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A REFINEMENT OF THE FUNCTION $g(x)$ ON GRIMM'S CONJECTURE

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ABSTRACT. In this paper, we refine the function $g(x)$ on Grimm's conjecture and improve a result of Erdös and Selfridge without using Hall's theorem.

1. Some basic notation. Let $\pi(x)$ be the prime counting function which represents the number of primes not exceeding the real number x. We write $f(x) = O(g(x))$, or equivalently $f(x) \ll g(x)$ when there is a constant C such that $|f(x)| \leq Cg(x)$ for all values of x under consideration. We write $f(x) = o(g(x))$ when $\lim_{x\to\infty} f(x)/g(x) = 0$. We let $[x]$ denote the largest integer not exceeding the real number x.

Let $\binom{m+n}{n}$ be the binomial coefficient. For a prime p and a positive integer $n > 1$, we define $v_p(n)$ to be the largest exponent of p that divides n. In this paper, we always denote by p a prime number. Denote the set of all prime numbers by P, and denote the set of all composite numbers by C. Let $H_n = \{x : x \in C, \text{ for all } p \mid x, p^{v_p(x)} \leq n\}$.

2. Introduction. In 1969, Grimm [**9**] made an important conjecture that if $m+1, \ldots, m+n$ are consecutive composite numbers, then there exist n distinct prime numbers $p_1, \ldots, p_n$ such that $m+j$ is divisible by p_j, $1 \leq j \leq n$. This implies that the product of any n consecutive composite numbers must have at least n distinct prime factors.

Grimm proved the conjecture for two special cases: (i) For all n in the sequence of consecutive composites, $\{n! + i\}$, $i = 2, \ldots, n$. (ii) For all m, when $m > n^{n-1}$. This was improved to $m > n^{\pi(n)}$ by Erdös and

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Selfridge [**7**] who used Hall's theorem [**10**]. Moreover, they obtained the following typical theorem:

Let $v(m,n)$ be the number of distinct prime factors of $\prod_{i=1}^{i=n}(m+i)$, and let $f(m)$ be the largest n for which $v(m,n) \geq n$, then $f(m) < c(m/\log m)^{1/2}$ for all m, where c is a positive absolute constant.

Thus, Grimm's conjecture implies $p_{r+1} - p_r \ll (p_r/\log p_r)^{1/2}$ which is out of bounds for even the Riemann hypothesis [**14**] which implies that $p_{r+1} - p_r \ll p_r^{1/2} \log p_r$, where p_{r+1} and p_r are consecutive primes. It implies particularly that there are primes between n^2 and $(n+1)^2$ for all sufficiently large n, a conjecture which is still open. For the details of its proof, see also [**17**]. Furthermore, by [**7**, Theorem 1], it is not difficult to prove that Grimm's conjecture implies that there are primes between x^2 and for $x^2 + x$ all sufficiently large x and there are primes between $y^2 - y$ and y^2 for all sufficiently large y. Let $x = n$ and $y = n + 1$; then, Grimm's conjecture implies that, for all sufficiently large n, there are two primes between n^2 and $(n+1)^2$ which implies that there are four primes between n^3 and $(n+1)^3$ for all sufficiently large n [**18**].

These surprising consequences motivate the study of the function $g(m)$ which is the largest integer n such that Grimm's conjecture holds for the interval $[m+1, m+n]$. Theorem 1 in [**7**] said that $g(m) = O(\sqrt{m/\log m})$. By using a result of Ramachandra [**15**], Erdös and Pomerance [**6**] pointed out that $g(m) < m^{(1/2)-c}$ for some fixed $c > 0$ and all large m. Thus, an interesting problem of research has been to obtain upper and lower bounds for $g(m)$.

In 1971, Erdös and Selfridge [**7**] proved $g(m) \geq (1 + o(1)) \log m$. In 1975, as an improvement of results of Cijsouw, Tijdeman [**3**] and Ramachandra [**14**], Ramachandra, Shorey and Tijdeman [**16**] obtained an important result which states that $(\log m/\log\log m)^3 \ll g(m)$ by using Gelfond-Baker's theory. This implies that Grimm's conjecture would follow from Cramér's conjecture [**5**] which states that $p_{r+1} - p_r = O((\log p_r)^2)$. In 2006, Laishram and Shorey [**12**] confirmed that Grimm's conjecture is true for $m \leq 1.9 \times 10^{10}$ and for all n by using Mathematica.

