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PERMANENCE IN MULTI-SPECIES COMPETITIVE SYSTEMS WITH DELAYS AND FEEDBACK CONTROLS

LINFEI NIE, JIGEN PENG AND ZHIDONG TENG

ABSTRACT. In this paper, we consider whether or not the feedback controls have influence on a multi-species Kolmogorov type competitive system with delays. The general criteria of integrable form on the ultimate boundedness and permanence are established. When these results are applied to some population models, some new results can be obtained, and some known results also can be generalized and easily verified.

1. Introduction. In this paper, we consider the following nonautonomous n-species Kolmogorov type competitive systems of functional differential equations with finite delays and feedback controls:

$$
\begin{aligned}
\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} &= x_i(t)f_i(t, x(t), x_t, u_i(t), u_{it}) \\
\frac{\mathrm{d}u_i(t)}{\mathrm{d}t} &= -e_i(t)u_i(t) + g_i(t, x_i(t), x_{it}),
\end{aligned}
\tag{1.1}
$$

where $i = 1, 2, \ldots, n$, $t \in R$, $x_i(t)$ is the density of competitive species, $u_i(t)$ is the control variable, see [**27, 29**], $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))$, $x_t(s) = x(t+s)$ and $u_{it}(s) = u_i(t+s)$ for all $s \in [-\tau, 0]$, $\tau \geq 0$ is a constant.

In the theory of mathematical biology, system (1.1) is a very important mathematical model which describes population dynamics of the multi-species in a time-fluctuating environment and the effects of time delays, and has been extensively investigated in literature as biomathematics models. This system contains many bio-mathematics

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models of delay differential equations with feedback controls, for example, the following well-known multi-species systems with feedback controls and finite delays.

(1) Nonautonomous n-species competitive Lotka-Volterra system with delays and feedback controls (see [**13, 27**] and references cited therein):

(1.2)
$$\begin{aligned}\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} &= \Big[r_i(t) - a_{ii}(t)x_i(t) - \sum_{\substack{j=1\\ j\neq i}}^{n} a_{ij}(t)\int_0^{\omega} K_{ij}(s)x_j(t-s)\,\mathrm{d}s \\ &\qquad - \alpha_i(t)\int_0^{\omega} H_i(s)u_i(t-s)\,\mathrm{d}s\Big] \\ \frac{\mathrm{d}u_i(t)}{\mathrm{d}t} &= -\eta_i(t)u_i(t) + a_i(t)\int_0^{\omega} K_{ij}(s)x_j(t-s)\,\mathrm{d}s.\end{aligned}$$

(2) Nonautonomous food limited Michaelis-Menton system with feedback controls and delays (see [**10**] and references cited therein):

(1.3)
$$\begin{aligned}\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} &= r_i(t)x_i(t)\Big[1 - \sum_{j=1}^{n}\frac{a_i(t)x_i(t)}{b_i(t)+c_i(t)x_i(t)} - d_i(t)u_i(t-\tau_i(t))\Big] \\ \frac{\mathrm{d}u_i(t)}{\mathrm{d}t} &= -\eta_i(t)u(t) + e_i(t)x_i(t-\sigma_i(t)).\end{aligned}$$

(3) Nonautonomous Allee-effect system with delays and feedback controls (see [**4, 5**] and references cited therein):

(1.4)
$$\begin{aligned}\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} &= x_i(t)\Big[r_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j^{\alpha_{ij}}(t) - \sum_{j=1}^{n}\sum_{l=1}^{m} b_{ijl}(t)x_j^{\beta_{ij}}(t-\tau_{ijl}) \\ &\qquad - \sum_{j=1}^{n}\int_{-\tau_{ij}}^{0} c_{ij}(t,s)x_j^{\gamma_{ij}}(t+s)\,\mathrm{d}s - d_i(t)u_i(t) \\ &\qquad - e_i(t)\int_{-\tau_i}^{0} H_i(t,s)u_i(t+s)\,\mathrm{d}s\Big] \\ \frac{\mathrm{d}u_i(t)}{\mathrm{d}t} &= f_i(t) - g_i(t)u_i(t) + h_i(t)\int_{-\eta_i}^{0} K_i(t,s)x_i^{\vartheta_i}(t+s)\,\mathrm{d}s.\end{aligned}$$

As is known, ecosystem in the real world is continuously distributed by unpredictable forces which can result in changes in the biological parameters such as survival rates. In ecology, we know that the practical interest question is whether or not an ecosystem can withstand those unpredictable disturbances which persist for a finite period of time. In the language of control variables, we call the disturbance functions *control variables*, whereas, the control variables discussed in most literature are constants or time dependent [**10**–**12**].

Recently, we have seen that some special population equations with delays and feedback controls are studied in [**2**–**5, 7, 9, 13, 15, 17, 18, 27**–**29**]. In particular, Li and Zhu [**13**] investigated the existence and nonexistence of positive periodic solutions of an infinite delay functional differential system with parameter and feedback controls. Weng [**27**] considered a class of periodic integrodifferential systems with feedback controls and established sufficient conditions for the existence and global stability of a positive periodic solution. Chen [**3**] considered a periodic multi-species Kolmogorov type competitive system with delays and feedback controls, and established sufficient conditions for the existence of the positive periodic solution. Xia [**28**] considered an almost periodic n-species competitive system with feedback controls and established sufficient conditions for the existence of a unique almost periodic solution. However, we see that for general nonautonomous n-species Kolmogorov type competitive systems of functional differential equations with finite delays and feedback controls (1.1), until now there has not been any work on permanence of positive solutions. On the other hand, we also note that few authors consider whether or not feedback controls have influence on the permanence of system (1.1).

