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MULTIPLIERS IN LOCALLY CONVEX *-ALGEBRAS

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ABSTRACT. We consider a *complete locally m-convex *-algebra* with continuous involution, which is also a "*perfect*" *projective limit*, and describe its *multiplier algebra*, under a weaker topology, making it a *locally C^*-algebra*. The same is applied in the case of certain locally convex H^*-algebras.

1. Introduction and preliminaries. Multipliers play an important role in several areas of mathematics where an algebra structure appears (see, e.g., [**2, 3, 14, 25**]; for (non-normed) topological algebras cf., e.g., [**12, 21**]). Due to important applications of non-normed topological $*$-algebras in other fields, we consider the *multiplier algebra* of certain locally convex *-algebras. A brief account on the origins of the theory, as well as further references on the subject of multipliers can be found in [**10**].

The algebras considered throughout are over the complexes **C**. Let E be an algebra. If $(\varnothing \neq) S \subseteq E$, $\mathcal{A}_l(S)$ (respectively, $\mathcal{A}_r(S)$) denotes the left (right) annihilator of S. $\mathcal{A}_l(S)$ (respectively, $\mathcal{A}_r(S)$) is a left (right) ideal of E, which in particular, is a two-sided ideal, if S is a left (right) ideal. An algebra E is called *left* (respectively, *right*) *preannihilator* if $\mathcal{A}_l(E) = (0)$ (respectively, $\mathcal{A}_r(E) = (0)$). If $\mathcal{A}_l(E) = \mathcal{A}_r(E) = (0)$, E is called *preannihilator* (see [**8**, page 149]). A left (right) ideal I of an algebra E is called *essential* in E if $I \cap J \neq (0)$ whenever J is a non-zero left (right) ideal in E.

An involutive locally (m-)convex algebra $(E, (p_\alpha)_{\alpha \in \Lambda})$, for which each p_α, $\alpha \in \Lambda$, is a C^*-seminorm, namely, $p_\alpha(x^*x) = p_\alpha(x)^2$ for every $x \in E$ [**24**, page 1, Definition 1], is called a *locally pre-C^*-algebra*. In the case E is complete, we use the term *locally C^*-algebra* [**13**, page

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198, Definition 2.2]. A *locally m-convex H^*-algebra* is an algebra E equipped with a family $(p_\alpha)_{\alpha\in\Lambda}$ of *Ambrose seminorms* in the sense that p_α, $\alpha \in \Lambda$, arises from a positive semi-definite (pseudo-)inner product $\langle,\rangle_\alpha$, such that the induced topology makes E into a locally m-convex topological algebra. Moreover, the following conditions are satisfied:

For any $x \in E$, there is an $x^* \in E$, such that

$$\begin{aligned}\langle xy, z\rangle_\alpha &= \langle y, x^* z\rangle_\alpha \\ \langle yx, z\rangle_\alpha &= \langle y, zx^*\rangle_\alpha\end{aligned} \tag{1.1}$$

for any $y, z \in E$ and $\alpha \in \Lambda$. x^* is not necessarily unique. In the case E is *proper* (viz., $Ex = (0)$, implies $x = 0$), then x^* is unique and $* : E \to E : x \mapsto x^*$ is an involution (see [**7**, page 451, Definition 1.1 and page 452, Theorem 1.3]).

To fix notation, we recall the following: let $(E, (p_\alpha)_{\alpha\in\Lambda})$ be a complete locally m-convex algebra and

$$\begin{aligned}\rho_\alpha : E \to E/\ker(p_\alpha) \equiv E_\alpha : x \mapsto \rho_\alpha(x) := x + \ker(p_\alpha) \\ \equiv x_\alpha, \quad \alpha \in \Lambda\end{aligned} \tag{1.2}$$

