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STRONGLY EXTREME POINTS IN KÖTHE-BOCHNER SPACES

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ABSTRACT. The Kadec-Klee property with respect to a measure is discussed. A characterization of strongly extreme points of the unit sphere in certain Köthe-Bochner spaces is given.

1. Introduction. Let (Ω, Σ, μ) denote a measure space with σ-finite and complete measure μ and $L^0 = L^0(\Omega)$ denote the space of all (equivalence classes of) Σ-measurable real-valued functions, equipped with the topology of convergence in measure on μ-finite sets. In what follows, if $x, y \in L^0$, then $x \le y$ means $x(t) \le y(t)$ μ-almost everywhere in Ω.

For any Banach space X we denote by S_X the unit sphere of X.

A Banach subspace E of L^0 is said to be a *Köthe function* space (over (Ω, Σ, μ)) if

(i) $|x| \le |y|$, $x \in L^0$, $y \in E$ imply $x \in E$ and $||x|| \le ||y||$,

(ii) $\operatorname{supp} E := \cup\{\operatorname{supp} x : x \in E\} = \Omega$, where $\operatorname{supp} x = \{t \in \Omega : x(t) \neq 0\}$.

A Köthe function space E is said to be *order continuous* (respectively, *monotone complete*) provided $x_n \downarrow 0$ implies $||x_n|| \to 0$ (respectively $0 \le x_n \uparrow x$, $x \in E$ imply $||x_n|| \to ||x||$).

Let E be a Köthe function space on (Ω, Σ, μ), X a Banach space. By $E(X)$ we denote the Banach space of all (equivalence classes of) strongly measurable functions $f : \Omega \to X$ such that $\bar{f} = ||f(\cdot)||_X \in E$ equipped with the norm $||f|| = ||\bar{f}||_E$.

Let E be a Köthe function space over (Ω, Σ, μ). E is said to have the (*positive*) *Kadec-Klee property* with respect to the measure μ (simply property (H_μ^+), respectively, (H_μ)), whenever $(x_n \overset{\mu}{\to} x, x_n, x \in E^+)$ $x_n \overset{\mu}{\to} x$ and $||x_n|| \to ||x||$ imply $x_n \to x$ strongly. Here

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$x_n \overset{\mu}{\to} x$ means that $x_n \to x$ in L^0 and if A is a subset of L^0, then $A^+ = \{x \in A : x \geq 0\}$.

Note that E has the (H_μ) (respectively, (H_μ^+)) property if and only if norm and measure convergence coincide on the unit sphere S_E of E (respectively on S_E^+).

In the above definitions measure convergence may be replaced by μ-almost everywhere convergence.

A Banach space X is said to be *locally uniformly rotund* if $||x_n|| \to ||x||$ and $||x_n + x|| \to 2||x||$ imply that $x_n \to x$ strongly.

We say that $\varphi : \mathbf{R} \to \mathbf{R}_+ = [0, \infty)$ is an *Orlicz function* if φ is convex and even, $\varphi(0) = 0$ and $\varphi(t) \to \infty$ as $t \to \infty$. We say that an Orlicz function satisfies the Δ_2-condition for all $t \in \mathbf{R}$ (at infinity) [at zero] if there are positive constants K and t_0 such that $\varphi(t_0) > 0$ and the inequality $\varphi(2t) \leq K\varphi(t)$ is satisfied for all $t \in \mathbf{R}$ (for $t \in \mathbf{R}$ with $|t| \geq t_0$) [for $t \in \mathbf{R}$ with $|t| \leq t_0$].

For any Orlicz function φ the statement "φ satisfies the suitable Δ_2-condition" means that:

φ satisfies the Δ_2-condition for all t if μ is atomless and infinite.

φ satisfies the Δ_2-condition at infinity if μ is atomless and finite.

φ satisfies the Δ_2-condition at zero if μ is counting measure.