Motivated by the above questions, we study the permanence of positive solutions for general n-species Kolmogorov type competitive systems with delays and feedback controls, and establish the general criteria of integrable form on the ultimate boundedness and permanence of all positive solutions. This paper is organized as follows. In the next section, as preliminaries some useful lemmas are presented. We will state and prove sufficient conditions on the ultimately bounded and permanence of any positive solutions for system (1.1) in Section 3. In the last section, we deduce criteria to some well-known special cases of system (1.1) to illustrate the generality of our results.

2. Preliminaries. Let τ be a nonnegative constant. For any positive integer m, we denote by C^m the Banach space of bounded continuous functions $\phi : [-\tau,\ 0] \to R^m$ with the supremum norm defined by

$$\|\phi\|_c = \sup_{-\tau \le s \le 0} |\phi(s)|,$$

where $\phi = (\phi_1, \dots, \phi_m)$ and $|\phi(s)| = \sum_{i=1}^n |\phi_i(s)|$. Let $R_+^m = \{x = (x_1, \dots, x_m) \in R^m : x_i > 0, i = 1, 2, \dots, m\}$ and $C_+^m = \{\phi = (\phi_1, \dots, \phi_m) \in C^m : \phi_i(s) \ge 0$, for all $s \in [-\tau,\ 0]$ and $\phi_i(0) > 0$, for $i = 1, \dots, m\}$. For any $\psi_1 = (\psi_{11}(t), \psi_{12}(t), \dots, \psi_{1m}(t))$, $\psi_2 = (\psi_{21}(t), \psi_{22}(t), \dots, \psi_{2n}(t)) \in C_+^m$, $\psi_1 \le \psi_2$ denotes that $\psi_{1i}(t) \le \psi_{2i}(t)$ for any $t \in R_{+0}$ and $i = 1, 2, \dots, m$. When $m = 1$ and $m = n$, we have the definition of C_+^1, C_+^n, R_+^n and R_+, respectively. Particularly, let $R_{+0} = [0,\ \infty)$. For any point $x \in R_+^n$ we will use $\hat{x}$ to denote the constant function $\phi(s) \equiv x$ for all $s \in [-\tau,\ 0]$; for any point $y \in R$ we will use y^* to denote the constant function $\psi(s) \equiv y$ for all $s \in [-\tau,\ 0]$, where $\psi \in C_+^1$.

As usual, if $x(t) : [-\tau,\ \alpha) \to R_+^n$ is a continuous function, $\alpha \ge 0$ and $t \in [0,\ \alpha)$, then $x_t(s)$ is defined by $x_t(s) = x(t+s)$ for all $s \in [-\tau,\ 0]$.

Firstly, we consider the following single-species nonautonomous Kolmogorov system

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} = y(t) f(t, y(t)), \tag{2.1}$$

where $f(t, y)$ is a continuous function defined on $(t, y) \in R_{+0} \times R_+$. We assume that for any $(t_0, y_0) \in R_{+0} \times R_+$, system (2.1) has a unique solution $y(t)$ satisfying $y(t_0) = y_0$. If $y(t) > 0$ on the interval of existence, then $y(t)$ is said to be a positive solution. It is easy to prove that for all $t \ge t_0$, $y(t) > 0$ if the initial value $y_0 > 0$ and $y(t) \ge 0$ if the initial value $y_0 \ge 0$. For system (2.1) we introduce the following assumptions.

($\mathbf{A}_1$) For any constant $\sigma > 1$, $f(t, y)$ is bounded on $R_{+0} \times [0,\ \sigma]$.

($\mathbf{A}_2$) There are positive constants k_1, ω_1, such that

$$\limsup_{t \to \infty} \int_t^{t+\omega_1} f(\tau, k_1)\, \mathrm{d}\tau < 0.$$

($\mathbf{A}_3$) Partial derivative $\partial f(t,y)/\partial y$ exists for all $(t,y) \in R_{+0} \times R_+$, and there is a nonnegative continuous function $q(t)$ and a constant $\omega_2 > 0$ which satisfy $\liminf_{t\to\infty} \int_t^{t+\omega_2} q(\tau)\,\mathrm{d}\tau > 0$, and a continuous function $p(y)$, which satisfies $p(y) > 0$ for all $u \in R_+$, such that

$$\frac{\partial f(t,y)}{\partial y} \leq -q(t)p(y) \quad \text{for all } (t,y) \in R_{+0} \times R_+.$$

We notice that assumptions (A_1)–(A_3) are quite weak and can be satisfied for wide classes of ecologically reasonable functions.

Let $y^*(t)$ be a fixed positive solution of system (2.1) defined on R_{+0}. We say that $y^*(t)$ is globally uniformly attractive on R_{+0}, if for any constants $\eta > 1$ and $\varepsilon > 0$ there is a constant $T = T(\eta,\varepsilon) > 0$ such that for any initial time $t_0 \in R_{+0}$ and any solution $y(t)$ of system (2.2) with $y(t_0) \in [\eta^{-1}, \eta]$, one has $|y(t) - y^*(t)| < \varepsilon$ for all $t \geq t_0 + T$. By Lemma 1 given in [**23**], we have the following result.

Lemma 2.1. *Suppose that assumptions* (A_1)–(A_3) *hold. Then*

(a) *there is a constant* $M > 0$ *such that*

$$\limsup_{t\to\infty}\ y(t) \leq M$$

for any positive solution $y(t)$ *of system* (2.1).

(b) *each fixed positive solution* $y^*(t)$ *of system* (2.1) *is globally uniformly attractive on* R_{+0}.

Next, let us consider the following first order differential equations with a parameter:

$$\frac{\mathrm{d}v(t)}{\mathrm{d}t} = g(t,\beta) - b(t)v(t), \tag{2.2}$$

where $g(t,\beta)$ is a continuous function defined on $(t,\beta) \in R_{+0} \times [0,\ \beta_0]$, β_0 is a constant, and $b(t)$ is a continuous function defined on R. For system (2.2) we introduce the following assumptions.

