ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 37, Number 3, 2007

CHARACTERS ON ALGEBRAS OF VECTOR-VALUED CONTINUOUS FUNCTIONS

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Dedicated to the memory of my grandmother Zzi Ballouk

ABSTRACT. Let A be a topological algebra with continuous multiplication, X a completely regular Hausdorff space and $C(X,A)$ the algebra of all A-valued continuous functions on X. We describe the characters on subalgebras of $C(X,A)$ by means of those of A and evaluations at points of the Stone-Čhech compactification of X.

1. Introduction and preliminaries. Let A be a topological algebra with continuous multiplication, X a completely regular Hausdorff space and $C(X,A)$ the algebra of all A-valued continuous functions on X. It is known [**6, 12**] that, if $C(X,A)$ is equipped with the compact open topology, every continuous character on it has the form $f \mapsto \tau(f(x))$, where τ is a character on A and $x \in X$. Moreover, the topological equality $\mathrm{Hom}\,(C(X,A)) = X \times \mathrm{Hom}\,(A)$ holds whenever $\mathrm{Hom}\,(A)$ is locally equicontinuous. Here, Hom stands for all the continuous characters. When A is a metrizable topological algebra, it is shown in [**2**] that every (even not continuous) character of $C(X,A)$ has the form $f \mapsto \tau(f^{\upsilon}(x))$, x running over the real compactification υX of X and $f^{\upsilon} : \upsilon X \to A$ being the unique continuous extension of f. However, when dealing with a subalgebra E of $C(X,A)$, the expression above fails to describe all the characters on E. Actually, Govaerts gave in [**6**] an example of a $\mathbf{C}^*$-algebra A, a completely regular Hausdorff space X and a character χ on the Banach algebra $C_b(X,A) \subset C(X,A)$ of all bounded functions, equipped with the uniform norm, such that no character τ on A and no $x \in \beta X$ satisfy $\chi(f) = (\tau \circ f)^{\beta}(x)$ for every $f \in C_b(X,A)$. Here, βX designates the Stone-Čhech compactification of X [**5**] and, for a bounded function g on X, g^{β} the Stone extension of g. In the scalar case, using a property introduced in [**1**], we could determine all the characters of any weighted algebra which is a $C_b(X)$-module [**10**]. Here, in Section 2, we make use of a vector valued version

Received by the editors on July 7, 2003, and in revised form on October 1, 2004.

of that property in order to give the expression of characters on (some subalgebras of) $C(X,A)$. Section 3 is devoted to the local equicontinuity of $\mathrm{Hom}\,(A)$ for a topological algebra A. Although this condition occurs in most of the papers on $\mathrm{Hom}\,(C(X,A))$, only Q-algebras were considered as examples of algebras enjoying it [**3, 4, 7, 8, 12**]. Here, we characterize the weighted algebras A with $\mathrm{Hom}\,(A)$ locally equicontinuous yielding examples of (even) complete locally m-convex algebras [**9**] with or without this property.

Henceforth, we will call a topological algebra any associative algebra A over the complex field $\mathbf{C}$ equipped with a linear Hausdorff topology such that the multiplication of A is separately continuous. By a character on A, we mean an algebra morphism from A onto $\mathbf{C}$. The set of all characters on A will be denoted by $\mathrm{hom}\,(A)$ and, whenever A is a topological algebra, the continuous ones will be designated by $\mathrm{Hom}\,(A)$. We will consider

$$r(A) := \{a \in A : \tau(a) = 0,\ \tau \in \mathrm{hom}\,(A)\}.$$

A linear involution on A is any mapping $^* : x \mapsto x^*$ defined from A onto itself such that $(x^*)^* = x$, $(x+y)^* = x^* + y^*$, $(\lambda x)^* = \overline{\lambda} x^*$. It is an algebra involution if, in addition, $(xy)^* = y^* x^*$. A character τ on A is said to be Hermitian if $\tau(a^*) = \overline{\tau(a)}$ for all $a \in A$ and A is called Hermitian if all its characters are.

