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ABELIAN GROUP THEORY IN ITALY

LUIGI SALCE

1. Introduction. The development of group theory in Italy, after the pioneering contributions at the end of 1800 and in the first three decades of 1900 by Betti, Frattini, Cipolla and Gaetano Scorza, was stimulated at the end of the 40's by Guido Zappa, who was full professor first in Napoli for some years and later in Florence for the rest of his academic life. He wrote about a hundred papers on noncommutative groups and a book in two volumes [**92**] on group theory. Chapter VII in this book contained the classical results on finitely generated and divisible abelian groups and, mostly without proofs, more advanced topics, as Kulikov's and Prüfer's criteria for direct sums of cyclic p-groups, Ulm's and Zippin's theorems, basic subgroups, pure-projectivity of direct sums of cyclics, Baer's theory of types, and Hajos's theorem on a conjecture of Minkowski. Furthermore, Chapter IX contained a description of the group of extensions of two abelian groups. Zappa did not do research in abelian group theory.

Two young algebraists started to work with Zappa in Napoli in the 50's, Giovanni Zacher and Mario Curzio. Zacher was the founder and the leader of the algebra school on noncommutative groups in Padova since the beginning of the 60's. His contribution to abelian groups is a recent paper with Costantini and Holmes on the challenging problem of describing the groups of autoprojectivities of the modular groups, modulo the automorphism groups; they dealt also specifically with the case of bounded abelian p-groups (see [**8**]). Curzio, after a period spent in Bari, became the leader of the algebra school in Napoli, where he is still teaching. His scientific activity was devoted to noncommutative groups, but he also write a paper [**9**] on the connection between an abelian group A and a group G such that the lattices of all subgroups of A and the lattice of normal subgroups of G are isomorphic.

Once these contributions coming from the noncommutative area of group theory are recalled, it might be pointed out that abelian

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group theory got its major impulse in Italy from the work of four researchers, each of whom dedicated at least ten years to this subject: in chronological order, Adalberto Orsatti, myself, Claudia Metelli and Dikran Dikranjan. About ten collaborators or students wrote papers on different topics in abelian groups, often as developments of their doctoral thesis. Among them, Gabriella D'Este, Elisabetta Monari-Martinez and Nicola Rodinó have a more consistent production.

The next section is dedicated to reconstructing the history of abelian group theory in Italy in connection with these researchers, and to synthetically outlining the main themes they investigated, often in collaboration with other leading research centers, as Tulane University in New Orleans, Charles University in Praha and the University of Essen.

2. People and main research themes. Everybody agrees that abelian group theory in Italy started in 1965 when Adalberto Orsatti, three years after his doctoral thesis on hypergroups and after a one-year employment in Milano at Olivetti (the Italian computer firm) returned to the Seminario Matematico of the University of Padova and began self-taught research in abelian groups. His first paper on abelian groups with local endomorphism ring [**67**] was published in 1963, the year in which Zappa's book quoted in the introduction appeared.

Orsatti was active in abelian group theory – his first mathematical love – during ten years between 1963 and 1973. He was first full professor in Perugia from 1970 to 1971, then in Ferrara from 1971 to 1974, finally in Padova since 1974. The core of Orsatti's work in abelian groups was a successful attempt to extend Corner's results on endomorphism rings, starting with the generalization from the countable to the locally countable case (see [**71**]). Even if this generalization does not greatly enlarge the class of involved rings, credit must be given to Orsatti since he made the first attempt to extend Corner's results. Orsatti was definitely fascinated by the Corner papers [*Every countable torsion-free ring is an endomorphism ring*, Proc. London Math. Soc. **3** (1963), pp. 687–710] and [*Endomorphism rings of torsion-free abelian groups*, Proc. Internat. Conf. Theory of Groups (Austral. Nat. Univ. Canberra 1965), 1967, pp. 59–69]. In that period everyone at the Seminario Matematico in Padova was fully informed of the relevance and beauty of Corner's work. In every office the blackboards were full of

Adalberto's handwriting; his reproaches to whomever didn't pay him due attention earned him the well-deserved name, "Il Maestro".

I would like to recall the first meeting of Tony Corner and Adalberto. It was in Padova in 1973, when Tony made a trip to visit the Bizantine monuments and mosaics in Ravenna. Adalberto received an unexpected call: "Sono Tony Corner, sono a Padova". Inconceivable surprise. They met at the "Gigi Bar", a pub well known to many algebraists in Padova, where they drank some beers while talking of endomorphism rings. Then they went to a restaurant for a dinner, still spiced with endomorphism rings, that I had the pleasure to attend.