The object of this section is to study a stronger function $w(m)$ which is the largest integer n such that the binomial coefficients $\binom{m+n}{n}$

may be written as $\binom{m+n}{n} = \prod_{i=1}^{i=n} a_i, a_i \mid (m+i),\ a_i \in N,\ a_i > 1,$ $(a_i, a_j) = 1,\ 1 \le i \ne j \le n$. By a result of the author [**20**], we see that every binomial coefficient has the representation: $\binom{m+n}{n} = \prod_{i=1}^{i=n} a_i,$ $a_i \mid (m+i),\ a_i \in N,\ (a_i, a_j) = 1,\ 1 \le i \ne j \le n$. Naturally, we want to know what condition m satisfies so that each $a_i > 1$ since it will imply that $\binom{m+n}{n}$ must have at least n distinct prime factors in this case. In this paper, we will prove the following theorem without using Hall's theorem:

Theorem 1. *When $m > \prod_{p \le n} p^{[\log_p n]}$, the binomial coefficient $\binom{m+n}{n}$ has the representation:*

$$\binom{m+n}{n} = \prod_{i=1}^{n} a_i,\ a_i \mid (m+i),\ a_i \in N,\ a_i > 1,\ (a_i, a_j) = 1,$$
$$1 \le i \ne j \le n.$$

Note that $w(m) \le g(m)$, $n^{\pi(n)} > \prod_{p \le n} p^{[\log_p n]}$, and it is easy to show by [**19**] that $\pi(n) \log n \ge n$ when $n \ge 17$. Hence ,we have $\log m \ll w(m) \ll \sqrt{m/\log m}$. So, by this lower bound, we have obtained an analogical result of Theorem 3 in [**7**].

In Section 3, we will give the proof of Theorem 1. In Section 4, we will try to point out that it is not easy to improve the lower bound of $w(m)$ to $(\log m)^2$.

3. Proof of Theorem 1. To prove the theorem, we need some lemmas which reflect that the binomial coefficients have some fascinating and remarkable arithmetic properties again.

Lemma 1. *Let $\binom{m+n}{n}$ be the binomial coefficient with $m, n \in N$. Then, for any prime p, $v_p\left(\binom{m+n}{n}\right) \le t$, where $t = \max_{1 \le i \le n}\{v_p(m+i)\}$.*

Proof. For the proof of Lemma 1, see [**20**].

Lemma 2. *If $m + i \notin H_n$ for some $1 \le i \le n$, then there is a prime number p such that $p^{v_p(m+i)} > n$. Moreover, $1 \le v_p\left(\binom{m+n}{n}\right) \le v_p(m+i)$.*

Proof. By our assumption and the definition of H_n, clearly, there is a prime p such that $p^{v_p(m+i)} > n$. Note that $v_p(m+j) < v_p(m+i)$ when $j \neq i$. Otherwise, $p^{v_p(m+i)} | (j-i)$. But $|j-i| < n$, which is impossible. Thus, $v_p(m+i) = \max_{1\le j\le n}\{v_p(m+j)\}$ and $v_p\left(\binom{m+n}{n}\right) \le v_p(m+i)$ by Lemma 1. On the other hand, if we write $m = p^{v_p(m+i)}x + y$, where $x, y \in N \cup \{0\}$ with $0 \le y < p^{v_p(m+i)}$, then we have $p^{v_p(m+i)} | (y+i)$ and $[(m+n)/p^{v_p(m+i)}] - [m/p^{v_p(m+i)}] = 1$ since $[n/p^{v_p(m+i)}] = 0$ and $n + y \ge i + y$. Hence, $\sum_{j=1}^{\infty}([(m+n)/p^j] - [m/p^j] - [n/p^j]) \ge 1$, and $1 \le v_p(\binom{m+n}{n})$. This completes the proof of Lemma 2. □

It is worthwhile pointing out that H_n has many interesting properties which should be given further consideration. For example, H_2 is an empty set, $H_3 = \{6\}$, $H_4 = \{4, 6, 12\}$ and so on. $H_n \subseteq H_{n+1}$; $|H_n| = \prod_{p\le n}(1 + [\log_p n]) - 1 - \pi(n)$; $H_n \subseteq \psi(x, n)$, where $\psi(x, n)$ is the set of n-smooth integers in $[1, x]$ with $x = \prod_{p\le n} p^{[\log_p n]}$. For some details on smooth integers which relate to factorization of integers, see [**4**]. For applications of smooth numbers to various problems in different areas of number theory, see [**2, 8, 11, 13**].