Now, let X be a completely regular Hausdorff space and $C(X,A)$ the space of all A-valued continuous functions on X. When $A = \mathbf{C}$, we will write $C(X)$ instead of $C(X,\mathbf{C})$. If * is a linear involution on A, for every $f \in C(X,A)$ and $x \in X$, we will set $f^*(x) = f(x)^*$. If * is continuous on A, then $f \mapsto f^*$ defines an involution on $C(X,A)$. We will say that an algebra $E \subset C(X,A)$ is self-adjoint if f^* belongs to E for every $f \in E$.

For every $x \in \beta X$, let δ_x denote the evaluation $f \mapsto f^\beta(x)$ at x, f being a continuous function on X. For any $f \in C(X,A)$ and $g \in C(X)$, set

$$\upsilon_f X = \{x \in \beta X; \exists a \in A : f(x_i) \longrightarrow a, \quad \text{for all} \quad (x_i)_i \subset X \quad \text{with} \quad x_i \to x \quad \text{in} \quad \beta X\},$$

and

$$\beta_g X = \{x \in \beta X; \exists \alpha \in \mathbf{C} : g(x_i) \longrightarrow \alpha, \quad \text{for all} \quad (x_i)_i \subset X \quad \text{such that} \quad x_i \to x \quad \text{in} \quad \beta X\}.$$

The vector a and the scalar α will be denoted respectively by $f^{\upsilon}(x)$ and $g^{\beta}(x)$.

2. Characters on subalgebras of $C(X,A)$**.** In the sequel, E will denote an algebra contained in $C(X,A)$. We will assume in addition that E is either an A-bimodule or a $C_b(X)$-module. For every $f \in E$ and $a \in A$, fa and af will denote respectively the functions $x \mapsto f(x)a$ and $x \mapsto af(x)$. These are elements of E. We do not assume any essentiality condition on E. Its cozero $\operatorname{coz}(E)$ may be arbitrary.

For such an algebra E, let $\operatorname{Hom}_E(A)$ be the set of all characters $\tau \in \hom(A)$ such that $\tau \circ f \in C(X)$ for all $f \in E$ and set

$$\upsilon_E X := \Big\{ x \in \bigcap_{f \in E} \upsilon_f X ; \exists f_x \in E : f_x^{\upsilon}(x) \notin r(A) \Big\},$$

$$\beta_E X := \Big\{ x \in \bigcap \big\{ \beta_{(\tau\circ f)} X, f \in E,\ \tau \in \operatorname{Hom}_E(A) \big\} ; \\ \exists f_x \in E,\ \tau_x \in \operatorname{Hom}_E(A) : (\tau_x \circ f_x)^{\beta}(x) \neq 0 \Big\}.$$

The following result shows that $\hom(E)$ is large enough whenever $\hom(A) \neq \varnothing$.

Lemma 2.1. *The mappings*

$$\begin{aligned} G_1 : \upsilon_E X \times \hom(A) &\longrightarrow \hom(E) \\ (x,\tau) &\longmapsto \tau \circ \delta_x \end{aligned}$$

and

$$\begin{aligned} G_2 : \beta_E X \times \operatorname{Hom}_E(A) &\longrightarrow \hom(E) \\ (x,\tau) &\longmapsto \delta_x((\tau \circ .)^{\beta}) \end{aligned}$$

are one to one and relatively open. Moreover, if $\hom(A)$ *is locally equicontinuous, then* G_1 *is also continuous.*

Proof. Let us denote by G indifferently G_1 or G_2. It is easy to see that, in both cases, G is well defined. Now, if $G((x,\tau)) = G((x',\tau'))$ and

$x \neq x'$, since E is a $C_b(X)$-module, we can choose $g \in C_b(X) = C(\beta X)$ such that $g(x) = 1$ and $g(x') = 0$. Then, for every $f \in E$ with $G((x,\tau))(f) = 1$, we have $1 = G((x,\tau))(gf) = g(x)G((x,\tau))(f) = G((x',\tau'))(gf) = 0$ which is absurd. In the same way, we show that $\tau = \tau'$. For the relative openness of G, consider a net $G((x_i,\tau_i))$ converging in $\hom(E)$ to $G((x,\tau))$ and arbitrary $g \in C_b(X)$ and $a \in A$. Choose $f \in E$ such that $G((\tau,x))(f) = 1$. Then

$$g(x_i) = \frac{G((x_i,\tau_i))(gf)}{G((x_i,\tau_i))(f)} \quad \text{tends to} \quad \frac{G((x,\tau))(gf)}{G(x,\tau))(f)} = g(x).$$

Since X is completely regular, $x_i \to x$. Similarly,

$$\tau_i(a) = \frac{G((x_i,\tau_i))(fa)}{G(x_i,\tau_i))(f)} \quad \text{tends to} \quad \frac{G((x,\tau))(fa)}{G((x,\tau))(f)} = \tau(a).$$

Whereby $\tau_i \to \tau$ since a was arbitrary.