Let E be a Köthe function space, and let φ be an Orlicz function. The functional

$$\rho(x) = \begin{cases} ||\varphi(x)||_E & \text{if } \varphi(x) \in E, \\ \infty & \text{if } \varphi(x) \notin E \end{cases}$$

is a convex modular, i.e., $\rho(0) = 0$ and $x = 0$ whenever $\rho(\alpha x) = 0$ for any $\alpha > 0$, $\rho(x) = \rho(-x)$, $\rho(\alpha x + \beta y) \leq \alpha\rho(x) + \beta\rho(y)$ for any x, $y \in L^0$ and $\alpha, \beta \geq 0$ such that $\alpha + \beta = 1$.

Let E_φ be the space generated by the modular ρ, i.e.,

$$E_\varphi = \{x \in L^0 : \rho(\lambda x) < \infty \text{ for some } \lambda > 0\}.$$

As a modular space E_φ is equipped with the so-called *Luxemburg norm*

$$||x||_\varphi = \inf\{\lambda > 0 : \rho(x/\lambda) \leq 1\}$$

under which it is a Köthe function space.

For the theory of modular spaces, we refer to [**12**]. It is clear that for $E = L_1$, E_φ becomes an ordinary Orlicz space L^φ (cf. [**10** and **12**]).

Note that E_φ is a special case of the Calderón-Lozanovskii space (see [**11, 1**]).

Suppose f belongs to L^0. The *nonincreasing rearrangement* of f is the function f^* defined on $[0,\infty)$ by

$$f^*(t) = \inf\{\lambda > 0 : \mu_f(\lambda) \le t\}$$

(by the convention $\inf \varnothing = \infty$), where μ_f is the *distribution function* of f defined by

$$\mu_f(t) = \mu(\{\omega \in \Omega : |f(\omega)| > t\}), \qquad t \ge 0.$$

By $w : [0,\gamma) \to \mathbf{R}_+$, $\gamma \le \infty$, denote a nonincreasing locally integrable function with respect to the Lebesgue measure m, called a *weight function.*

Recall that the *Lorentz space* Λ_w is defined as follows:

$$\Lambda_w = \{f \in L^0 : ||f||_{\Lambda_w} = \int_0^\gamma f^*(t)w(t)\,dt < \infty\}, \qquad \gamma = \mu(\Omega).$$

Now, if for a given Orlicz function φ, we take $E = \Lambda_w$, then the space E_φ, denoted by $\Lambda_{\varphi,w}$, is called the *Orlicz-Lorentz space* (cf. [**6**]).

An easy proof of the following lemma, useful in the sequel, can be found in [**8**, Lemma 2, p. 141].

Lemma 1. *Let E be a Köthe function space. If $x_n \to x$ in E, then there exist $y \in E^+$, $(x_{n_k}) \subset (x_n)$ and $(\varepsilon_{n_k}) \subset \mathbf{R}_+$ with $\varepsilon_{n_k} \downarrow 0$ such that $|x_{n_k} - x| \le \varepsilon_{n_k} y$.*

It is proved in [**13**] that, for any p in $(1,\infty)$ and any separable Banach space X, a point f of the unit sphere of the Lebesgue-Bochner space $L^p(\mu, X)$ is strongly extreme if and only if the values $f(s)/||f(s)||_X$ are strongly extreme points of the unit ball of X for μ-almost everywhere $s \in \operatorname{supp} f$.

Recall that an element x of the unit sphere of a Banach space X is called *strongly extreme* if any sequence $(x_n) \subset X$ such that $||x_n + x|| \to 1$ and $||x_n - x|| \to 1$ converges to zero.

In the proof of this theorem the facts that for any p in $(1, \infty)$, the Lebesgue space $L^p(\mu)$ is locally uniformly rotund (in fact, it is uniformly rotund) and that, for any Banach space X, $L^p(\mu, X)$ has the Kadec-Klee property with respect to the measure μ were applied.