Lemma 3. *If $m + i \notin H_n$ for every $1 \le i \le n$, then $\binom{m+n}{n}$ has the representation:*

$$\binom{m+n}{n} = \prod_{i=1}^{n} a_i,\ \ a_i \mid (m+i),\ \ a_i \in N,\ \ a_i > 1,\ \ (a_i, a_j) = 1,$$
$$1 \le i \neq j \le n.$$

Proof. Write $B = \binom{m+n}{n} = \prod_{i=1}^{i=k} p_i^{e_i}$ by the fundamental theorem of arithmetic, where k is the number of distinct prime factors of B. Clearly, for every prime factor p_i of B, there must be a number i_j with $1 \le i_j \le n$, such that $v_{p_i}(m+i_j) = \max_{1\le r\le n}\{v_{p_i}(m+r)\}$. By Lemma 1, we have $e_i \le v_{p_i}(m+i_j)$ for every $1 \le i \le n$. Therefore, we can choose the number $m + i_j$ such that

$$\frac{(m+1)\cdots(m+n)}{n!}$$
$$= \frac{\frac{m+1}{p_i^{v_{p_i}(m+1)}}\cdots\frac{m+i_j-1}{p_i^{v_{p_i}(m+i_j-1)}}\frac{m+i_j}{p_i^{v_{p_i}(m+i_j)-e_i}}\frac{m+i_j+1}{p_i^{v_{p_i}(m+i_j+1)}}\cdots\frac{m+n}{p_i^{v_{p_i}(m+n)}}}{n!/p_i^{v_{p_i}(n!)}}.$$

Notice that $(m+1)/p_i^{v_{p_i}(m+1)}, \ldots, (m+i_j-1)/p_i^{v_{p_i}(m+i_j-1)}, (m+i_j)/p_i^{v_{p_i}(m+i_j)-e_i}, (m+i_j+1)/p_i^{v_{p_i}(m+i_j+1)}, \ldots, (m+n)/p_i^{v_{p_i}(m+n)}, n!/p_i^{v_{p_i}(n!)}$ are all integers. Thus, by reduction, B has the representation:

$$B = \prod_{i=1}^{n} a_i, \ a_i \mid (m+i), \ a_i \in N, \ (a_i, a_j) = 1, \quad 1 \le i \ne j \le n.$$

On the other hand, since $m + i \notin H_n$ for every $1 \le i \le n$, there is a prime number p_i such that $p_i^{v_{p_i}(m+i)} > n$. By Lemma 2, we have $v_{p_i}(m+i) = \max_{1 \le r \le n} v_{p_i}(m+r)$ and $p_i \mid a_i$. Note that prime numbers $p_1, \ldots, p_n$ are distinct. This completes the proof of Lemma 3.

Proof of Theorem 1. For every $1 \le i \le n$, we write $m + i = (\prod_{p^{v_p(m+i)} > n} p^{v_p(m+i)})(\prod_{p^{v_p(m+i)} \le n} p^{v_p(m+i)})$. If $m + i \in H_n$ for some $1 \le i \le n$, then we have $m + i = \prod_{p^{v_p(m+i)} \le n} p^{v_p(m+i)} \le \prod_{p^{v_p(m+i)} \le n} p^{[\log_p n]}$. It is a contradiction by our assumption. Therefore, $m+i \notin H_n$ for all $1 \le i \le n$, and by Lemma 3, we get the assertion of Theorem 1.

Corollary 1. *If* $2 \le n \le 7$ *and* $m+1, \ldots, m+n$ *are consecutive composite numbers, then* $\binom{m+n}{n}$ *can be written as*

$$\binom{m+n}{n} = \prod_{i=1}^{n} a_i, \ a_i \mid (m+i), \ a_i \in N, \ a_i > 1, \ (a_i, a_j) = 1,$$
$$1 \le i \ne j \le n.$$

Namely, the conjecture in [**20**] is true when $2 \le n \le 7$.