($\mathbf{B}_1$) Function $g(t,\beta)$ is a nonnegative bounded on $R_{+0} \times [0,\ \beta_0]$ and satisfies the Lipschitz condition with $\beta \in [0,\ \beta_0]$, i.e., there is a constant

$L = L(\beta_0) > 0$ such that $|g(t, \beta_1) - g(t, \beta_2)| \le L|\beta_1 - \beta_2|$ for all $t \in R$ and $\beta_1,\ \beta_2 \in [0,\ \beta_0]$.

($\mathbf{B}_2$) Function $b(t)$ is nonnegative bounded on R_{+0} and there is a constant $\omega_3 > 0$ such that

$$\liminf_{t\to\infty} \int_t^{t+\omega_3} b(s)\,\mathrm{d}s > 0.$$

In system (2.2), when parameter $\beta = 0$ we obtain the following system

$$\frac{\mathrm{d}v(t)}{\mathrm{d}t} = g(t, 0) - b(t)v(t). \tag{2.3}$$

By Lemma 3 given in [**23**], we have the following result.

Lemma 2.2. *Suppose that the conditions* (B_1) *and* (B_2) *hold. Then*

(a) *there is a constant* $M > 0$ *such that*

$$\limsup_{t\to\infty} v(t) \le M$$

for any positive solution $v(t)$ *of system* (2.3).

(b) *If there is a constant* $\omega_4 > 0$ *such that*

$$\liminf_{t\to\infty} \int_t^{t+\omega_4} g(s, 0)\,\mathrm{d}s > 0,$$

then there is a constant $\eta > 1$ *such that*

$$\eta^{-1} \le \liminf_{t\to\infty} v(t) \le \limsup_{t\to\infty} v(t) \le \eta$$

for any positive solution $v_\beta(t)$ *of system* (2.3).

(c) *Each fixed positive solution* $v^*(t)$ *of system* (2.3) *is globally uniformly attractive on* R_{+0}.

Let $v_0 \in R_+$, $t_0 \in R_{+0}$ and $\beta \in [0,\ \beta_0]$, and further let $v_\beta(t)$ and $v_0(t)$ be solutions of systems (2.2) and (2.3) with initial values $v_\beta(t_0) = v_0$

and $v_0(t_0) = v_0$, respectively. By Lemma 2.2 given in [**17**], we further have the following result.

Lemma 2.3. *Suppose that assumptions* (A_1) *and* (A_2) *hold. Then* $v_\beta(t)$ *converges to* $v_0(t)$ *uniformly for* $t \in [t_0, \infty)$ *as* $\beta \to 0$.

Remark 2.1. In system (2.3), if function $g(t, 0) \equiv 0$, then system (2.3) has a trivial equilibrium $E = 0$, and E is globally asymptotically stable. For any constant $\Gamma > 0$ and $t_0 \in R_{+0}$, let $\beta \in [0, \beta_0]$ and $v_\beta(t)$ be the positive solutions of system (2.2) with initial value $v_\beta(t_0) \in [0, \Gamma]$. By Lemmas 2.2 and 2.3, we further have the following result: the solution $v_\beta(t)$ converges to 0, as $\beta \to 0$ and $t \to \infty$, i.e., for any $\varepsilon > 0$, there are positive constants $T = T(\varepsilon, \Gamma)$ and $\delta = \delta(\varepsilon)$ such that $v_\beta(t) < \varepsilon$ for all $t \geq t_0 + T$ and $\beta < \delta$.

In order to show the convenience of the statement in the beginning of this paper, we introduce the following definition on persistence.

Definition 2.1. System (1.1) is said to be persistent if there are positive constants m and M such that

$$m \leq \liminf_{t\to\infty} x_i(t) \leq \limsup_{t\to\infty} x_i(t) \leq M, \quad i = 1, 2, \ldots, n,$$

for any positive solution $(x_1(t), x_2(t), \ldots, x_n(t), u_1(t), u_2(t), \ldots, u_n(t))$ of system (1.1).

Remark 2.2. In system (1.1), $u_i(t)$, $i = 1, 2, \ldots, n$, is a control variable, so we do not consider the permanence of control variables.

Main results. In system (1.1), we introduce the following assumptions.

$(\mathbf{H}_1)$ For each $1 \leq i \leq n$, the function $f_i(t, x(t), x_t, u_i(t), u_{it})$ satisfies the following conditions.

(1) For any constant $K > 0$, the function $f_i(t, x, \phi, u_i, \psi_i)$ satisfies

$$\sup\big\{|f_i(t, x, \phi, u_i, \psi_i)| : \ (t, x, \phi, u_i, \psi_i) \in R_{+0} \times R_+^n \times C_+^n \times R_+ \times C_+^1, \\ |x_j| \leq K, \ \|\phi_j\|_c \leq K, \ |u_i| \leq K, \ \|\psi_i\|_c \leq K, \ j = 1, 2, \ldots, n.\big\} < \infty.$$

(2) The function $f_i(t, x, \phi, u_i, \psi_i)$ is decreasing with respect to $(x, \phi, u_i, \psi_i) \in R_+^n \times C_+^n \times R_+ \times C_+^1$, i.e., for any $(x_1, \phi_1, u_{i1}, \psi_{i1})$, $(x_2, \phi_2, u_{i2}, \psi_{i2}) \in R_+^n \times C_+^n \times R_+ \times C_+^1$, if $x_1 \le x_2$, $\phi_1 \le \phi_2$, $u_{i1} \le u_{i2}$, and $\psi_{i1} \le \psi_{i2}$. then $f_i(t, x_1, \phi_1, u_{i1}, \psi_{i1}) \ge f_i(t, x_2, \phi_2, u_{i2}, \psi_{i2})$ for all $t \in R$.