In the present paper, it is proved that Smith's characterization of strongly extreme points of the unit sphere of $L^p(\mu, X)$ remains valid if we replace the Lebesgue space $L^p(\mu)$, $1 < p < \infty$, by any Köthe function space E which is locally uniformly rotund. In Proposition 1 the (H_μ) property is discussed. Moreover, it is proved (cf. Lemma 2) that the spaces E_φ, where E is a monotone complete Köthe function space with the (H_μ^+) property, have the (H_μ) property if the Orlicz function φ satisfies the suitable Δ_2-condition. It is proved also (cf. Theorem 1 and Remark 2) that in the case of an atomless finite measure μ the Orlicz-Lorentz space $\Lambda_{\varphi,w}$ has the (H_μ) property if and only if the Orlicz function φ satisfies the suitable Δ_2-condition.

2. Results. We start with the following

Proposition 1. *Let E be an order continuous Köthe function space. The following statements are equivalent*:

(a) *E has the (H_μ) property.*

(b) *E has the (H_μ^+) property.*

(c) *For any (f_n) and f in $E(X)$, if $f_n \to f$ μ-almost everywhere and $||f_n|| \to ||f||$, then $f_n \to f$ strongly for any Banach space X.*

Proof. The implications (a) $\Rightarrow$ (b) and (c) $\Rightarrow$ (a) are obvious. In order to finish the proof, it suffices to show (b) $\Rightarrow$ (c). Suppose (b) holds. Let X be any Banach space, and let f_n, $f \in E(X)$ satisfy $f_n \to f$ μ-almost everywhere and $||f_n|| \to ||f||$. Clearly,

$$\bar{f}_n \xrightarrow{\mu} \bar{f} \quad \text{and} \quad ||\bar{f}_n||_E \to ||\bar{f}||_E.$$

Hence, $\bar{f}_n \to \bar{f}$ strongly in E, by (b). Thus, passing to a subsequence

(f_{n_k}) and applying Lemma 1, we have

$$(*) \qquad |\bar{f}_{n_k}| = \bar{f}_{n_k} \le x$$

for some $x \in E^+$. Let

$$g_m(\cdot) = ||f_{n_m}(\cdot) - f(\cdot)||_X$$

and

$$x_k(\cdot) = \sup\{g_m(\cdot) : m \ge k\}.$$

The sequence (x_k) is nonincreasing, $x_k \to 0$ μ-almost everywhere (because $f_n \to f$ μ-almost everywhere) and, by virtue of $(*)$, we have

$$0 \le x_k \le x + \bar{f} \in E^+.$$

Thus $x_k \in E$ and, in view of the order continuity of E, $x_k \to 0$ strongly in E. Since $0 \le g_k \le x_k$, we have $g_k \to 0$ in E. This implies, of course, that $f_n \to f$ strongly in $E(X)$. The proof is finished. □

It is well known that the space L^1 has the (H_μ) property. This fact will be used to show that some class of Köthe function spaces have the (H_μ) property too.

Lemma 2. *Let E be a monotone complete Köthe function space with the (H_μ^+) property such that $L^\infty \subset E$ in the case of an atomless finite measure, and $E \subset l^\infty$ in the case of counting measure. Then E_φ has the (H_μ) property whenever φ satisfies the suitable Δ_2-condition.*

Proof. First observe that, if E is monotone complete with the (H_μ^+) property, then E is order continuous. In fact, if $x_n \downarrow 0$, $x_n \in E$, then $x_1 - x_n \uparrow x_1$ which yields $||x_1 - x_n|| \to ||x_1||$. Thus $||x_n|| \to 0$ by the (H_μ^+) property.

Suppose that φ satisfies the suitable Δ_2-condition. Then $x \in E_\varphi$ if and only if $\rho(x) < \infty$. This easily implies that E_φ is order continuous. Therefore, in view of Proposition 1, in order to prove that E_φ has the (H_μ) property, it suffices to show that it has the (H_μ^+) property.

We observe now that $||x||_\varphi = 1$ implies $\rho(x) = 1$. Let $||x||_\varphi = 1$. Take any sequence (ε_n), $\varepsilon_n \downarrow 0$. Then it follows that $||y_n||_E \le 1$, where $y_n = \varphi(x/(1+\varepsilon_n))$ for $n \in \mathbf{N}$.