Assume now that $\hom(A)$ is locally equicontinuous and that $\tau_i \to \tau$ in $\hom(A)$ and $x_i \to x$ in $\upsilon_E X$. Choose an equicontinuous neighborhood V of τ. There is some i_0 such that $\tau_i \in V$ for all $i \geq i_0$. Moreover, for an arbitrary $f \in E$ and $i \geq i_0$, we have

$$|\tau_i(f(x_i)) - \tau(f(x))| \leq |\tau_i(f(x_i)) - \tau_i(f(x))| + |\tau_i(f(x)) - \tau(f(x))|$$

Since $f(x_i) \to f(x)$, the first term of the right-hand side converges to zero. As to the second, it also converges to zero since τ_i converge to τ and $f(x) \in A$. $\square$

Remark 2.2. 1. It is clear that, whenever A is algebraically strongly semi-simple, i.e., $r(A) = \{0\}$, $\upsilon_E X$ contains $\operatorname{coz}(E)$. This inclusion need not hold if $r(A) \neq \{0\}$. For such an example, let A be a non semi-simple commutative Banach algebra and X its closed unit ball. Take E the algebra consisting of all the finite combinations $\sum g_i p_i a_i$, where $g_i \in C_b(X)$, p_i a polynomial in $x \in X$ and $a_i \in A$. This is a subalgebra of $C(X,A)$ which is also an A-bimodule and a $C_b(X)$-module. If $0 \neq x \in r(A) \cap X$, $\tau \circ f(x) = 0$ for all $f \in E$ and all $\tau \in \hom(E)$. However, $x \in \operatorname{coz}(E)$.

2. We were not able to show the continuity of G_2 in Lemma 2.1 whenever $\operatorname{Hom}_E(A)$ is locally equicontinuous.

In [**1, 10**] the following property (P) was used to describe characters of some algebras of scalar-valued functions. In the following, we will extend this property to the vector-valued case and make use of it to establish some new results. A character χ on an algebra F of scalar-valued continuous functions on X is said to satisfy property (P) if, for every $f \in F$, $\chi(f) \in \overline{f(X)}$. There are different ways to extend this property to the vector-valued case as the following definition shows.

Definition 2.3. A character χ on E will be said to satisfy the spectral property (sp), respectively the continuous spectral property (csp), if there is some $\tau \in \hom(A)$, respectively $\tau \in \mathrm{Hom}_E(A)$ such that:

$$\chi(f) \in \overline{\tau \circ f(X)}, \quad \text{for all} \quad f \notin \ker \chi.$$

It is said to satisfy the weak spectral property (wsp), respectively the weak continuous spectral property (wcsp), if

$$\chi(f) \in \bigcup \left\{ \overline{\tau \circ f(X)}, \tau \in \hom(A) \right\}, \quad \text{for all} \quad f \notin \ker \chi,$$

respectively

$$\chi(f) \in \bigcup \left\{ \overline{\tau \circ f(X)}, \tau \in \mathrm{Hom}_E(A) \right\}, \quad \text{for all} \quad f \notin \ker \chi.$$

Recall that A is said to satisfy the Wiener property if $a \in A$ is quasi-invertible if and only if $\tau(a) \neq 1$ for all $\tau \in \hom(A)$. If B is another algebra containing A as a subalgebra, we will say that A is quasi-inverse closed in B if an element of A is quasi-invertible in A provided it is in B.

Lemma 2.4. *Let χ be a character on E.*

1. *If $EA \not\subset \ker \chi$, then there is exactly one character τ on A such that*

$$\chi(f) \in \overline{\tau \circ f(X)}, \quad \textit{for all} \quad f \notin \ker \chi.$$

2. *If A has continuous quasi-inverse and satisfies the Wiener property and E is quasi-inverse closed in $C(X, A)$, then every character on E satisfies* (wsp).