Proof. Corollary 1 holds when $2 \le n \le 6$ by computations since $m \le 2^2 \times 3 \times 5 = 60$ by Theorem 1. Next, we consider the case of $n = 7$. By Theorem 1, when $m > 2^2 \times 3 \times 5 \times 7 = 420$, Corollary 1 holds. When $m \le 420$, by the table of prime numbers, the set of 7 consecutive composite numbers must be a subset of one of the following 13 sets:

$\{90, 91, 92, 93, 94, 95, 96\}$,
$\{114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126\}$,
$\{182, 183, 184, 185, 186, 187, 188, 189, 190\}$,

$$\{200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210\},$$
$$\{212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222\},$$
$$\{242, 243, 244, 245, 246, 247, 248, 249, 250\},$$
$$\{284, 285, 286, 287, 288, 289, 290, 291, 292\},$$
$$\{318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330\},$$
$$\{338, 339, 340, 341, 342, 343, 344, 345, 346\},$$
$$\{360, 361, 362, 363, 364, 365, 366\},$$
$$\{390, 391, 392, 393, 394.395, 396\},$$
$$\{402, 403, 404, 405, 406, 407, 408\},$$
$$\{410, 411, 412, 413, 414, 415, 416, 417, 418\}.$$

We have $204 \times 205 \times 206 \times 207 \times 208 \times 209 \times 210/1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 = 17 \times 41 \times 103 \times 207 \times 208 \times 209 \times 5$. The remaining cases satisfy $m + i \notin H_7$ for every $1 \leq i \leq 7$ when $n = 7$. By Lemma 3, Corollary 1 holds.

4. Remarks. In this section, we will try to explain that it is not easy to improve the lower bound of $w(m)$ to $(\log m)^2$. On one hand, we found two exceptions to the conjecture in [**20**] when $m = 116$, $n = 10$ or $m = 118$, $n = 8$. This implies that the conjecture in [**20**] is not always true. On the other hand, if the lower bound of $w(m)$ can be improved to $(\log m)^2$, then assuming Cramér's conjecture, there are only finitely many exceptions to the conjecture in [**20**]. which implies Grimm's conjecture (note that Grimm's conjecture follows from Cramér's conjecture). Namely, one would guess the following:

Conjecture 1. *For every sufficiently large integer n, the product of n consecutive composite numbers $m+1, \ldots, m+n$ may be written as*

$$\prod_{i=1}^{n}(m+i) = n! \times \prod_{i=1}^{n} a_i,\ a_i \mid (m+i),\ a_i \in N,\ a_i > 1,\ (a_i, a_j) = 1,$$
$$1 \leq i \neq j \leq n.$$

Obviously, if Conjecture 1 holds, then it also implies a surprising consequence that there are only finitely many exceptions to the following Conjecture 2.

Conjecture 2. (i) *For every $n > 1$, there are primes in the intervals $(d-n, d+n]$ and $(d-q, d+q)$, respectively, where $d > 1$ is a factor of $\prod_{n<p<2n} p$, and q is the least prime factor of d.*

(ii) *For every $n > 2$, let $d > 1$ be a factor of $\prod_{n/2<p<n} p$. If $d = nt+r$ and d is coprime to each of $nt+1, \ldots, nt+r-1, nt+r+1, \ldots, nt+n$, then there are primes in the interval $[nt+1, nt+n]$.*

Proof. (i) By the Bertrand-Chebyshev theorem which states that there exists a prime in interval $(n, 2n)$ when $n > 1$, we see that $\prod_{n<p<2n} p$ has a prime factor $> n$. Let $d > 1$ be a factor of $\prod_{n<p<2n} p$. If there are no primes in the interval $(d-n, d+n]$, then $d-n+1, \ldots, d-n+2n$ are all composite numbers. By the assumption that Conjecture 1 holds, we have that $\prod_{i=1}^{i=2n}(d-n+i) = (2n)! \times \prod_{i=1}^{i=2n} a_i$ with $a_i|(d-n+i)$, $a_i \in N$, $a_i > 1$, $(a_i, a_j) = 1$, $1 \le i \ne j \le 2n$ for a sufficiently large integer n. But this is impossible since we obtain $a_n = 1$ from $v_p((2n)!) = 1$ and $v_p((d-n+1)\cdots(d-n+2n)) = 1$.

Let q be the least prime factor of d. If d is prime, then $q = d$. Clearly, there are primes, for example q, in the interval $(d-q, d+q) = (0, 2q)$. Now, we assume that d is not prime. If there are no primes in the interval $(d-q, d+q)$, then $d-q+1, \ldots, d-q+2q-1$ are all composite numbers. Conjecture 1 holds, so we have that $\prod_{i=1}^{i=2q-1}(d-q+i) = (2q-1)! \times \prod_{i=1}^{i=2q-1} a_i$ with $a_i \mid (d-q+i)$, $a_i \in N$, $a_i > 1$, $(a_i, a_j) = 1$, $1 \le i \ne j \le 2q-1$ for a sufficiently large integer n. Note that, for any prime divisor r of d/q, $r > q$ since $d > 1$ is a factor of $\prod_{n<p<2n} p$ and is square-free. Notice also that $r < 2q$ since $q > n$ and $r < 2n$. Therefore, $a_q = 1$ since $v_r((2q-1)!) = 1$ and $v_r((d-q+1)\cdots(d-q+2q-1)) = 1$. This contradiction shows that case (i) holds.