(3) The partial derivative $\partial f_i(t, 0, \dots, 0, x_i, 0, \dots, \hat{0}, 0, 0^*)/\partial x_i$ exists for any $x_i \in R_+$, and there exist a nonnegative continuous function $q_i(t)$ and a constant $\vartheta_i > 0$, which satisfy $\liminf_{t\to\infty} \int_t^{t+\vartheta_i} q_i(\tau)\,\mathrm{d}\tau > 0$, and a continuous function $p_i(x_i)$, which satisfy $p_i(x_i) > 0$ for all $x_i \in R_+$, such that

$$\frac{\partial f_i(t, 0, \dots, 0, x_i, 0, \dots, 0, \hat{0}, 0, 0^*)}{\partial x_i} \le -q_i(t)p_i(x_i)$$

for all $(t, x_i) \in R \times R_+$.

(4) There are positive constants k_i and ν_i such that

$$\limsup_{t\to\infty} \int_t^{t+\nu_i} f_i(s, 0, \dots, 0, k_i, 0, \dots, 0, \hat{0}, 0, 0^*)\,\mathrm{d}s < 0.$$

($\mathbf{H}_2$) For each $1 \le i \le n$, the function $g_i(t, x_i, \phi_i)$ satisfies the following conditions.

(1) For any constant $G > 0$, satisfies

$$\sup\{g_i(t, x_i, \psi_i) : (t, x_i, \psi_i) \in R_{+0} \times R_+ \times C_+^1,\ |x_i| \le G,\ \|\psi_i\|_c \le G\} < \infty.$$

Also, for $x_1 \ge 0$ and $x_2 \ge 0$, $g_i(t, x_1, x_2) \ge 0$.

(2) The function $g_i(t, x_i, \phi_i)$ is increasing with respect to $(x_i, \phi_i) \in R_+ \times C_+^1$.

(3) The function $g_i(t, x_i, \phi_i)$ satisfies the Lipschitz condition with respect to $(x_i, \phi_i) \in R_+ \times C_+^1$.

($\mathbf{H}_3$) For each $1 \le i \le n$, the function $e_i(t)$ is nonnegative continuous and bounded on R_{+0}, and there is a constant $\alpha_i > 0$ such that

$$\liminf_{t\to\infty} \int_t^{t+\alpha_i} e_i(s)\,\mathrm{d}s > 0.$$

For any $(t_0, \phi, \psi) \in R_+ \times C_+^n \times C_+^n$, by the fundamental theory of functional differential equations (see [**8, 10**]), it is well known that system (1.1) has a unique solution $X(t) = (x(t), u(t))$ through (t_0, ϕ, ψ) which is continuous. It is easy to verify that solutions of system (1.1) are defined on $[0, \infty)$ and remain positive for all $t \geq 0$ if the initial value $(t_0, \phi, \psi) \in R_+ \times C_+^n \times C_+^n$.

Firstly, on the boundedness and ultimate boundedness of all positive solutions of system (1.1), we can get

Theorem 3.1. *Suppose that assumptions* (H_1)–(H_3) *hold. Then system* (1.1) *is ultimately bounded in the sense that there are positive constants* M *and* T *such that, if* $t > T$*, then* $x_i(t) \leq M$ *and* $u_i(t) \leq M$*,* $i = 1, 2, \dots, n$*, for all positive solutions* $X(t) = (x(t), u(t))$ *of system* (1.1).

Proof. Let $X(t) = (x(t), u(t))$ be any positive solution of system (1.1). We first prove that the components x_i, $i = 1, 2, \dots, n$, of system (1.1) are ultimately bounded. From condition (2) of (H_1) and the ith equation of system (1.1) we have

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} \leq x_i(t) f_i(t, 0, \dots, 0, x_i(t), 0, \dots, 0, \hat{0}, 0, 0^*).$$

By the comparison theorem and conclusion (a) of Lemma 2.1, we can obtain that there is constant M_{i1} such that for any positive solution $(x(t), u(t))$ of system (1.1), there is a $T_{i1} > 0$ such that $x_i(t) < M_{i1}$ for all $t \geq T_{i1}$. Now, let $M_1 = \max_{1 \leq i \leq n}\{M_{i1}\}$ and $T_1 = \max_{1 \leq i \leq n}\{T_{i1}\}$. We have

$$x_i(t) \leq M_1 \text{ for all } t \geq T_1, \quad i = 1, 2, \dots, n. \tag{3.1}$$

Further, from (3.1), condition (2) of (H_2) and the $(n+i)$th equation of system (1.1), we have

$$\frac{\mathrm{d}u_i(t)}{\mathrm{d}t} \leq -e_i(t) u_i(t) + g_i(t, M_1, M_1)$$

for all $t \geq T_1 + \tau$. Hence, by the comparison theorem and conclusion (a) of Lemma 2.2, we can obtain that there is a constant $M_{i2} > 0$ such that for any positive solution $(x(t), u(t))$ of system (1.1), there is a $T_{i2} \geq T_1 + \tau$ such that $u_i(t) < M_{i2}$ for all $t \geq T_{i2}$.

Finally, we let $M = \max\{M_1, M_{12}, \dots, M_{n2}\}$ and $T = \max\{T_1, T_{12}, \dots, T_{n2}\}$. Then, for all $t \geq T$

$$x_i(t) \leq M, \qquad u_i(t) \leq M, \quad i = 1, 2, \dots, n.$$

Therefore, the solution $X(t) = (x(t), u(t))$ is ultimately bounded. This completes the proof of this theorem. $\square$

In order to obtain the permanence of system (1.1), we first consider the following single-species nonautonomous Kolmogorov system

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = x_i(t) f_i(t, 0, \dots, 0, x_i(t), 0, \dots, 0, \hat{0}, 0, 0^*), \tag{3.2}$$

where $i = 1, 2, \dots, n$. By conditions (1), (3) and (4) of (H_1), we see that system (3.2) satisfies all conditions of Lemma 2.1. Hence, by Lemma 2.1, each positive solution of system (3.2) is globally asymptotically stable. Let $x_{i0}(t)$ be some fixed positive solution of system (3.2) and $x_0(t) = (x_{10}(t), x_{20}(t), \dots, x_{n0}(t))$.