Proof. 1. For $a \in A$, put

$$\tau_\chi(a) := \frac{\chi(fa)}{\chi(f)}, \quad f \notin \ker \chi.$$

Then $\tau_\chi(a)$ does not depend on f and τ_χ satisfies

$$\chi(fa) = \chi(af) = \chi(f)\,\tau_\chi(a), \quad f \in E, \quad a \in A.$$

Since $EA \not\subset \ker \chi$, τ_χ does not vanish identically on A and then τ_χ belongs to $\hom(A)$. Now, assume that there is some $\tau \in \hom(A)$ such that

$$\chi(f) \in \overline{\tau \circ f(X)}, \quad \text{for all} \quad f \notin \ker \chi.$$

If $\tau \neq \tau_\chi$, choose $a \in A$ and $f \in E$ such that $\tau_\chi(a) = 1$, $\tau(a) = 0$ and $\chi(f) = 1$. Then

$$1 = \chi(fa) \in \overline{\tau(fa)(X)} = \{0\}.$$

This is absurd.

2. Suppose that E is quasi-inverse closed in $C(X, A)$ and that A enjoys the Wiener property and has continuous quasi-inverse. If $\chi \in \hom(E)$ fails to satisfy (wsp), then there exists $f \in E$ such that $0 \neq \chi(f) \notin \overline{\tau \circ f(X)}$ for every $\tau \in \hom(A)$. Therefore, for each $\tau \in \hom(A)$, there is an $\varepsilon_\tau > 0$ such that

$$|\tau \circ f(x) - \chi(f)| > \varepsilon_\tau, \quad \text{for all } x \in X.$$

This gives $\tau((f/\chi(f))(x)) \neq 1$ for every $x \in X$. By our assumptions, $f/\chi(f)$ is quasi-invertible in E. But this contradicts the fact that $\chi(f)$ belongs to the spectrum of f. □

Theorem 2.5. *Assume that A is Hermitian and that E is self-adjoint. If $\chi \in \hom(E)$ satisfies* (csp)*, then there is some $z_0 \in \beta(X)$ and $\tau \in \mathrm{Hom}_E(A)$ so that $\chi(f) = (\tau \circ f)^\beta(z_0)$ for every $f \in E$.*

Proof. Notice first that, if χ satisfies (sp), then it is Hermitian. Indeed, if $g \in E$, then both its real and imaginary parts also belong E, where

$$\mathrm{Re} := \frac{g + g^*}{2} \quad \text{and} \quad \mathrm{Im}\, g := \frac{g - g^*}{2i}.$$

Since these parts are Hermitian, by (sp), $\chi(\operatorname{Re} g)$ and $\chi(\operatorname{Im} g)$ are real numbers and the linearity of χ leads to the conclusion. Next, given $\chi \in \hom(E)$ satisfying (csp). Then there is a $\tau \in \operatorname{Hom}_E(A)$ such that $\chi(f) \in \overline{\tau \circ f(X)}$, for every $f \notin \ker\chi$. For such an f and arbitrary $\varepsilon > 0$, set

$$F(f,\varepsilon) := \{x \in X : |\tau(f(x)) - \chi(f)| \le \varepsilon\}$$

and

$$G(f,\varepsilon) := \{x \in \beta(X) : |(\tau \circ f)^{\beta}(x) - \chi(f)| \le \varepsilon\}.$$

Then $F(f,\varepsilon) \subset G(f,\varepsilon)$ and, by (csp), $F(f,\varepsilon) \neq \varnothing$. By compactness of $\beta(X)$, the set $I_f := \cap_{\varepsilon>0} G(f,\varepsilon)$ is not empty. Obviously, for every $x \in I_f$, one has $\chi(f) = (\tau \circ f)^{\beta}(x)$. Furthermore, if $f_1, f_2, \dots, f_n$ are elements of $E \setminus \ker\chi$ and $\varepsilon > 0$, since E is self-adjoint,

$$h := \sum_{i=1}^{n} \left(f_i f_i^* - \overline{\chi(f_i)} f_i - \chi(f_i) f_i^* \right)$$