Since $0 \le y_n \uparrow y = \varphi(x)$ and $y \in E_\varphi$, we have

$$\rho(x) = ||y||_E = \lim_{n\to\infty} ||y_n||_E \le 1.$$

Assume that

$$(+) \qquad \rho(x) < 1.$$

Since φ satisfies the suitable Δ_2-condition, the function

$$f(t) = \rho(tx), \qquad t > 0,$$

is an Orlicz function. Condition (+) means that $f(1) < 1$. Thus, by continuity of f, it follows that there exists $\lambda > 1$ such that $f(\lambda) = \rho(\lambda x) \le 1$. This yields $||x||_\varphi \le 1/\lambda < 1$, a contradiction.

In order to finish the proof, suppose that E_φ does not have the (H_μ^+) property. Thus, passing to a subsequence, if necessary, we can assume that, for some x_n, $x \ge 0$ in E_φ, we have

$$(**) \qquad ||x_n||_\varphi = ||x||_\varphi = 1, \qquad x_n \xrightarrow{\mu} x \quad \text{and} \quad ||x_n - x||_\varphi > \varepsilon$$

for some $\varepsilon > 0$ and any $n \in \mathbf{N}$. As we have just proved, from $(**)$ it follows that $||y_n||_E = ||y||_E = 1$, where $y_n = \varphi(x_n)$ and $y = \varphi(x)$. Since $y_n \xrightarrow{\mu} y$ in E, by the (H_μ^+) property and Lemma 1, we obtain that

$$0 \le y_{n_k} \le w$$

for some subsequence (y_{n_k}) and $w \in E^+$. Notice that if an Orlicz function φ satisfies the Δ_2-condition at zero, then it satisfies the Δ_2-condition on the interval $[0, t_0]$ for any positive constant t_0.

By virtue of the assumption $E \subset l^\infty$, in the case of counting measure, putting $t_0 = \max\{\sup_n \varphi^{-1}(w(n)), \sup_n \varphi^{-1}(x(n))\}$, where $w(n)$ and $x(n)$ denote, respectively, the n-th coordinate of w and x, we get for some constant $K > 0$ depending on t_0

$$\begin{aligned}\varphi(x_{n_k} - x) &\le K\varphi((x_{n_k} - x)/2) \le K(\varphi(x_{n_k}) + \varphi(x))/2 \\ &\le K(w + y)/2 \in E^+.\end{aligned}$$

This inequality holds also in the case of an atomless infinite measure. In the case of an atomless finite measure, in view of $L^\infty \subset E$ and the Δ_2-condition at infinity, we have

$$\varphi(x_{n_k} - x) \le K(w+y)/2 + \varphi(t_0)\chi_\Omega \in E^+.$$

In consequence, $\varphi(x_{n_k} - x) \xrightarrow{\mu} 0$ and $0 \le \varphi(x_{n_k} - x) \le r$ for some $r \in E^+$. Thus, by the order continuity of E, we have

$$\rho(x_{n_k} - x) \to 0.$$

Since φ satisfies the suitable Δ_2-condition, we have $x_{n_k} - x \to 0$ in E_φ, and that contradicts $(**)$. Thus, E_φ has the (H_μ) property, and the proof is finished. □

It is obvious that $L^\varphi = (L^1)_\varphi$ and $L^\infty \subset L^\varphi$ if μ is atomless and finite, $l^\varphi \subset l^\infty$ if μ is counting measure and both L^1 and l^1 have the (H_μ) property. Thus, by Lemma 2, we obtain the following result.

Corollary 1. *An Orlicz space $L^\varphi(\mu)$ has the (H_μ) property if and only if φ satisfies the suitable Δ_2-condition.*

The necessity follows from the fact that if φ does not satisfy the suitable Δ_2-condition then L^φ contains an isometric copy of l^∞ (cf. [**2, 5, 14**]). Indeed, any Orlicz space L^φ is monotone complete, whence it follows that the (H_μ) property of L^φ implies order continuity. Therefore, assuming that φ does not satisfy the suitable Δ_2-condition, we obtain a contradiction.