the respective quotient maps. Then, $\|x_\alpha\|_\alpha := p_\alpha(x)$, $x \in E$, $\alpha \in \Lambda$, defines on E_α an algebra norm, so that E_α is a normed algebra and the morphisms ρ_α, $\alpha \in \Lambda$, are continuous. $\widetilde{E}_\alpha$, $\alpha \in \Lambda$, denotes the completion of E_α (with respect to $\|\cdot\|_\alpha$). Λ is endowed with a partial order by putting $\alpha \leq \beta$ if and only if $p_\alpha(x) \leq p_\beta(x)$ for every $x \in E$. Thus, $\ker(p_\beta) \subseteq \ker(p_\alpha)$, and hence the continuous (onto) morphism

$$f_{\alpha\beta} : E_\beta \longrightarrow E_\alpha : x_\beta \mapsto f_{\alpha\beta}(x_\beta) := x_\alpha, \quad \alpha \leq \beta \tag{1.3}$$

is defined. Moreover, $f_{\alpha\beta}$ is extended to a continuous morphism

$$\overline{f}_{\alpha\beta} : \widetilde{E}_\beta \longrightarrow \widetilde{E}_\alpha, \quad \alpha \leq \beta. \tag{1.4}$$

Thus, $(E_\alpha, f_{\alpha\beta})$, $(\widetilde{E}_\alpha, \overline{f}_{\alpha\beta})$, $\alpha, \beta \in \Lambda$, with $\alpha \leq \beta$ are projective systems of normed (respectively, Banach) algebras, so that

$$E \cong \varprojlim E_\alpha \cong \varprojlim \widetilde{E}_\alpha \ (\textit{Arens-Michael decomposition}), \tag{1.5}$$

within topological algebra isomorphisms (cf., for instance, [**19**, page 88, Theorem 3.1 and page 90, Definition 3.1] and/or [**20**, page 20, Theorem 5.1]).

2. Multipliers as a pure algebraic notion. Denote by $L(E)$ the algebra of all linear operators on an algebra E.

Definition 2.1. An element T in $L(E)$ is called a *left* (*right*) *multiplier* on E if $T(xy) = T(x)y$ (respectively, $T(xy) = xT(y)$ for all $x, y \in E$; it is called a *two-sided multiplier* on E if it is both a left and a right multiplier.

It is known that *if E is a proper algebra, then any two-sided multiplier on E is a linear mapping* [**18**].

In the sequel, we use the term *multiplier* in place of a two-sided multiplier. Let us denote by $M_l(E)$ the set of all left multipliers on E, by $M_r(E)$ the set of all right multipliers on E and by $M(E)$ that of all multipliers on E. Note that, by definition, $M(E) = M_l(E) \cap M_r(E)$.

It is easily checked that *$M_l(E)$ is a subalgebra of $L(E)$*. The same holds for $M_r(E)$ and $M(E)$. Now, for $x \in E$, the operator l_x on E given by $l_x(y) = xy$, $y \in E$, is, due to the associativity of E, a left multiplier. Similarly, we can also define the right multiplier attached to $x \in E$, say r_x.

Proposition 2.2. *Let E be a preannihilator algebra. Then the following hold*:

(i) *The mapping*

$$L : E \longrightarrow M_l(E) \text{ given by } x \longmapsto l_x \tag{2.1}$$

defines an algebra monomorphism which identifies E with a subalgebra of $M_l(E)$.

(ii) *E is a left ideal of the algebra $M_l(E)$.*

(iii) *If F is a subalgebra of $M_l(E)$ such that $E \subset F$, then $I \cap E \neq (0)$ for every non-zero right ideal I of F.*

Proof. (i) Since the (partial) left multiplication satisfies the relations $l_{(x+\lambda y)} = l_x + \lambda l_y$ and $l_{xy} = l_x \circ l_y$ for every $x, y \in E$ and $\lambda \in \mathbf{C}$, we get that L is a homomorphism. If $L(x) = 0$, then $l_x(y) = 0$ for all $y \in E$. Namely, $xy = 0$ for all $y \in E$. By hypothesis, $x = 0$ and L is finally a monomorphism.