belongs to $E \setminus \ker\chi$. Moreover, for $x \in F(h,\varepsilon^2)$, we have

$$\sum_{i=1}^{n} |\tau \circ f_i(x) - \chi(f_i)|^2 \le \varepsilon^2,$$

whence

$$F(h,\varepsilon^2) \subset \bigcap_{i=1}^{n} G(f_i,\varepsilon).$$

This shows that the family $\{I_f,\ f \in E\}$ satisfies the finite intersection property. Again, by compactness of $\beta(X)$,

$$I = \bigcap \{I_f, f \in E \setminus \ker\chi\} \neq \varnothing.$$

Finally, $\chi(f) = (\tau \circ f)^{\beta}(x)$ for any $x \in I$ and $f \notin \ker\chi$. Since $f \mapsto (\tau \circ f)^{\beta}(x)$ belongs to $\hom(E)$, it equals χ. □

In order to obtain applications of Theorem 2.5, we give instances in which every character $\chi \in \hom(E)$ satisfying (wsp) must enjoy (sp) too.

Proposition 2.6. *In the following instances* (wsp) *implies* (sp) *for* $\chi \in \hom(E)$: $EA \not\subset \ker \chi$ *and the characteristic function of any singleton of* $\hom(A)$ *is the Gelfand transform of some* $a \in A$.

Proof. Since $EA \not\subset \ker \chi$, $\tau_\chi \in \hom(A)$ and the characteristic function of $\{\tau_\chi\}$ is the Gelfand transform of some $b \in A$. Take $f \notin \ker \chi$ and $\tau \in \hom(A)$ such that $\chi(fb) \in \overline{\tau \circ (fb)(X)}$. If $\tau \neq \tau_\chi$, then

$$\chi(f) = \chi(fb) \in \tau(b)\overline{\tau \circ f(X)} = \{0\}.$$

This is impossible.

Remark 2.7. 1. Most of the sequence algebras satisfy $\mathrm{Hom}(A) = \hom(A)$ and $\mathrm{Hom}(A)$ satisfies the second condition above. For instance, the algebra $\mathbf{C}^{(\mathbf{N})}$ of all sequences with finite support equipped with its strongest locally convex topology, the algebra $\mathbf{C}^{\mathbf{N}}$ of all sequences with its natural Fréchet algebra topology, c_0 and all the ℓ^p's, $0 < p < \infty$. Now, applications are obtained by combining Theorem 2.5, Proposition 2.6 and the foregoing remark.

2. We think that Theorem 2.5 should hold without any involution on A. However, we do not know whether or not a character on $C(X, A)$ must satisfy (wsp).

3. Local equicontinuity of $\mathrm{Hom}(A)$**.** The local equicontinuity of $\mathrm{Hom}(A)$ occurs naturally when dealing with the ideals of $C(X, A)$ [**7**] or with the topological equality $X \times \mathrm{Hom}(A) = \mathrm{Hom}(C(X, A))$ [**3, 4, 6, 8, 12**]. In the following, we will study this property in the Nachbin algebras. Before that, it is worth pointing out that $\mathrm{Hom}(A)$ is (even) equicontinuous whenever A is a P-algebra, i.e., the set $\{a \in A : a^n \to 0\}$ is a zero neighborhood. Now, recall that a Nachbin family on X is any collection V of nonnegative upper semi-continuous functions on X such that for every $v_1, v_2 \in V$, $x \in X$ and $\lambda > 0$, there exists $v \in V$ with $\lambda v_i \leq v$, $i = 1, 2$ and $v(x) > 0$. With each Nachbin family V on X is associated the so-called weighted locally convex space

$$CV(X) := \left\{ f \in C(X) : |f|_v := \sup_{x \in X} v(x)\,|f(x)| < +\infty,\ v \in V \right\}$$

with its natural topology given by the semi-norms $(|\ |_v)_{v \in V}$. In general, this space need not be an algebra, but it always contains many interesting ones, cf. [**11**]. As shown there, every locally convex algebra $E \subset CV(X)$, for the induced topology, which is a $C_b(X)$-module is contained in

$$C_\ell V(X) := \{f \in CV(X) : |f|\, V \le V\}\,.$$

Moreover, hom$\,(E)$ as well as Hom$\,(E)$ are homeomorphic to subspaces of βX. This means that every character on E is the evaluation at a point of βX. Precisely, we have the identifications