Corollary 2. *Let $L^\varphi(\mu)$ be an Orlicz space with φ satisfying the suitable Δ_2-condition and X a Banach space. Then, for (f_n) and f in the Orlicz-Bochner space $L^\varphi(\mu, X)$, if $||f_n|| \to ||f||$ and $f_n \to f$ μ-almost everywhere, then $f_n \to f$ in $L^\varphi(\mu, X)$.*

The proof follows by applying Corollary 1 and Proposition 1.

Remark 1. In the case of $\varphi(t) = |t|^p$, $1 \le p < \infty$, the above result was proved by Smith in [**13**] by a quite different method.

Next we will give an example of Köthe function spaces with the (H_μ) property, namely Orlicz-Lorentz spaces (cf. [**6**]).

Theorem 1. *Let $\Lambda_{\varphi,w}$ be an Orlicz-Lorentz space over a finite atomless measure space (Ω, Σ, μ) with the weight function w. Then $\Lambda_{\varphi,w}$ has the (H_μ) property if φ satisfies the suitable Δ_2-condition.*

Proof. Since Λ_w is monotone complete, in view of Lemma 2, it suffices to show that Λ_w has the (H_μ^+) property. In order to prove that it is enough to establish that, if x_n, $x \geq 0$, and

$$x_n \overset{\mu}{\to} x \quad \text{and} \quad ||x_n||_{\Lambda_w} \to ||x||_{\Lambda_w}, \tag{++}$$

then (x_n) contains a subsequence convergent strongly to x.

Since $x_n \overset{\mu}{\to} x$, we have $x_n^* \to x^*$ m-almost everywhere (cf. [**9**, p. 93]). Thus

$$x_n^* w \to x^* w \qquad m\text{-a.e.}$$

and

$$\int x_n^* w\, dt \to \int x^* w\, dt,$$

by the assumption that $||x_n||_{\Lambda_w} \to ||x||_{\Lambda_w}$. This yields, by the (H_μ) property of L^1 that

$$\int |x_n^* - x^*| w\, dt \to 0.$$

Thus, in virtue of Lemma 1, this implies that

$$|x_{n_k}^* - x^*| w \leq y$$

for some subsequence $(x_{n_k}^*)$ of (x_n^*) and $y \in (L^1)^+$. In consequence, by the assumption on the weight function w, we obtain

$$\begin{aligned} (x_{n_k} - x)^*(t) w(t) &\leq x_{n_k}^*(t/2) w(t) + x^*(t/2) w(t) \\ &\leq x_{n_k}^*(t/2) w(t/2) + x^*(t/2) w(t/2) \\ &\leq y(t/2) + 2x^*(t/2) w(t/2) = z(t). \end{aligned}$$

We have $z \in L^1$ and $(x_{n_k} - x)^* \to 0$ m-almost everywhere. Thus, the Lebesgue dominated convergence theorem yields

$$||x_{n_k} - x||_{\Lambda_w} = \int (x_{n_k} - x)^* w \, dt \to 0.$$

This finishes the proof. □

Remark 2. In the case of an atomless measure, the assumption concerning the Δ_2-condition for φ is necessary in Theorem 1, because, in the opposite case, the Orlicz-Lorentz space $\Lambda_{\varphi,w}$ contains an isometric copy of l^∞ (cf. [**6**]).

Now we will consider the problem of a characterization of strongly extreme points of the unit sphere of Köthe-Bochner spaces.

Remark 3. Every locally uniformly rotund Banach space X has the *Kadec-Klee property*, i.e., norm and weak convergence of sequences coincide on the unit sphere of X. Therefore, locally uniformly convex Köthe function spaces are order continuous (cf. [**10**, p. 28]).