(ii) Under the identification $x \equiv l_x$, we only have to show that E absorbs multiplication on the left: If $x \in E$ and $T \in M_l(E)$, then, for each $y \in E$, we have $Tl_x(y) = T(xy) = T(x)y = l_{T(x)}(y)$, and therefore $Tl_x = l_{T(x)}$.

(iii) See [**10**, Proposition 2.2]. □

A similar result is also valid for right multipliers; however, (ii) for (two-sided) multipliers is somehow stronger. Namely, the algebra E can then be identified with a two-sided ideal in $M(E)$. Thus, we have:

Corollary 2.3. *Every preannihilator algebra E is an essential two-sided ideal in its multiplier algebra $M(E)$.*

3. The multiplier algebra of a locally m-convex algebra with an approximate identity. We describe the multiplier algebra $M(E)$ in the case where E is a certain complete locally m-convex algebra with an approximate identity. It is shown that $M(E)$ is a subalgebra of $\mathcal{L}(E)$, the algebra of all continuous linear operators in E.

Recall that an *approximate identity* in a topological algebra E is a net $\{e_\delta\}_{\delta\in\Delta}$ such that, for each $x \in E$, we have:

$$(x - xe_\delta) \underset{\delta}{\longrightarrow} 0 \quad \text{and} \quad (x - e_\delta x) \underset{\delta}{\longrightarrow} 0 \text{ for all } x \in E.$$

An algebra with an approximate identity is proper.

Theorem 3.1. *Let $(E, (p_\alpha)_{\alpha\in\Lambda})$ be a complete locally m-convex algebra with an approximate identity $\{e_\delta\}_{\delta\in\Delta}$. Suppose that each factor $E_\alpha = E/\ker p_\alpha$ in the Arens-Michael decomposition of E is complete. Then each (two-sided) multiplier T of E is continuous, viz., $M(E)$ is a subalgebra of $\mathcal{L}(E)$.*

Proof. Let T be an element in $M(E)$, and $\alpha \in \Lambda$. Take $x \in \ker p_a$. For $\varepsilon > 0$, there exists an index $\delta_0 \in \Delta$ such that $p_\alpha(T(x) - T(x)e_\delta) < \varepsilon$ whenever $\delta \geq \delta_0$.

We have:

$$\begin{aligned} p_\alpha(T(x)) &= p_\alpha(T(x - xe_{\delta_0} + xe_{\delta_0})) \\ &= p_\alpha(T(x) - T(xe_{\delta_0}) + T(xe_{\delta_0})) \\ &\leq p_\alpha(T(x) - T(xe_{\delta_0})) + p_\alpha(T(xe_{\delta_0})) \\ &= p_\alpha(T(x) - T(x)e_{\delta_0}) + p_\alpha(xT(e_{\delta_0})) \\ &\leq p_\alpha(T(x) - T(x)e_{\delta_0}) + p_a(x)p_\alpha(T(e_{\delta_0})) < \varepsilon. \end{aligned}$$

Since this is true for an arbitrary $\varepsilon > 0$, we conclude that $p_\alpha(T(x)) = 0$, that is, $T(x) \in \ker p_\alpha$. Then the initial multiplier $T : E \to E$ has projections $T_\alpha : E_\alpha \to E_\alpha$, where $T_\alpha(x + \ker p_\alpha) = T(x) + \ker p_\alpha$; multipliers of the proper normed algebras E_α, which by hypothesis, are Banach algebras for every $\alpha \in \Lambda$.