Next, we consider the following auxiliary system

$$\frac{\mathrm{d}u_i(t)}{\mathrm{d}t} = -e_i(t) u_i(t) + g_i(t, 0, 0^*), \tag{3.3}$$

where $i = 1, 2, \dots, n$. By conditions (1) and (3) of (H_2) and (H_3), we note that system (3.3) satisfies all conditions of Lemma 2.3. Hence, by Lemma 2.3, each positive solution of system (3.3) is globally asymptotically stable. Let $u_{i0}(t)$ be some fixed positive solution of system (3.3).

Remark 3.1. If $g_i(t, 0, 0^*) \equiv 0$, then system (3.3) has a trivial equilibrium $E_0 = 0$, and E_0 is globally asymptotically stable. In this case, we let $u_{i0}(t) = 0$.

On the permanence of system (1.1), we have the following result.

Theorem 3.2. *Suppose that assumptions (H_1)–(H_3) hold. Assume further that*

($\mathbf{H}_4$) *for each* $1 \le i \le n$, *there is a constant* $\gamma_i > 0$ *such that*

$$\liminf_{t\to\infty} \int_t^{t+\gamma_i} f_i(\mu, x_{10}(\mu), \ldots, x_{i-10}(\mu), 0, x_{i+10}(\mu), \ldots, x_{n0}(\mu), x_{0\mu}, u_{i0}(\mu), u_{i0\mu})\, \mathrm{d}\mu > 0,$$

where $x_{0\mu} = x_0(\mu + s)$ *and* $u_{i0\mu} = u_{i0}(\mu + s)$ *for all* $s \in [-\tau,\ 0]$.

Then system (1.1) *is permanent.*

Proof. Let $X(t) = (x(t), u(t))$ be any positive solution of system (1.1). From Theorem 3.1, there is a constant $M > 0$ such that for any positive solution $X(t)$ of system (1.1), there is a $T_1 \ge 0$ such that, for all $t \ge T_1$,

$$x_i(t) < M, \qquad u_i(t) < M, \quad i = 1, 2, \ldots, n.$$

Therefore, from condition (2) of (H_1) and the ith equation of system (1.1), we have

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} \ge x_i(t) f_i(t, U, \hat{U}, M, M^*) \ge -\alpha_i x(t) \tag{3.4}$$

for all $t \ge T_1 + \tau$, where $U = (M, \ldots, M) \in R^n$, $\alpha_i = \sup_{t\in R_{+0}} \times \{|f_i(t, U, \widehat{U}, M, M^*)|\}$. For any $t \ge T_1 + \tau$ and $s \in [-\tau,\ 0]$, integrating (3.4) from $t + s$ to t we obtain

$$x_i(t + s) \le x_i(t) \exp(-\alpha_i s) \le x_i(t) \exp(\alpha_i \tau) \le x_i(t) \exp(\alpha\tau), \tag{3.5}$$

where $\alpha = \max_{1\le i\le n}\{\alpha_i\}$ and $i = 1, 2, \ldots, n$.

On the other hand, from condition (2) of (H_1) and the ith equation of system (1.1), we have

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} \le x_i(t) f_i(t, 0, \ldots, 0, x_i(t), 0, \ldots, 0, \hat{0}, 0, 0^*).$$

By the comparison theorem and since $x_{i0}(t)$ is a globally uniformly attractive positive solution of system (3.2), we obtain for any $\epsilon > 0$ there is a constant $t_{i1} = t_{i1}(\epsilon) > 0$ such that

$$x_i(t) \le x_{i0}(t) + \epsilon \quad \text{for all } t \ge t_{i1}.$$

Now, let $t_1 = \max_{1\le i\le n}\{t_{i1}\}$. Then, for all $t \ge t_1$,

$$x_i(t) \le x_{i0}(t) + \epsilon, \quad i = 1, 2, \dots, n. \tag{3.6}$$

For any t_2, t_3 and $t_3 \ge t_2 \ge 0$, integrating system (1.1) directly we have

$$x_i(t_3) = x_i(t_2)\exp\int_{t_2}^{t_3} f_i(t, x(t), x_t, u_i(t), u_{it})\,\mathrm{d}t. \tag{3.7}$$

In the following, we will use two propositions to complete the proof of Theorem 3.2.

Proposition 3.1. *There is a constant $\beta > 0$ such that* $\limsup_{t\to\infty} \times$ $x_i(t) > \beta$, *$i = 1, 2, \dots, n$, for any positive solution $X(t) = (x(t), u(t))$ of system* (1.1).

In fact, by (H_4), we can choose positive constants $T_2 \ge T_1$, ε and δ such that

(3.8)
$$\int_t^{t+\gamma_i} f_i(\mu, x_{10}(\mu) + \varepsilon, \dots, x_{i-10}(\mu) + \varepsilon, \varepsilon, x_{i+10}(\mu) + \varepsilon, \dots, x_{n0}(\mu) + \varepsilon, x_{0\mu} + \overline{\varepsilon}, u_{i0}(\mu) + \varepsilon, u_{i0\mu} + \varepsilon)\,\mathrm{d}\mu > \delta.$$

for all $t \ge T_2$ and $i = 1, 2, \dots, n$, where $\overline{\varepsilon} = (\varepsilon, \dots, \varepsilon) \in R^n$.