The second love of Orsatti was general topology, another subject that he made popular in Padova. This is testified by two papers [**70, 73**] where compact and locally compact abelian groups are investigated. De Marco was the first talented student of Orsatti with a main interest in general topology. Their frequent discussions on topological aspects of various algebraic structures had as a byproduct a joint paper [**11**] on the algebraic structure of the completion of abelian groups with respect to linear topologies, published in 1974.

After this paper Orsatti moved to different kinds of algebraic structures, but he left an abelian group theory heritage: many students and the book, *Gruppi abeliani astratti e topologici*, which appeared in 1978 in the series "Quaderni dell' Unione Matematica Italiana". This book developed the basic results on abstract and topological abelian groups and discussed the deep results by Haar, Peter-Weyl and Pontryagin on compact groups and the theories of algebraically compact and cotorsion groups. Furthermore, three special sections were devoted to homomorphic images of direct products, slender groups and Corner's theorems on endomorphism rings. Some of these topics were presented in a talk in 1975 in Cagliari at the national conference of the Unione Matematica Italiana. That lecture popularized the subject of abelian groups for the first time to a large audience in Italy.

As pointed out above, the leader of the algebra school in Padova was at that time Giovanni Zacher. I asked him to give me a research problem for my doctoral thesis in 1968. Since he was leaving for Urbana, Illinois to spend a sabbatical year, Zacher sent me to the outstanding young algebraist who was doing excellent research in the new area of abelian groups. Orsatti gave me the problem of generalizing the first of Corner's theorems on the realization of reduced countable torsion-free rings as rings of endomorphisms of abelian groups. In my thesis [**80**] I was able only to give a simpler proof of Orsatti's generalization to the locally countable case.

A few years later a second student wrote a thesis on abelian groups with Orsatti, namely Gabriella D'Este. The subject of her thesis, topological properties of endomorphism rings of abelian groups, was suggested by Orsatti, but I became her supervisor since Orsatti was already out of the abelian group theory world. Gabriella continued for a few years (see [**12, 13, 17**]) with Orsatti's work of elaborating on Corner's realization theorems of abstract and topological rings as endomorphism rings of abelian groups, using also Corner's idea of residually controlled rings. She also investigated ([**16**]) the $\oplus_c$-topology of p-groups, obtaining some nice results partly invalidated by a subtle mistake detected by Adolf Mader.

In the Seminario Matematico of the University of Padova, among people working on noncommutative groups, there was Claudia Metelli. In 1972 I sold her my old car (a white Fiat 500, the smallest car existing at that time). Very soon she had a crash, breaking her hip. She spent some boring months in a body-long cast and I went often to her house to distract her by discussing mathematics. When Orsatti, reading the paper by G.J. Hauptfleisch [*Torsion-free abelian groups with isomorphic endomorphism rings*, Archiv. Math. **24** (1973), 269–273], where it is proved that homogeneous separable torsion-free groups are determined by their endomorphism rings, gave me the idea of trying to characterize these rings in a similar way as Wolfgang Liebert characterized endomorphism rings of separable p-groups [*Endomorphism rings of abelian p-groups*, Studies on Abelian Groups (Symposium Montpellier, 1967), Springer, pp. 239–258], I involved Claudia who was ready for that job. Our joint paper on endomorphism rings of separable homogeneous groups [**63**] was submitted to Reinhold Baer, who appreciated the use of the subgroup denoted by G_R. That same technique was used recently by Lutz Strüngmann and myself [**90**] to solve a problem on stacked bases of homogeneous completely decomposable groups. Later on, Metelli extended the results to nonhomogeneous separable groups – where the situation is much more complicated and Hauptfleisch's result fails – in a joint paper with a young new student of Orsatti, Silvana Bazzoni, when they were both in New Orleans at Tulane University in

1978–1979.

As I already recalled in [**88**], the bridge between Italy and New Orleans was already set at the end of 1975, when I went to Tulane to spend one year with Laszlo Fuchs. After me, some other people went from Padova to New Orleans to benefit from the scientific leadership of Laszlo Fuchs, who has had a strong influence on the growth of the Padova Algebra school. My experience in Tulane was decisive for my scientific activity; I learned so many things from Laszlo that my knowledge of abelian groups quickly improved. During my ten months' stay in Tulane we wrote two joint papers on almost totally injective p-groups and on p-groups of nonlimit length, and one more paper on monotone subgroups of the Baer-Specker group (see [**46, 47, 48**]). I also read a wide portion of the existing literature on p-groups so that, once back in Italy, it was not difficult for me to prepare a course on p-groups in Ferrara that I gave in 1976–77. The notes of that course formed the core of my book, *Struttura dei p-gruppi abeliani*, written in Italian and published by the Unione Matematica Italiana in 1980, in the same series as Orsatti's book. In this book there is an intensive use of valued vector spaces, influenced by Fuchs's point of view. Furthermore, there is a homological classification of p-groups via the functors $p^\sigma\mathrm{Ext}$ (σ is an ordinal); hence, a central part of the book is the illustration of p^σ-projective and p^σ-injective p-groups for various ordinals σ, and of totally projective and (almost) totally injective p-groups.