Similarly, one could deduce case (ii) in Conjecture 2 assuming Conjecture 1. In fact, if there are no primes in the interval $[nt+1, nt+n]$, then by the assumption that Conjecture 1 holds, we have that $\prod_{i=1}^{i=n}(nt+i) = n! \times \prod_{i=1}^{i=n} a_i$ with $a_i \mid (nt+i)$, $a_i \in N$, $a_i > 1$, $(a_i, a_j) = 1$, $1 \le i \ne j \le n$ for every sufficiently large integer n. However, by our assumption and the known conditions, for any prime divisor p of $d = nt+r$, we have $v_p(n!) = 1$ and $v_p((nt+1)\cdots(nt+n)) = 1$ since $d = nt+r$ is square-free and coprime to each of $nt+1, \ldots, nt+r-1, nt+r+1, \ldots, nt+n$. Therefore, $a_r = 1$ since $v_p(\binom{nt+n}{n}) = 0$, which is a contradiction.

Conjecture 2 also implies that, for every $n > 1$, there are primes in the interval $(d-q, d+q)$, where $d = \prod_{n<p<2n} p$, and q is the least prime factor of d. Of course, if d is prime, then there are primes in the interval $(d-q, d+q) = (0, 2q)$. When d is not prime, or equivalently, when $n = 4$ or $n > 5$ (by the refined Bertrand-Chebyshev theorem which states that there exist at least two distinct primes in interval $(n, 2n)$ when $n = 4$ or $n > 5$), we have $d - q \neq 0$. Moreover, by the Chebyshev theorem which states that $0.92129x/\log x < \pi(x) < 1.1056x/\log x$, we have that $2n < 2q < 4n < (((0.92 \times 2n/\log 2n) - (1.11n/\log n)) \log n)^2 < ((\pi(2n) - \pi(n)) \log q)^2 < (\log(\prod_{n<p<2n} p - q))^2$ for every sufficiently large integer n. Thus, $2q < (\log(d - q))^2$ which is out of bounds for even Cramér's conjecture, and it is hard to prove that there are primes in the interval $(d - q, d + q)$, say nothing of Conjecture 1. Therefore, we think that it is not easy to improve the lower bound of $w(m)$ to $(\log m)^2$. Is Conjecture 1 true?

Based on Conjecture 2, one could obtain some interesting results. For instance, below is a fast algorithm for generating large primes by using small known primes.

An algorithm for generating an m-bit prime number.

Input: A natural number m

Step 1: Choose $p_1 < \cdots < p_r < 2p_1$ such that r is appropriately large and p_i are all primes (not necessarily consecutive), for $1 \leq i \leq r$ and $2^{m-1} < k = \prod_{i=1}^{i=r} p_i < 2^m$.

This step can be finished easily by pre-computing.

Step 2: Test whether each of $k \pm 2, \ldots, k \pm 2[p_1/2]$ is prime. If, for some i, $k + i$ or $k - i$ is prime, terminate the algorithm. Otherwise, Conjecture 2 does not hold for $n = 2[p_1/2] + 1$.

Such a prime can be found quickly when r is appropriately large. On one hand, primality testing is comparatively easy since its running time is polynomial [**1**]. On the other hand, since k has many prime divisors, there is a high probability that either $k + 2i$ or $k - 2i$ is prime for some $1 \leq i \leq [p_1/2] < p_1/2 < 2^{(m/r)-1}$.

Output: An m-bit prime number. Due to the fact that it lies outside the scope of this paper, we omitted more details of this algorithm. As a toy example, we can generate a 32-bit prime using small primes 29, 31, 37, 41, 43, 47. Note that $2^{31} < 29 \times 31 \times 37 \times 41 \times 43 \times 47 < 2^{32}$. By an exhaustive search, or a simple sieve, we find that $29 \times 31 \times 37 \times 41 \times 43 \times 47 + 6 = 2756205449$ is a 32-bit prime. Unfortunately, we do not know whether Conjecture 2 is true or not, although we can test whether Conjecture 2 holds by the aforementioned algorithm. Moreover, we have not been able to work out a complete proof of Conjecture 2, still less Conjecture 1. But, all of these and related questions, specially, the lower bound of $w(m)$, we hope to investigate.

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