Consider the following system with a parameter

$$\frac{\mathrm{d}u_i(t)}{\mathrm{d}t} = -e_i(t)u_i(t) + g_i(t, \beta, \beta\exp(\alpha\tau)), \tag{3.9}$$

where $\beta \in [0,\ \beta_0]$ and $i = 1, 2, \dots, n$. Let $u_{i\beta}(t)$ be the solution to system (3.9). By conditions (H_1) and (H_2), we see that system (3.9) satisfies all conditions of Lemmas 2.2 and 2.3. By Lemma 2.2, $u_{i\beta}(t)$ is globally asymptotically stable. Further, by Lemma 2.3, we obtain that $u_{i\beta}(t)$ converges uniformly for $t \in R_+$ to $u_{i0}(t)$, as $\beta \to 0$. Hence, there are constants $\beta > 0$ and $\beta < \varepsilon$ such that for all $t \ge T_2$,

$$u_{i\beta}(t) \le u_{i0}(t) + \frac{\varepsilon}{2}, \quad i = 1, 2, \dots, n. \tag{3.10}$$

If Proposition 3.1 is not true, then there are an integer $k \in \{1, 2, \dots, n\}$ and a positive solution $(x(t), u(t))$ of system (1.1) such that $\limsup_{t\to\infty} x_k(t) < \beta$. Hence, there is a constant $T_3 > T_2$ such that $x_k(t) < \beta$ for all $t \geq T_3$. From condition (2) of (H_2), (3.5) and the $(n+k)$th equation we obtain

$$\frac{\mathrm{d}u_k(t)}{\mathrm{d}t} \leq -\eta_k(t) + g_k(t, \beta, \beta \exp(\alpha\tau)) \text{ for all } t \geq T_3.$$

Using the comparison theorem and globally asymptotically stable of solution $u_{k\beta}(t)$, we obtain that there is a $T_4 \geq T_3$ such that

$$u_k(t) \leq u_{k\beta}(t) + \frac{\varepsilon}{2} \quad \text{for all } t \geq T_4. \tag{3.11}$$

Hence, from (3.10) and (3.11) it follows that

$$u_k(t) \leq u_{k0}(t) + \varepsilon \quad \text{for all } t \geq T_4. \tag{3.12}$$

On the other hand, by (3.6) there is a $T_5 \geq T_4$ such that

$$x_i(t) \leq x_{i0}(t) + \varepsilon \quad \text{for all } t \geq T_5, \tag{3.13}$$

where $i = 1, 2, \dots, n$ and $i \neq k$.

By (3.7), (3.12), (3.13) and condition (2) of (H_1), we obtain

$$\begin{aligned} x_k(t) &= x_k(T_5) \exp \int_{T_5}^{t} f_k(\mu, x(\mu), x_\mu, u_k(\mu), u_{k\mu})\, \mathrm{d}\mu \\ &\geq x_k(T_5) \exp \int_{T_5}^{t} f_k(\mu, x_{10}(\mu) + \varepsilon, \dots, x_{k-10}(\mu) + \varepsilon, \\ &\qquad \varepsilon, x_{k+10}(\mu) + \varepsilon, \dots, +x_{n0}(\mu) + \varepsilon, x_{0\mu} + \overline{\varepsilon}, u_{i0}(\mu) + \varepsilon, \\ &\qquad\qquad u_{i0\mu} + \varepsilon)\, \mathrm{d}\mu \end{aligned}$$

for all $t \geq T_5$. Thus, from (3.8) we finally obtain $\lim_{t\to\infty} x_k(t) = \infty$ which leads to a contradiction. Therefore, Proposition 3.1 is true.

Proposition 3.2. *There is a constant* $\gamma > 0$ *such that* $\liminf_{t\to\infty} x_i(t) > \gamma$, $i = 1, 2, \dots, n$, *for any positive solution* $X(t)$ *of system* (1.1).

In fact, if Proposition 3.2 is not true, then there is an integer $j \in \{1, 2, \dots, n\}$ and a sequence of initial value $\{X_m = (\phi_m, \psi_m)\} \subset C_+^n \times C_+^n$ such that, for the solution $(x(t, X_m), u(t, X_m))$ of system (1.1),

$$\liminf_{t\to\infty} x_j(t, X_m) < \frac{\beta}{m^2}, \quad m = 1, 2, \dots ,$$

where constant β is given in Proposition 3.1. By Proposition 3.1, for every m there are two time sequences $\{s_q^{(m)}\}$ and $\{t_q^{(m)}\}$, which satisfy $0 < s_1^{(m)} < t_1^{(m)} < s_2^{(m)} < t_2^{(m)} < \cdots < s_q^{(m)} < t_q^{(m)} < \cdots$ and $\lim_{q\to\infty} s_q^{(m)} = \infty$, such that

$$x_j(s_q^{(m)}, X_m) = \frac{\beta}{m}, \qquad x_j(t_q^{(m)}, X_m) = \frac{\beta}{m^2} \tag{3.14}$$

and

$$\frac{\beta}{m^2} < x_j(t, X_m) < \frac{\beta}{m} \quad \text{for all } t \in (s_q^{(m)},\ t_q^{(m)}). \tag{3.15}$$

From the ultimate boundedness of system (1.1), we can choose a positive constant $T^{(m)}$ for every m such that $x_i(t, X_m) < M$ and $u_i(t, X_m) < M$ for all $t > T^{(m)}$ and $i = 1, 2, \dots , n$. Further, there is an integer $K_1^{(m)} > 0$ such that $s_q^{(m)} > T^{(m)} + \tau$ for all $q > K_1^{(m)}$. Let $q > K_1^{(m)}$, for any $t \in [s_q^{(m)},\ t_q^{(m)}]$; by condition (2) of (H_1) we have

$$\begin{aligned}\frac{\mathrm{d}x_j(t, X_m)}{\mathrm{d}t} &\geq x_j(t, X_m) f_j(t, U, \widehat{U}, M, M^*)\\ &\geq -\gamma_0 x_j(t, X_m),\end{aligned}$$

where $U = (M, \dots , M) \in R^n$ and $\gamma_0 = \sup_{t\in R_{+0}}\{|f_j(t, U, \widehat{U}, M, M^*)|\}$. Integrating the above inequality from $s_q^{(m)}$ to $t_q^{(m)}$, we further have

$$x_j(t_q^{(m)}, X_m) \geq x_j(s_q^{(m)}, X_m) \exp[-\gamma_0(t_q^{(m)} - s_q^{(m)})].$$