Between 1974 and 1978 four more people made incursions into the abelian group theory world. Three of them were students of Orsatti; each one wrote a paper inspired by him. The first one was Alberto Zelger who was an assistant of Orsatti in Ferrara; his paper [**93**] on completions of abelian groups in the topology of subgroups of finite index was published in 1975. The second was Attilio Le Donne who came from Palermo to Padova to work in general topology. He received from Orsatti the problem of extending Corner's '63 theorem to torsion-free algebras over Dedekind domains, which he solved in [**53**]. This job was continued later in 1988 by Giorgio Piva [**79**], the last of Orsatti's students with a thesis on abelian groups. He extended Orsatti's version of Corner's result in the locally countable case to Dedekind domains. The third was Florida Girolami, a student of Orsatti's in Perugia. She obtained a characterization of groups with vanishing first Ulm subgroup [**52**]. The last one was Federico Menegazzo, a top researcher in noncommutative group theory in Padova. I involved him during the Christmas vacation of 1974 in a search for a characterization of abelian groups whose endomorphism ring is linearly compact in the finite topology [**89**], a problem also suggested by Orsatti.

In 1978 Paolo Zanardo wrote his doctoral thesis under my supervision on $p^{\omega+2}$-projective p-groups [**91**]. After that experience, Zanardo and I dedicated our interest for awhile to the investigation of modules over valuation domains. In 1979 I suggested a problem to Elisabetta Monari-Martinez, a student from Bologna University. The problem was to extend to mixed groups the construction of the groups P_β (which form building blocks for totally projective p-groups), introduced by Elbert Walker in the paper [*The groups* P_β, Symposia Math. XII, Academic Press, London, 1974, pp. 245–255] presented at the INDAM (Istituo Nazionale di Alta Matematica) Conference in Rome in 1972. She derived some results on balanced projective groups [**64**] and then established some properties of local Warfield groups in 1984 [**65**]. We also wrote a joint paper in 1982 [**66**] on a homological characterization of λ-large subgroups of p-groups.

In 1981 Ladislav Bican came to Padova as a visiting professor. I met him for the first time in Rome in 1977 at that INDAM Conference. He gave a seminar on torsion-free abelian groups of finite rank. At that time I was interested in balanced-exact sequences and was aware of the paper by R. Hunter on this subject [*Balanced subgroups of abelian groups*, Trans. Amer. Math. Soc. **21** (1976), 81–98]. I realized that one of Bican's characterizations of Butler groups was in terms of balanced-exact sequences. The generalization to groups of arbitrary rank via this homological characterization by the functor Bext was natural. The new classes of B_1-groups and B_2-groups were defined and investigated in the countable case (see [**3**] and [**4**]). Everybody working in abelian groups knows how much the theory of Butler groups of infinite rank has developed in the last 20 years.

The collaboration with Bican and other algebraists of the Praha Algebra school grown around Procházka continued for some years. Nowadays, it is blooming again on different algebraic subjects which often have their roots in abelian group theory. One of these subjects is cotorsion theories that I introduced in the abelian groups setting in a paper [**85**] presented at the INDAM Conference of 1977 in Rome (where I first met Rüdiger Göbel). This subject originated from a curiosity in seeing what happens to cotorsion groups C ($\mathrm{Ext}^1(\mathbf{Q}, C) = 0$) if one replaces the group $\mathbf{Q}$ of rational numbers by subgroups R of $\mathbf{Q}$. Since these subgroups R are called *rational* groups, one gets the so-called *rational cotorsion theories*. The following problem arose: does a rational cotorsion theory have enough projectives or (equivalently, as shown in [**85**]) enough injectives?