By definition we have $T_\alpha \circ \varrho_\alpha = \varrho_\alpha \circ T$, where $\varrho_\alpha : E \to E_\alpha$, $\alpha \in \Lambda$, are the canonical quotient maps. Moreover, $f_{\alpha\beta} \circ T_\beta = T_\alpha \circ f_{\alpha\beta}$, for all $\alpha \leq \beta$ in Λ. Here $f_{\alpha\beta}$, $\alpha \leq \beta$, denote the connecting maps (of the projective system; see (1.3)). Namely, $(T_\alpha)_{\alpha\in\Lambda}$ *is a projective system of maps with respect to* $\{(E_\alpha, f_{\alpha\beta})\}$, $\alpha \leq \beta$ in Λ, so that $T = \varprojlim T_\alpha$ (cf., [**4**, page 77, Proposition 1]; see also [**19**, page 89]). Denote by f_α the restrictions of $\pi_\alpha : \Pi_{\alpha\in\Lambda} E_\alpha \to E_\alpha$ to the projective limit $\varprojlim E_\alpha$. Since $f_\alpha \circ \varphi = \varrho_\alpha$, where φ is the topological algebra isomorphism identifying E with $\varprojlim E_\alpha$, we set $f_\alpha = \varrho_\alpha$.

Since multipliers on proper Banach algebras are bounded (equivalently continuous) (see [**17**, page 20, Theorem 1.1.1]), T_α is continuous on E_α. Therefore, $T_\alpha \circ f_\alpha$ is continuous, as well. Since $T_\alpha \circ f_\alpha = f_\alpha \circ T$, for all $\alpha \in \Lambda$, T is continuous. □

We know that every locally C^*-algebra E has an approximate identity (bounded by 1) (see [**13**, page 208, Theorem 2.6; see also its proof]) and each factor E_α is complete. Yet, E is proper, and thus we get the next corollary. The same provides an alternative setting and proof of a result of Weinder (see [**26**] and [**15**, pages 74–75]).

Corollary 3.2. *Let E be a locally C^*-algebra. Then, the algebra of multipliers of E is a subalgebra of the algebra of continuous linear operators on E.*

The proof of Theorem 3.1 suggests the next lemma, cf., also [**4**, page 78, Corollary 1 and subsequent remarks]. We are indebted to the referee whose relevant remarks led us to its formulation.

Lemma 3.3. *Let $E = \varprojlim E_\alpha$ be a projective limit algebra. Then, the respective multiplier algebras yield a projective limit, as well, such that*

$$M(E) = M(\varprojlim E_\alpha) = \varprojlim M(E_\alpha) \quad (\textit{set-theoretically}).$$

4. The multiplier algebra of a locally m-convex H^*-algebra. We present here the relation between the multiplier algebra of a certain locally convex H^*-algebra and the respective ones in the factors of its Arens-Michael decomposition (under a weaker topology than the initial one). To proceed, we use the notion of a perfect projective system as it appeared in [**9**, page 199, Definition 2.7]. To fix notation, we repeat it.

Definition 4.1. A projective system $\{(E_\alpha, f_{\alpha\beta})\}_{\alpha\in\Lambda}$ of topological algebras is called *perfect*, if the restrictions to the projective limit algebra

(4.1)
$$E = \varprojlim E_\alpha = \left\{(x_\alpha) \in \prod_{\alpha\in\Lambda} E_\alpha : f_{\alpha\beta}(x_\beta) = x_\alpha, \quad \text{if } \alpha \leq \beta \text{ in } \Lambda\right\}$$

of the canonical projections $\pi_\alpha : \prod_{\alpha\in\Lambda} E_\alpha \to E_\alpha$, $\alpha \in \Lambda$, namely, the (continuous algebra) morphisms

$$f_\alpha = \pi_\alpha\big|_{E=\varprojlim E_\alpha} : E \longrightarrow E_\alpha, \quad \alpha \in \Lambda, \tag{4.2}$$

are onto maps. The resulting projective limit algebra $E = \varprojlim E_\alpha$ is then called a *perfect* (*topological*) *algebra.*

We note that *every Fréchet locally m-convex algebra* $(E, (p_n)_{n\in\mathbf{N}})$ *gives a perfect projective system of normed algebras, and thus it is a perfect algebra* (see [**9**, page 199, Lemma 2.8] and [**16**, page 229, Theorem 8]).