Consequently, by (3.14),

$$\frac{\beta}{m^2} \geq \frac{\beta}{m} \exp[-\gamma_0(t_q^{(m)} - s_q^{(m)})].$$

Hence,

$$t_q^{(m)} - s_q^{(m)} \geq \frac{\ln m}{\gamma_0} \quad \text{for all } q > K_1^{(m)}. \tag{3.16}$$

By (3.8), there are positive constants P and ϱ such that

(3.17)
$$\int_t^{t+\kappa} f_j(\mu, x_{10}(\mu)+\varepsilon, \dots, x_{i-10}(\mu)+\varepsilon, \varepsilon, x_{i+10}(\mu)+\varepsilon, \dots, x_{n0}(\mu) + \varepsilon, x_{0\mu}+\overline{\varepsilon}, u_{i0}(\mu)+\varepsilon, u_{i0\mu}+\varepsilon)\,\mathrm{d}\mu > \varrho$$

for all $t \geq 0$ and $\kappa \geq P$.

Let $\widetilde{u}_{j\beta}(t)$ be the solution of system (3.9) with the initial condition $\widetilde{u}_{j\beta}(s_q^{(m)}) = u_j(s_q^{(m)}, X_m)$. By (3.5), (3.15) and condition (2) of (H_2), we have

$$\frac{\mathrm{d}u_j(t,X_m)}{\mathrm{d}t} \leq -e_j(t)u_j(t,X_m) + g_j(t,\beta,\beta\exp(\alpha\tau))$$

for any m, q and $t \in [s_q^{(m)},\ t_q^{(m)}]$. Using the comparison theorem it follows that

$$u_j(t,X_m) \leq \widetilde{u}_{j\beta}(t) \quad \text{for all } t \in [s_q^{(m)},\ t_q^{(m)}]. \tag{3.18}$$

Further, by Lemma 2.2, the solution $u_{j\beta}(t)$ of system (3.9) is globally uniformly attractive on R_{+0}. We obtain that there is a constant $T_2 \geq P$, and T_2 is independent of any m and $q \geq K^{(m)}$, such that

$$\widetilde{u}_{j\beta}(t) \leq u_{j\beta}(t) + \frac{\varepsilon}{2} \quad \text{for all } t \geq s_q^{(m)} + T_2. \tag{3.19}$$

On the other hand, by (3.6) there is a $T_3 \geq T_2$ and T_3 is independent of any m and $q \geq K^{(m)}$, such that

$$x_i(t,X_m) \leq x_{i0}(t) + \varepsilon \quad \text{for all } t \geq T_3, \tag{3.20}$$

where $i = 1, 2, \dots, n$ and $i \neq j$. Choose an integer $N_0 > 0$ such that, when $m \geq N_0$ and $q \geq K^{(m)}$,

$$t_q^{(m)} - s_q^{(m)} > T_3 + P.$$

Further, from (3.10), (3.18) and (3.19) we obtain

$$(3.21) \qquad u_j(t, X_m) \le u_{j0}(t) + \varepsilon \quad \text{for all } t \in [s_q^{(m)} + T_3,\ t_q^{(m)}].$$

Hence, when $m \ge N_0$ and $q \ge K^{(m)}$, by (3.7), (3.20), (3.21) and condition (2) of (H_1), it follows that

$$\begin{aligned}\frac{\beta}{m^2} &= x_j(s_q^{(m)} + T_3, X_m) \exp \int_{s_q^{(m)}+T_3}^{t_q^{(n)}} f_j(t, x(t), x_t, u_j(t), u_{jt})\, \mathrm{d}t \\ &\ge x_j(s_q^{(m)} + T_3, X_m) \exp \int_{s_q^{(m)}+T_3}^{t_q^{(n)}} f_j(t, x_{10}(t) + \varepsilon, \dots, x_{j-10}(t) + \varepsilon, \\ &\qquad \varepsilon, x_{j+10}(t), \dots, x_{n0}(t) + \varepsilon, x_{0t} + \overline{\varepsilon}, u_{j0}(t) + \varepsilon, u_{j0t} + \varepsilon)\, \mathrm{d}t \\ &> \frac{\beta}{m^2},\end{aligned}$$

which leads to a contradiction. Therefore, Proposition 3.2 is true.

Finally, from Propositions 3.1 and 3.2 we complete the proof of this theorem. □

Remark 3.2. From the proof of Theorem 3.2, we note that if function $g_i(t, 0, 0) \equiv 0$, then let $u_{i0}(t) = 0$, $i = 1, 2, \dots, n$. In this case, the feedback controls are harmless to the permanence of system (1.1).

If system (1.1) is T-periodic, then we can take constant ϑ_i in condition (3) of (H_1), ν_i in condition (4) of (H_1), α_i in (H_3) and γ_i in (H_4), $i = 1, 2, \dots, n$, all as T, and particularly let $x_{i0}(t)$ and $u_{i0}(t)$ be a fixed positive T-periodic solution of systems (3.2) and (3.3), respectively. Further, for any T-periodic continuous function $f(t)$ on R, we can obtain that $\liminf_{t\to\infty} \int_t^{t+T} f(s)\, \mathrm{d}s$ and $\limsup_{t\to\infty} \int_t^{t+T} f(s)\, \mathrm{d}s$ are equal to $\int_0^T f(s)\, \mathrm{d}s$.

Using Theorem 1 given by Teng and Chen in [**21**] on the existence of positive periodic solutions for the general n-species periodic Kolmogorov type systems with delays, we have the following theorem on the existence of positive periodic solutions for the periodic system (1.1).

Theorem 3.3. *If system* (1.1) *is T-periodic and assumptions* (H_1)–(H_4) *hold, and further assumption* $\int_0^T g_i(s, 0, 0^*)\, \mathrm{d}s > 0$, $i =$

$1, 2, \ldots, n$, *holds, then system* (1.1) *has at least a positive T-periodic solution.*

The proof of Theorem 3.3 is similar to that of Theorem 3 in [**18**], so we omit it here.