Cotorsion theories slept for 20 years in the woods as the sleeping beauty, till a prince kissed them. This happened when Göbel-Shelah solved in the paper [*Cotorsion theories and splitters*, Trans. Amer. Math. Soc. **352** (2000), 5357–5379] a problem on splitter groups G ($\mathrm{Ext}^1(G, G) = 0$) posed by Schultz in 1980, and the problem on rational cotorsion theories mentioned above. The Göbel-Shelah paper stimulated the paper [*How to make Ext vanishing*, Bull. London Math. Soc. **33** (2001), 41–51] by Eklof-Trlifaj, where a crucial theorem on the vanishing of Ext^1 over general rings was obtained. This theorem and the discovery of connections between cotorsion theories and the theory of covers and envelopes enabled Enochs to solve the long-standing question of the existence of flat covers over any ring (see [L. Bican, R. El Bashir and E. Enochs, *All modules have flat covers*, Bull. London Math. Soc., **33** (2001), 385–390]). Nowadays, cotorsion theories and their princes live happily and are very prolific, both in the abelian group theory world (see papers by Göbel, Shelah, Strüngmann and Wallutis in various combinations) and in the outside worlds of modules and model structures in abelian categories (see papers by Eklof, Trlifaj, Enochs, Hovey, Bazzoni and myself).

At the beginning of the 80's we started a fruitful scientific relationship with Rüdiger Göbel and the Essen University. Metelli and I went often to meet Göbel and his school (Manfred Dugas was still there), and people from Essen came often to Padova.

In 1981 the INDAM organized a summer school in Cortona. Fuchs and Orsatti were the lecturers. One of the students was Nicola Rodinó, who soon became a close friend of Orsatti. They often worked together and, several years later during and immediately after the 1984 Conference on Topology in Primorsko, Bulgaria, they solved a problem in general topology on powers of connected compact topological spaces posed by Vera Trnková at that conference. Using Pontryagin duality, Corner's results on endomoprhism rings of torsion-free groups and on fully rigid systems [*Fully rigid systems of modules*, Rend. Sem. Mat.

Univ. Padova **82** (1989), 55–66] and Scheinberg's theorem on homeomorphisms between locally compact, connected topological abelian groups [*Homemorphisms and isomorphisms of abelian groups*, Canad. J. Math. **6** (1974), 1515–1519] they provided a spectacular solution to Trnková's problem (see [**77**]). The results in this paper have been extended by Dikranjan-Rodinó some years later [**28**] to pseudocompact totally minimal groups.

Among all of the people mentioned above, Claudia Metelli was the only one who continued to work successfully in abelian groups in the last two decades. The exceptions of the 1986 paper by Orsatti-Rodinó [**77**] and of the joint paper by Strüngmann and myself [**90**] quoted above can be considered as "backfires", as I said in [**87**]. But, while Orsatti's research since 1973 focused mainly on modules over noncommutative rings, my activity since 1979 has been devoted to a long-term project with Laszlo Fuchs. The project is to set a bridge from abelian groups to modules over general domains. The pioneering work was done mainly by Kaplansky, Matlis and Warfield between the fifties and the seventies. In this project I had the strong collaboration of Paolo Zanardo and Silvana Bazzoni in Padova. The first step was a deep investigation of modules over valuation domains that absorbed about fifteen years. The passage to general domains is in progress. Two books have been written by Fuchs and myself: *Modules over valuation domains* in 1985 and *Modules over non-Noetherian domains*, which just appeared. I am confident that people working in abelian group theory will, in the future, enlarge their horizon to include the large and fascinating territory of modules over general domains.

Going back to Claudia Metelli, she first dedicated herself to torsion-free abelian groups of infinite rank [**54**–**59**], studying the classes of coseparable groups (dual to separable groups), and of almost separable groups, a wide generalization where splitting properties are replaced by type-related properties. The results obtained by Metelli on these subjects are relevant in view of the difficulty of finding good classes of torsion-free groups that can be described in a reasonable way. In a revisitation to the Zacher school, in [**50**] with Emanuela Gasparini, they determined the projectivities of torsion-free rank one groups, a prelude to the work by Zacher quoted above. A Tulane sabbatical in 1989 started a fruitful collaboration with Laszlo Fuchs, producing indecomposable Butler groups of arbitrary rank with $\mathbf{Z}$ as an endomorphism ring, and super-decomposable Butler groups of countable rank [**42**]. Their notion of prebalanced subgroup allows a simpler development of the theory [**43**]. Their new approach to finite rank Butler $B(1)$-groups [**41**] started an in-depth investigation of this widely studied class. This line of research is still pursued in Napoli (where Metelli has been a full professor since 1990), both by herself [**61, 62**] and in collaboration with Clorinda De Vivo, a researcher of the Curzio school. In a series of five papers [**18**–**22**] they associated to a $B(1)$-group an order-morphism called a "tent". Then they realized all $B(1)$-groups on which a $\mathbf{Q}$-matrix acts as a base-change and developed a finite algorithm to decompose a $B(1)$-group into inner direct summands. Finally, they proved results needed to approach the open problem of establishing all base-changes of $B(1)$-groups. Complex problems on $(0,1)$-matrices have prompted a collaboration with Francesco Barioli, a former student of mine in combinatorical matrix theory.

