JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 17, Number 2, Summer 2005

BOOK REVIEW

Collocation Methods for Volterra Integral and Related Functional Equations, by Hermann Brunner,
Cambridge Monographs on Applied and Computational Mathematics,
vol. 15, Cambridge University Press, Cambridge, U.K.
$120. xiv + 597 pp., hardcover. ISBN 0-521-80615-1.

By Volterra Functional Equations is usually meant a broad class of time-dependent equations with memory terms, that is, equations whre, at any time t, the unknown function $y(t)$ depends on some (possibly infinitely many) past values of the function y itself. This class includes, as special instances, integral equations and integro-differential equations of first and second kind, delay differential equations, integral and integro-differential equations with delay, delay equations with neutral terms, etc.

The properties of the kernel (regular, weakly singular, of convolution type, etc.) in the integral equations and the particular form of the delay function (constant, time dependent, state dependent, vanishing, proportional, unbounded, etc.) give rise to a variety of equations that require specific approaches but share the need of continuous approximations for their numerical integration. Consequently, "collocation" and, in particular, "piecewise polynomial collocation" methods turn out to provide the most natural approach for integrating such a class of equations.

On the basis of this fact the author, who, since the early seventies, has made significant theoretical and numerical contributions to this area, provides in this expository research monograph a well-balanced combination of the basic knowledge about several functional equations and the extensive analysis of the collocation method for their discretization. The author begins the book by introducing the collocation method applied to initial value problems for ODEs and then uses it throughout the book as leit-motif for deriving the discrete analogue of several Volterra Functional Equations and for outlining the intrinsic difficulties in their numerical integration inherited by the presence of the memory terms.

As stated by the author in the preface, the principal aims of the monograph are: "(i) to serve as an introduction and a guide to the basic principles and the analysis of the collocation methods for a broad range of functional equations...; (ii) to describe the current 'state of the art' of the field; (iii) to make the reader aware of the many (often very challenging) problems that remain open and which represent a rich source for future research; (iv) to showthat Volterra equations are not simply an 'isolated' small class of functional equations but that they play an (increasingly) important—and often unexpected!—role in time-dependent PDEs, boundary integral equations, and in many other areas of analysis and applications."

Actually, the author fully achieves his aims through nine chapters and a comprehensive bibliography that is enriched by many annotated references that makes it much more effective and eases bibliographic searches. The chapters may be clustered into four parts. Part I, including Chapters 1, 2 and 3, concerns the basic theory of collocation in continuous and smoother piecewise polynomial spaces and its application to Volterra integral and integro-differential equations. Part II, including Chapters 4 and 5, extends the analysis to Volterra and more general functional equations with delay. The case of proportional (vanishing) delay, that includes the pantograph equation and its generalizations, is widely developed and takes up the whole of the Chapter 5. Part III, including the subsequent Chapters 6 and 7, deals with Volterra integral and integro-differential equations with weakly singular kernels. Part IV, including the last two Chapters 8 and 9, ends up the book with an outlook of the application of collocation to differential-algebraic equations and more general functional equations coupled with algebraic components, followed by an exploration of new frontiers for the research on the use of collocation as well as other general approaches for the numerical integration of functional equations.

The problem of the asymptotic stability and other stability properties of the collocation solution go beyond the scope of the book and are considered only peripherally in the monograph. For these and other topics, such as contractivity, local error estimation and adaptive collocation in the numerical integration of equations with delay terms, the author refers to the recent book by Bellen and Zennaro [**1**] and the references therein.

As a rule, each chapter starts with a review of the basic theory concerning the equation dealt with in that chapter and ends with two sections devoted to Exercises, including unsolved problems, and to Notes, including additional remarks and references to papers on related topics not included in the book. The central part of each chapter is spent to describe and to develop the error analysis, and to determine the global convergence and superconvergence order of the collocation method, as well as the fully discretized and iterated collocation methods, when applicable.

In summary, this book gives a rather comprehensive treatment of collocation methods and their application to a wide class of functional equations. Even though it is centered on the use of collocation, this book also provides an introductory survey on theoretical and practical problems related to several kinds of Volterra Functional Equations and their numerical integration.

The clarity of the exposition, the completeness in the presentation of stated and proved theorems, and the inclusion of a long list of exercises and open problems, along with a wide and exhaustive annotated bibliography, make this monograph a useful and valuable reference book for a wide range of scientists and engineers. In particular, it can be recommended to advanced undergraduate and graduate students in mathematics and may also serve as a source of topics for M.Sc. and Ph.D. theses in this field.

REFERENCES

1. A. Bellen and M. Zennaro, *Numerical methods for delay differential equations*, Oxford University Press, Oxford, 2003.

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