Theorem 2. *Let E be a locally uniformly rotund Köthe function space over a measure space (Ω, Σ, μ), and let X be a Banach space. If $f \in S_{E(X)}$ is such that $f(s)/||f(s)||_X$ is a strongly extreme point of S_X for μ-almost everywhere $s \in \operatorname{supp} f$, then f is a strongly extreme point of $S_{E(X)}$.*

Proof (cf. [**13**]). The local uniform rotundity of E implies that E has the (H_μ) property (cf. [**4**]). Suppose f is in $S_{E(X)}$ and (g_n) is a sequence in $E(X)$ such that $||f \pm g_n|| \to 1 = ||f||$. Hence, it follows that $||2f + g_n|| \to 2$. By the triangle inequality in X and E, we have

$$\begin{aligned} ||2f \pm g_n|| &\le || \, ||f(\cdot)||_X + ||f(\cdot) \pm g_n(\cdot)||_X \, ||_E \\ &\le ||f|| + ||f \pm g_n||. \end{aligned} \tag{1}$$

Since the left side and the right side of (1) tend towards two, the local uniform rotundity of E yields that $||f(\cdot) \pm g_n(\cdot)||_X \to ||f(\cdot)||_X$ in E. Thus, passing to a subsequence, if necessary, and applying the continuous embedding of E into L^0, we can assume that

$$||f(s) \pm g_n(s)||_X \to ||f(s)||_X \qquad \mu\text{-a.e.} \tag{2}$$

Hence, for μ-almost every $s \in \operatorname{supp} f$, we have

$$||f(s) \pm g_n(s)||_X / ||f(s)||_X \to 1.$$

Thus, by the assumption that $f(s)/||f(s)||$ are strongly extreme points of S_X μ-almost everywhere in $\operatorname{supp} f$, it follows that $g_n(\cdot) \to 0$ μ-almost everywhere in $\operatorname{supp} f$. By (2), it follows also that $g_n(\cdot) \to 0$ μ-almost everywhere in $\Omega \backslash \operatorname{supp} f$. In consequence, we have

$$||f \pm g_n|| \to ||f||$$

(by the assumption) and

$$f + g_n \to f \qquad \mu\text{-a.e.} \quad \text{in } \Omega.$$

By Remark 3, E is order continuous. Thus, in view of Proposition 1, we get $g_n \to 0$ in $E(X)$, and the proof is finished. □

Corollary 3. *Let φ be a strictly convex Orlicz function satisfying the suitable Δ_2-condition, X a Banach space. If $f \in S_{L^\varphi(X)}$ is such that $f(s)/||f(s)||_X$ is a strongly extreme point of S_X for μ-almost everywhere s in* $\operatorname{supp} f$, *then f is a strongly extreme point of $S_{L^\varphi(X)}$.*

The proof follows immediately from Theorem 2 and the fact that the strict convexity of φ and the suitable Δ_2-condition for φ imply the local uniform rotundity of L^φ (cf. [**7**]). Note that, in the case of counting measure, it is enough to assume strict convexity of φ on the interval $[0, \varphi^{-1}(1)]$) (cf. [**6**]).

In the case of a nonatomic measure μ, strongly extreme points of the unit sphere of Orlicz spaces $L^\varphi = L^\varphi(\mu, \mathbf{R})$ were characterized in [**3**] in the case when φ satisfies the suitable Δ_2-condition.

Under the same assumption on φ as in Corollary 3, if X is a separable Banach space, then the sufficient condition for f to be an extreme point of $S_{L^\varphi(X)}$ is also necessary. This follows immediately from the following theorem.

Theorem 3. *Let E be as in Theorem* 2 *and X a separable Banach space. If $f \in S_{E(X)}$ is a strongly extreme point, then $f(s)/||f(s)||_X$ are strongly extreme points of S_X for μ-almost every $s \in$* $\operatorname{supp} f$.

The proof for $L^p(\mu, X)$, $1 < p < \infty$, in [**13**] can be repeated in our case.

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