The Italian abelian group theory community made an important new acquisition at the beginning of the 90's when Dikran Dikranjan, after some time spent as a visiting professor in Italy, came from Bulgaria to settle in Udine. He became an associate professor and then full professor of Algebra in Udine, and since 1999 he is an Italian citizen. He started collaborating in 1983 with Orsatti on topological rings and modules and with Rodinó on topological abelian groups. He has a wide production of results on topological questions on abelian groups, minimal groups and pseudocompact groups, essential subgroups (related to the open mapping theorem), weak and strong forms of completeness, dimension and connectedness, suitable sets of generators (a topic introduced in the 60's by Tate and Douady in the framework of Galois cohomology) and isomorphisms versus homeomorphisms. More recently, he wrote joint papers on powers of minimal ω-bounded abelian groups with Alberto Tonolo [**37**], a second generation pupil of Orsatti and with Elena Boschi [**5, 6**], a student of his, on essential subgroups of topological groups and suitable sets of locally compact groups.

I would like to conclude by recalling the contributions given by the Italian researchers in the promotion of abelian group theory. After the first visit by Corner in 1973, we had many other experts in abelian group theory as visitors, either for short periods, or for many months. They brought us their direct up-to-date knowledge of important topics in abelian groups. The long list of visitors – mostly in Padova, but also in Ferrara and Napoli – includes Mines, Fuchs, Bican, Becvar, Göbel, Dugas, Pierce, Goldsmith, Eklof, Fay, Rangaswamy, Keef, Faticoni, Benabdallah and Mader. These contacts put Italy on the abelian group theory map and produced many successful collaborations.

We also promoted abelian group theory by organizing many conferences, as illustrated in another part of this volume. The title of the 1977 INDAM Rome Conference was "Gruppi abeliani e loro relazioni con la teoria dei moduli". This title makes evident the intention that one can find in all the Italian conferences, of enlarging the horizon from abelian group theory to more general settings such as module theory, ring theory, representations of algebras and topological algebraic structures. This trend seems to be widely shared nowadays, as the title of this successful Honolulu Conference testifies. In another complementary direction, conferences organized in Germany focused on the interplay between abelian groups and model theory and infinite combinatorics. We are convinced that communication between different mathematical communities working on different algebraic and non-algebraic subjects is a fruitful experience which stimulates the development of mathematics and, in particular, Abelian group theory.

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Remark . The following is the complete (as far as I know) list of papers on abelian groups published by Italian researchers, including Dikranjan's papers related to his activity in Italy since 1993.

REFERENCES

1. S. Bazzoni and C. Metelli, *On abelian torsion-free separable groups and their endomorphism rings*, (Conf. on Abelian Groups and their Relationship to the Theory of Modules, INDAM, Rome, 1977), Symposia Math., vol. 23, Academic Press, London, 1979, pp. 259–287.

2. A. Berarducci, D. Dikranjan, M. Forti and S. Watson, *Cardinal invariants and independence results in the lattice of precompact group topologies*, J. Pure Appl. Algebra **126** (1998), 19–49.

3. L. Bican and L. Salce, *Butler groups of infinite rank*, Abelian Group Theory (Honolulu, Hawaii, 1983), Lecture Notes in Math., vol. 1006, Springer, Berlin, 1983, pp. 171–189.

4. L. Bican, L. Salce and J. Štěpan, *A characterization of countable Butler groups*, Rend. Sem. Mat. Univ. Padova **74** (1985), 51–58.

5. E. Boschi and D. Dikranjan, *Essential subgroups of topological groups*, Comm. Algebra **28** (2000), 4941–4970.

6. ———, *Locally compact abelian groups admitting totally suitable sets*, J. Group Theory **4** (2001), 59–73.

7. W. Comfort and D. Dikranjan, *On the poset of totally dense subgroups of compact groups*, Topology Proc. **24** (1999), 103–128.

8. M. Costantini, C.S. Holmes and G. Zacher, *A representation theorem for the group of auto-projectivities of an abelian p-group of finite exponent*, Ann. Mat. Pura Appl. **175** (4) (1998), 119–140.

9. M. Curzio, *Una caratterizzazione reticolare dei gruppi abeliani*, Rend. Mat. Appl. (5) **24** (1965), 1–10.

10. G. De Marco, *On the algebraic compactness of some quotients of product groups*, Rend. Sem. Mat. Univ. Padova **53** (1975), 320–333.

11. G. De Marco and A. Orsatti, *Complete linear topologies on abelian groups*, (Convegno di Gruppi Abeliani, INDAM, Rome, 1972), Symposia Math., vol. 13, Academic Press, London, 1974, pp. 153– 161.

