.

37. It is irrelevant to this whether Bob's reasoning is impeccable and its conclusion correct.

REFERENCES

[1] Burge, T., "Semantical paradox," *The Journal of Philosophy*, vol. 76 (1979), pp. 169–98; reprinted in Martin [6], pp. 83–114. 1, 2, 2, 2, 4, 4, 4, 4

[2] Burge, T., "The Liar Paradox: Tangles and chains," *Philosophical Studies*, vol. 41 (1982), pp. 367–81. MR 84j:03011 1, 2, 2, 2, 2, 3, 4, 4, 4, 4

[3] Burge, T., "Postscript to 'Semantical Paradox'," pp. 114–17 in *Recent Essays on Truth and the Liar Paradox*, edited by R. L. Martin, Oxford University Press, Oxford, 1984. 2, 3, 4, 4

[4] Gaifman, H., "Pointers to truth," *The Journal of Philosophy*, vol. 89 (1992), pp. 223–61. MR 93h:03002 4, 4, 4, 4, 4, 4

[5] Kripke, S., "Outline of a theory of truth," *The Journal of Philosophy*, vol. 72 (1975), pp. 690–716; reprinted in Martin [6], pp. 53–81. Zbl 0952.03513 4

[6] Martin, R. L., *Recent Essays on Truth and the Liar Paradox*, Oxford University Press, Oxford, 1984. Zbl 0623.03001 1, 5, 7

[7] Parsons, C., "The Liar Paradox," *The Journal of Philosophical Logic*, vol. 3 (1974), pp. 381–412; reprinted in Martin [6], pp. 9–47. Zbl 0296.02001 MR 58:21402 1, 4, 4

[8] Parsons, C., "Postscript to 'the Liar Paradox'," pp. 251–67 in *Mathematics in Philosophy* by C. Parsons, Cornell University Press, Ithaca, 1983. 4

[9] Skyrms, B., "Intensional aspects of semantical self-reference," pp. 119–31 in *Recent Essays on Truth and the Liar Paradox*, edited by R. L. Martin, Oxford University Press, Oxford, 1984. 4

[10] Yablo, S., "Paradox without self-reference," *Analysis*, vol. 53 (1993), pp. 251–2. Zbl 0943.03565 MR 94i:03008 4

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