To proceed, we need some more notation. Suppose E is a locally C^*-algebra. Denote by $(p_\alpha)_{\alpha\in\Lambda}$ the set of all continuous seminorms defining the topology on E. Then, $M_l(E)$ becomes a complete locally m-convex algebra with respect to the family of seminorms $(\widetilde{p}_\alpha)_{\alpha\in\Lambda}$, where

$$\widetilde{p}_\alpha(T_l) = \sup\{p_\alpha(T_l(x)), x \in E \text{ and } p_\alpha(x) \leq 1\}, \quad T_l \in M_l(E). \tag{4.3}$$

(See [**15**, page 75, Theorem 3.5]).

Theorem 4.2. *Let* $(E, (p_\alpha)_{\alpha\in\Lambda})$ *be a complete locally m-convex* **-algebra with continuous involution such that the respective projective* *system is perfect. Moreover, suppose that E can be made into a locally C^*-algebra under a weaker locally convex topology, than the initial one, denoted by $(E, (q_\alpha)_{\alpha\in\Lambda})$. Then,*

$$M_l((E, (q_\alpha)_{\alpha\in\Lambda})) \cong \varprojlim M_l(F_\alpha), \tag{4.4}$$

within an isomorphism, where F_α, $\alpha \in \Lambda$, are the normed factors in the Arens-Michael decomposition of E under the new topology.

Proof. By hypothesis, $(q_\alpha)_{\alpha\in\Lambda}$ is the family of seminorms defining the weaker locally convex topology on E (equivalently, $\ker(p_\alpha) \subseteq \ker(q_\alpha), \alpha \in \Lambda$). According to [**24**, page 2, Theorem 2] this topology is actually a locally m-convex one. Put $F_\alpha \equiv E/\ker(q_\alpha)$. By [**1**, page 32, Theorem 2.4], the factor normed algebras F_α, $\alpha \in \Lambda$, in the Arens-Michael decomposition (under the new topology) are C^*-algebras, and hence $\widetilde{F_\alpha} = F_\alpha$ (see also (1.5)). Take the respective analysis

$$E \cong \varprojlim E_\alpha,$$

with respect to $(p_\alpha)_{\alpha\in\Lambda}$. By hypothesis, each canonical projection map $f_\alpha : \varprojlim E_\alpha \to E_\alpha$ is onto (see (4.2)). Denote by

$$g_\alpha : \varprojlim F_\alpha \longrightarrow F_\alpha$$

the respective projection maps corresponding to the family $(q_\alpha)_{\alpha\in\Lambda}$. Since $\ker(p_\alpha) \subseteq \ker(q_\alpha)$, there is an induced surjective map $\phi_\alpha : E/\ker(p_\alpha) \to E/\ker(q_\alpha)$, given by $\phi_\alpha(x + \ker(p_\alpha)) = x + \ker(q_\alpha)$. It is easily checked that $\phi_\alpha \circ f_\alpha = g_\alpha$ (here, we use the identification $f_\alpha = \varrho_\alpha$; see the proof of Theorem 3.1). Thus g_α is onto as well. The assertion follows now from [**15**, page 75, Theorem 3.5]. See also [**23**, page 178, Theorem 3.14]. □

We proceed by presenting a concrete application of the previous theorem: Let $(E, (p_\alpha)_{\alpha\in\Lambda})$ be a proper complete locally m-convex H^*-algebra. Then, E can be made into a *locally pre-C^*-algebra*, via a family $(q_\alpha)_{\alpha\in\Lambda}$ of C^*-seminorms, given by

$$q_\alpha(x) = \sup\{p_\alpha(xy) : p_\alpha(y) \leq 1\}, \quad \alpha \in \Lambda, \tag{4.5}$$