12. G. D'Este, *Sui gruppi abeliani il cui anello degli endomorfismi è numerabile e non discreto nella topologia finita*, 1st Veneto Sci. Lett. Arti Atti Cl. Sci. Mat. Natur. **134** (1975/76), 239–243.

13. ———, *A theorem on the endomoprhism ring of reduced torsion-free abelian groups and some applications*, Ann. Mat. Pura Appl. (4) **116** (1978), 381–392.

14. ———, *Abelian groups with anti-isomorphic endomorphism rings*, Rend. Sem. Mat. Univ. Padova **60** (1978), 55–75.

15. ———, *Torsion-free abelian groups and completely decomposable p-adic modules*, Rend. Sem. Mat. Univ. Padova **62** (1980), 95–102.

16. ———, *The $\oplus_c$-topology on abelian p-groups*, Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) **7** (1980), 241–256.

17. G. D'Este, *On topological rings which are endomorphism rings of reduced torsion-free abelian groups*, Quart. J. Math. Oxford Ser. (2) **32** (1981), 303–311.

18. C. De Vivo and C. Metelli, *$B^{(1)}$-groups: Some counterexamples*, Abelian Groups and Modules (Colorado Springs, 1995), Lecture Notes Pure and Appl. Math., vol. 182, Dekker, New York, 1996, pp. 227–232.

19. ———, *Finite partition lattices and Butler groups*, Comm. Algebra **27** (1999), 1571–1590.

20. ———, *Admissible matrices as base changes of $B^{(1)}$-groups: A realizing algorithm*, Abelian Groups and Modules (Dublin, 1998), Trends in Math., Birkhauser, Basel, 1999, pp. 135–147.

21. ———, *Decomposing $B^{(1)}$-groups: An algorithm*, Comm. Algebra, to appear.

22. ———, $\mathbf{Z}_2$*-linear order-preserving transformations and tents*, Ricerche di Mat., to appear.

23. D. Dikranjan, *Dimension and connectedness in pseudo-compact groups*, Comp. Rend. Acad. Sci. Paris, Sér I **316** (1993), 309–314.

24. ———, *Chains of pseudocompact group topologies*, J. Pure Appl. Algebra **124** (1998), 65–100.

25. ———, *Recent advances in minimal topological groups*, Topology Appl. **85** (1998), 53–91.

26. ———, *Compactness and connectedness in topological groups*, Topology Appl. **84** (1998), 227–252.

27. ———, *Quotients of zero-dimensional precompact abelian groups*, Topology Appl. **86** (1998), 47–62.

28. D. Dikranjan and N. Rodinó, *On a property of the one-dimensional torus*, Rend. Sem. Mat. Univ. Padova **69** (1983), 277–288.

29. ———, *Isomorphism between finite powers of connected, totally minimal, pseudocompact, abelian groups*, Rend. Circ. Mat. Palermo (2), Suppl. no. 29 (1992), 385–398.

30. D. Dikran and D. Shakhmatov, *Compact-like totally dense subgroups of compact groups*, Proc. Amer. Math. Soc. **114** (1992), 1119–1129.

31. ———, *Pseudocompact and countably compact abelian groups*: *Cartesian products and minimality*, Trans. Amer. Math. Soc. **335** (1993), 775–790.

32. ———, *Algebraic structure of the pseudocompact groups*, Mem. Amer. Math. Soc. **133/633** (1998),

33. D. Dikranjan and M. Tkachenko, *Sequential completeness of quotient groups*, Bull. Austral. Math. Soc. **61** (2000), 129–151.

34. ———, *Weak completeness of free topological groups*, Topology Appl. **112** (2001), 259–287.

35. ———, *Sequentially complete groups*: *Dimension and minimality*, J. Pure Appl. Algebra **157** (2001), 215–239.

36. D. Dikranjan, M. Tkachenko and V. Tkachuk, *Topological groups with thin generating sets*, J. Pure Appl. Algebra **145** (2000), 123–148.

37. D. Dikranjan and A. Tonolo, *On the minimality of powers of minimal* ω*-bounded abelian groups*, Proc. Amer. Math. Soc. **127** (1999), 3101–3110.

38. D. Dikranjan and J. Trigos, *Suitable sets in some topological groups*, J. Group Theory **3** (2000), 293–321.

39. D. Dikranjan and V. Uspenskij, *Isomoprhism between homeomorphic topological groups*, Infinite Groups 1994 (Ravello), de Gruyter, Berlin, 1996, pp. 33–39.

40. ———, *Categorically compact topological groups*, J. Pure Appl. Algebra **126** (1998), 149–168.