$$\hom(E) = \left\{x \in \beta X : \forall\, f \in E, f^\beta(x) \ne \infty \text{ and } \exists g \in E,\ g^\beta(x) \ne 0\right\}.$$

and

$$\mathrm{Hom}\,(E) = \{x \in \hom(E) : \tilde{v}(x) \ne 0 \text{ for some } v \in V\}$$

where

$$\tilde{v}(x) := \frac{1}{\sup\{|f(x)|, f \in E \text{ and } |f|_v \le 1\}}, \quad \text{with} \quad \frac{1}{\infty} = 0.$$

For a subset Y of hom$\,(E)$, set $\Delta_Y := \{\delta_y, y \in Y\}$. We then get

Proposition 3.1. *Let $E \subset CV(X)$ be a locally convex algebra which is a $C_b(X)$-module and $Y \subset$ Hom$\,(E)$. Then Δ_Y is equicontinuous if and only if there is some $v \in V$ such that $\nu = \inf\{\tilde{v}(y),\ y \in Y\} > 0$.*

Proof. If Δ_Y is equicontinuous, there is some $v \in V$ such that

$$|\delta_y(f)| := \left|f^\beta(y)\right| \le |f|_v\,, \quad y \in Y, \quad f \in E. \tag{1}$$

If $\nu = \inf\{\tilde{v}(y),\ y \in Y\} = 0$, then the open subset

$$U_n := \left\{x \in \mathrm{Hom}\,(E) : \tilde{v}(x) < \frac{1}{n}\right\}$$

of Hom$\,(E)$ intersects Y for any integer $n \ge 1$. Choose $y_n \in U_n \cap Y$ and an open subset Ω_n of βX with $\Omega_n \cap \mathrm{Hom}\,(E) = U_n$. Take then $f_n \in C(\beta X) = C_b(X)$ with $f_n(y_n) = n$, $0 \le f_n \le n$ and $f_n = 0$ out of Ω_n. Since E is a $C_b(X)$-module and $y_n \in \mathrm{Hom}\,(E)$, we may assume that $f_n \in E$. We then get by (1), $n = |f_n(y_n)| \le |f_n|_v \le 1$

for every $n \geq 1$. This is impossible. The converse is easy since $|f|_v = \sup\{\tilde{v}(x)|f^\beta(x)|,\ x \in \mathrm{Hom}\,(E)\}$ and

$$\left|f^\beta(x)\right| \leq \frac{1}{\nu}\ |f|_v\,, \quad f \in E. \qquad \square$$

As a consequence of Proposition 3.1, we get

Corollary 3.2. *Let $E \subset CV(X)$ be a locally convex algebra which is a $C_b(X)$-module. Then* $\mathrm{Hom}\,(E)$ *is locally equicontinuous if and only if every $x \in \mathrm{Hom}\,(E)$ is contained in an open subset U of* $\mathrm{Hom}\,(E)$ *such that* $\inf\{\tilde{v}(x),\ x \in U\} > 0$ *for some $v \in V$.*

If $E = C(X)$ is endowed with the topology of uniform convergence on a family P of bounding sets as in [**13**], then

$$\mathrm{Hom}\,(E) = Y_P := \bigcup_{B \in P} \overline{B}^{\upsilon X}.$$

Hence $\mathrm{Hom}\,(E)$ is locally equicontinuous if and only if every $x \in Y_P$ is an interior point of $\overline{B}^{\upsilon X}$ for some $B \in P$. In particular, if $C(X)$ is equipped with the compact open, respectively simple, topology, then $\mathrm{Hom}\,(C(X))$ is locally equicontinuous if and only if X is locally compact, respectively X is discrete. This shows that $\mathrm{Hom}\,(C_c(\mathbf{R}))$ is locally equicontinuous although $C_c(\mathbf{R})$ is not a Q-algebra. On the other hand, if X is a nonlocally compact $k_{\mathbf{R}}$-space, then $E := C_c(X)$ is a complete locally m-convex algebra and $\mathrm{Hom}\,(E)$ is not locally equicontinuous. Here the subscript c stands for the compact open topology.

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