(*Gel'fand construction*; Mallios's terminology), so that,

$$q_\alpha(x) \leq p_\alpha(x), \quad \text{for every } x \in E,\ \alpha \in \Lambda. \tag{4.6}$$

Thus, *the respective topology on* E *is weaker than the given one.* Moreover,

$$p_\alpha(xy) \leq q_\alpha(x)p_\alpha(y), \quad \text{for every } x, y \in E,\ \alpha \in \Lambda. \tag{4.7}$$

(See [**6**, page 265, Proposition 2.3, Definition 2.1 and the comments before it]).

Corollary 4.3. *Let* $(E, (p_\alpha)_{\alpha\in\Lambda})$ *be a complete locally* m*-convex* H^**-algebra with continuous involution such that the respective projective system be perfect. Moreover, consider on* E *the locally* m*-convex topology, defined by* (4.5)*, and its completion* $\widetilde{E}$*. Then,*

$$M_l(\widetilde{E}) \cong \varprojlim M_l(F_\alpha),$$

within an isomorphism, where F_α*,* $\alpha \in \Lambda$*, are the normed factors in the Arens-Michael decomposition of* $\widetilde{E}$*.*

Proof. Since $*$ is an involution, E is a proper algebra (see [**7**, page 452, Theorem 1.3]). By the comments preceding the statement, $(E, (q_\alpha)_{\alpha\in\Lambda})$ is a locally pre-C^*-algebra. Hence, its completion $\widetilde{E}$ is a locally C^*-algebra, so Theorem 4.2 yields the assertion. We note that the factors of $\widetilde{E}$ are given by $F_\alpha = (\widetilde{E}, \widetilde{q_\alpha})/\ker(\widetilde{q_\alpha})$, where $\widetilde{q_\alpha}$, the extensions of $q_\alpha, \alpha \in \Lambda$, to $\widetilde{E}$. $\square$

The last two statements have a special bearing on relevant results in [**15, 23**].

Let $(K_\alpha)_{\alpha\in\Lambda}$ be a family of two-sided ideals in an algebra A. We recall that A is the "*algebraic*" *direct sum* of the K_α's, and we write $A = \bigoplus_{\alpha\in\Lambda} K_\alpha$, in the case where the sum is direct in the vector space sense (see e.g., [**11**, page 119]) and $K_\alpha K_\beta = (0)$, for all $\alpha \neq \beta$ (see e.g., [**22**, page 328]). *If* A *is preannihilator, every multiplier* T *"respects" the two-sided ideals, viz.,* $T(K_\alpha) \subseteq K_\alpha$*,* $\alpha \in \Lambda$: For normed algebras, cf., [**5**, page 391, Lemma 3]; its proof is entirely algebraic, offered here for completeness; fix $\alpha_0 \in \Lambda$. For $x \in K_{\alpha_0}$, $(Tx)_\alpha$ denotes the projection of Tx into K_α for some $\alpha \neq \alpha_0$. Suppose that $(Tx)_\alpha \neq 0$. In that case, there exists some $0 \neq y \in K_\alpha$ so that $(Tx)y = (Tx)_\alpha y \neq 0$. Otherwise, since A is the direct sum of the K_α's, one gets

$$(Tx)_\alpha A = (Tx)_\alpha \left(\bigoplus_{\alpha\in\Lambda} K_\alpha \right) = (Tx)_\alpha K_\alpha = 0.$$

This yields a contradiction, for A is preannihilator. Therefore, $(Tx)y = (Tx)_\alpha y = 0$, with y, as before. Now, since $\alpha \neq \alpha_0$, we get $Txy = T0 = 0$, while $Txy = (Tx)y \neq 0$, a contradiction, since T is a multiplier. So, finally, we get $(Tx)_\alpha = 0$, for any $\alpha \neq \alpha_0$. This implies the assertion. We intend to be more detailed on the subject elsewhere.

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