41. L. Fuchs and C. Metelli, *On a class of Butler groups*, Manuscripta Math. **71** (1991), 1–28.

42. ———, *Indecomposable Butler groups of large cardinalities*, Arch. Math. **57** (1991), 339–344.

43. ———, *Countable Butler groups*, Abelian Groups and Noncommutative Rings, Contemp. Math., vol. 130, Amer. Math. Soc., Providence, RI, 1992, pp. 133–143.

44. ———, *Butler quotients of torsion-free abelian groups modulo prebalanced subgroups*, Abelian Groups (Curacao, 1991), Lecture Notes Pure and Appl. Math., vol. 146, Dekker, New York, 1993, pp. 151–156.

45. L. Fuchs, C. Metelli and K.M. Rangaswamy, *Corank one subgroups of completely decomposable abelian groups*, Comm. Algebra **22** (1994), 1031–1036.

46. L. Fuchs and L. Salce, *Almost totally injective p-groups*, Quaestiones Math. **1** (1976), 225–234.

47. ———, *Abelian p-groups of not limit length*, Comment. Math. Univ. St. Paul **26** (1977/78), 25–33.

48. ———, *Gruppi monotoni di successioni intere*, Rend. Sem. Mat. Univ. Padova **56** (1979), 176–188.

49. ———, *On almost totally injective p-groups and cotorsion groups*, J. Algebra **58** (1979), 176–188.

50. E. Gasparini and C. Metelli, *On projectivities of abelian groups of torsionfree rank one*, Boll. Un. Mat. Ital. A (6) **3** (1984), 363–371.

51. A. Giovannitti, P. Goeters and C. Metelli, *Cobalanced exact sequences*, Comment. Math. Univ. Carolin. **30** (1989), 637–641.

52. F. Girolami, *On a class of abelian groups without elements of infinite height*, Boll. Un. Mat. Ital. A (5) **15** (1978), 455–459.

53. A. Le Donne, *Sull'algebra degli endomorfismi di una classe di moduli ridotti e senza torsione sui domini di Dedekind*, Rend. Sem. Mat. Univ. Padova **56** (1976), 215–226.

54. C. Metelli, *On type-related properties of torsion-free abelian groups*, Abelian Group Theory (Honolulu, Hawaii, 1983), Lecture Notes in Math., vol. 1006, Springer, Berlin, 1983, pp. 253–267.

55. ———, *On coseparable completely decomposable torsion free abelian groups*, Abelian Groups and Modules (Udine, 1984), CISM Courses and Lectures, vol. 287, Springer, Wien, 1984, pp. 215–220.

56. ———, *Coseparable torsion-free abelian groups*, Arch. Math. **45** (1985), 116–124.

57. ———, *Subgroups of almost separable torsionfree abelian groups*, Proc. Amer. Math. Soc. **106** (1989), 9–17.

58. ———, *Almost separable torsionfree abelian groups*, Abelian Group Theory (Oberwolfach, 1985), Gordon and Breach, New York, 1987, pp. 275–289.

59. ———, *Bihomogeneous groups*, Abelian Group Theory (Perth, 1987), Contemp. Math., vol. 87, Amer. Math. Soc., Providence, RI, 1989, pp. 161–169.

60. ———, *On automorphisms of completely decomposable torsionfree abelian groups*, Comm. Algebra **18** (1990), 529–549.

61. ———, *On direct sums of $B^{(1)}$-groups*, Comment. Math. Univ. Carolin. **34** (1993), 587–591.

62. ———, *An adjointness relation for finite partition lattices*, Abelian Groups, Module Theory and Topology (Padova, 1977), Lecture Notes Pure Appl. Math., vol. 201, Dekker, New York, 1998, pp. 301–310.

63. C. Metelli and L. Salce, *The endomorphism ring of an abelian torsionfree homogeneous separable group*, Arch. Math. **26** (1975), 480–485.

64. E. Monari Martinez, *Some classes of mixed abelian groups of rank* 1, Ann. Univ. Ferrara Sez. VII **25** (1979), 47–61.

65. ———, *Some properties of Warfield modules*, Comment. Math. Univ. St. Paul **33** (1984), 223–231.

66. E. Monari Martinez and L. Salce, *A homological characterization of certain subgroups of abelian p-groups*, Acta Math. Acad. Sci. Hungar. **39** (1982), 143–147.

67. A. Orsatti, *Alcuni gruppi abeliani il cui anello degli endomorfismi è locale*, Rend. Sem. Mat. Univ. Padova **35** (1965), 107–115.

68. ———, *Su di un problema de T. Szele e J. Szendrei*, Rend. Sem. Mat. Univ. Padova **35** (1965), 171–175.

69. ———, *Un lemma di immersione per i gruppi abeliani senza elementi di altezza infinita*, Rend. Sem. Mat. Univ. Padova **38** (1967), 1–13.

70. ———, *Una caratterizzazione dei gruppi abeliani compatti o localmente compatti nella topologia naturale*, Rend. Sem. Mat. Univ. Padova **39** (1967), 219–225.

71. ———, *A class of rings which are the endomorphism rings of some torsion-free abelian groups*, Ann. Scuola Norm. Sup. Pisa (3) **23** (1969), 143–153.

72. ———, *Una proprietà caratteristica dei gruppi abeliani torsionalmente completi*, Rend. Sem. Mat. Univ. Padova **42** (1969), 326–328.

73. ———, *Sui gruppi abeliani ridotti che ammettono una unica topologia compatta*, Rend. Sem. Mat. Univ. Padova **43** (1970), 341–347.

74. ———, *Anelli di endomorfismi di gruppi abeliani senza torsione*, (Convegno sulle Algebre Associative, INDAM, Rome, 1970), Symposia Math., vol. 8, Academic Press, London, 1972, pp. 179–191.

75. ———, *Alcuni aspetti della teoria dei gruppi abeliani*, Boll. Un. Mat. Ital. A (5) **13** (1976), 485–533.

76. ———, *Gruppi abeliani astratti e topologici*, Quaderno UMI N. **8**, Pitagora Ed., Bologna, 1979.

77. A. Orsatti and N. Rodinó, *Homeomorphisms between finite powers of topological spaces*, Topology Appl. **23** (1986), 271–277.

78. ———, *Isomorphisms between finite powers of abelian groups*, Rend. Sem. Mat. Fis. Milano **55** (1985), 181–188.

79. G. Piva, *On endomorphism algebras over admissible Dedekind domains*, Rend. Sem. Mat. Univ. Padova **79** (1988), 163–172.

80. L. Salce, *Sull'anello degli endomorfismi dei gruppi abeliani, ridotti e liberi da torsione*, 1st Veneto Sci. Lett. Arti Atti Cl. Sci. Mat. Natur. **128** (1969/70), 401–410.

81. ———, *Classi di gruppi abeliani chiuse rispetto alle immagini omomorfe ed ai limiti proiettivi*, Rend. Sem. Mat. Univ. Padova **49** (1973), 1–7.

82. ———, *Moduli slender su anelli di Dedekind*, Ann. Univ. Ferrara Sez. VII (N.S.) **20** (1975), 59–63.

83. ———, *The λ-inductive topology on Abelian p-groups*, Rend. Sem. Mat. Univ. Padova **59** (1978), 167–177.

84. ———, *I_λ-groups and almost totally injective groups*, Ann. Univ. Ferrara Sez. VII (N.S.) **25** (1979), 197–204.

85. ———, *Cotorsion theories for abelian groups*, (Conf. Abelian Groups and their Relationship to the Theory of Modules, INDAM, Rome, 1977), Symposia Math., vol. **23**, Academic Press, London, 1979, pp. 11–32.

86. ———, *Struttura dei p-gruppi abeliani*, Quaderno UMI N. **18**, Pitagora Ed., Bologna, 1980.

87. ———, *Adalberto Orsatti's contribution to abelian group theory*, Abelian Groups, Module Theory and Topology (Proc. in honor of A. Orsatti, Padova, 1997), Lecture Notes Pure Appl. Math., vol. 201, Dekker, New York, 1998, pp. 3–8.

88. ———, *László Fuchs and his "moddom" work*, Abelian Groups, Rings and Modules (AGRAM 2000 Conf., Perth), Contemp. Math., vol. 273, Amer. Math. Soc., Providence, RI, 2001, pp. 3–7.

89. L. Salce and F. Menegazzo, *Abelian groups whose endomorphism ring is linearly compact*, Rend. Sem. Mat. Univ. Padova **53** (1975), 315–325.

90. L. Salce and L. Strüngmann, *Stacked bases for homogeneous completely decomposable groups*, Comm. Algebra **29** (2001), 2575–2588.

91. P. Zanardo, *On $p^{\omega+2}$-projective p-groups*, Comment. Math. Univ. St. Paul **29** (1980), 45–53.

92. G. Zappa, *Fondamenti di teoria dei gruppi*, Vol. I and Vol. II, C.N.R., Monografie Matematiche **13**, Ed. Cremonese, Roma, 1965 and 1970.

93. A. Zelger, *Sul completamento di un gruppo abeliano nella topologia dei sottogruppi di indice finito*, Rend. Sem. Mat. Univ. Padova **52** (1974), 59–69.

Dipartimento di Matematica Pura e Applicata, Universitá di Padova, Italy
E-mail address: salce@math.unipd.it