Remark 3.3. Chen [**3**] studied the existence of the positive T-periodic solution of T-periodic system (1.1) by using the technique of Schauder's fixed point theorem (see [**3**, Theorem 2.1]). Obviously, this method is totally different from our method in this paper. We also note that assumption (A_8) in Theorem 2.1 of [**3**] clearly implies assumption (H_4) in Theorem 3.3. So, our result is very general and rather weak.

4. Applications. To illustrate generality of the results obtained, we will apply the results given in Section 3 to particular competition systems with delay and feedback controls or without delay and feedback controls, which have been studied extensively in the literature. The following four examples will show that the derived sufficient conditions are easily verifiable, more general, and weaker than those given in the literature; thus, we improve and generalize some well-known results.

Example 4.1. Consider the following nonautonomous Lotk-Volterra differential system with finite delays

(4.1)
$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = x_i(t)\left[c_i(t) - a_i(t)x_i(t) - \sum_{j=1}^{n}\sum_{l=0}^{m} a_{ij}^l(t)x_j(t-\tau_l)\right], \quad t \geq t_0,$$
$$x_i(t) = \phi_i(t) \geq 0, \quad t \leq t_0 \text{ and } \phi_i(t_0) > 0,$$

where $\tau_l \geq 0$, $0 \leq l \leq m$, $i = 1, 2, \ldots, n$, and each $\phi_i(t)$ is a continuous function for $t \leq t_0$, each $c_i(t)$, $a_i(t)$ and $a_{ij}^l(t)$ is a nonnegative bounded continuous function on $[t_0, \infty)$.

On the permanence of system (4.1), applying Theorems 3.1 and 3.2 we have the following result.

Theorem 4.1. *Suppose that there exist positive constants ω_i and λ_i such that for each $i = 1, 2, \ldots, n$,*

$$\liminf_{t\to\infty}\int_t^{t+\omega_i} a_i(s)\,\mathrm{d}s > 0,$$

and

$$\liminf_{t\to\infty}\int_t^{t+\lambda_i}\left[c_i(s)-\sum_{j=1,j\neq i}^{n}\sum_{l=0}^{m}a_{ij}^l(s)x_{j0}(s)\right]\mathrm{d}s>0,$$

where $x_{j0}(t)$ *is some fixed positive solution of system*

$$\frac{\mathrm{d}x_j(t)}{\mathrm{d}t}=x_j(t)[c_j(t)-a_j(t)x_j(t)],$$

then system (4.1) *is permanent.*

Remark 4.1 Obviously, the result in [**16**, Theorem 1.1] implies our Theorem 4.1.

Example 4.2. Consider the permanence of any positive solution for system (1.2).

In system (1.2), we assume that

($\mathbf{C}_1$) For each i, $j=1,2,\dots,n$, $r_i(t)$, $a_{ij}(t)$, $\alpha_i(t)$, $a_i(t)$ and $\eta_i(t)$ are nonnegative bounded continuous functions on $[0,\ \infty)$.

($\mathbf{C}_2$) For each i, $j=1,2,\dots,n$, $K_{ij}:[0,\ \omega]\to[0,\ \infty)$ is continuous and $\int_0^\omega K_{ij}(s)\,\mathrm{d}s=1$, K_i, H_i are provided with the same behaviors as K_{ij}.

($\mathbf{C}_3$) There is a constant $\gamma_i>0$, $i=1,\dots,n$, such that

$$\liminf_{t\to\infty}\int_t^{t+\gamma_i}\eta_i(s)\,\mathrm{d}s>0.$$

On the permanence of system (1.2), applying Theorems 3.1–3.3 we have the following result.

Theorem 4.2. *Suppose that assumptions* (C_1)–(C_3) *hold, assume further that there exist positive constants* ω_i *and* λ_i *such that for each*

$i = 1, 2, \ldots, n$,

$$\liminf_{t\to\infty} \int_t^{t+\omega_i} a_{ii}(s)\,\mathrm{d}s > 0$$

and

$$\liminf_{t\to\infty} \int_t^{t+\lambda_i} \left[r_i(\tau) - \sum_{j=1, j\neq i}^{n} a_{ij}(\tau) \int_0^{\omega} K_{ij}(s) x_{j0}(\tau - s)\,\mathrm{d}s \right] \mathrm{d}\tau > 0,$$

where $x_{j0}(t)$ is some fixed positive solution of system

$$\frac{\mathrm{d}x_j(t)}{\mathrm{d}t} = x_j(t)[r_j i(t) - a_{jj}(t) x_j(t)].$$

Then (a) *system* (1.2) *is permanent.*

(b) *If system* (1.2) *is T-periodic and $\int_0^T a_i(s)\,\mathrm{d}s > 0$, $i = 1, 2, \ldots, n$, then system* (1.2) *has at least a positive T-periodic solution.*

Remark 4.2. Weng [**27**] studied the existence of the positive ω-periodic solution of ω-periodic system (1.2) by using the technique of coincidence degree. Obviously, this method is totally different from our method in this paper, and our result can be more easily checked.

Remark 4.3. In system(1.2), we note that the feedback controls are harmless to the permanence of system (1.2).

Example 4.3. Consider the permanence of any positive solution for system (1.3).

For system (1.3), we introduce the following assumptions:

($\mathbf{D}_1$) Functions $r_i(t)$, $a_i(t)$, $b_i(t)$, $c_i(t)$, $d_i(t)$, $e_i(t)$, $\eta_i(t)$, $\tau_i(t)$ and $\sigma_i(t)$ are nonnegative bounded continuous functions on $[0, \infty)$, and $\inf_{t\geq 0} b_i(t) > 0$, $i = 1, 2, \ldots, n$.

($\mathbf{D}_2$) There is a constant $\mu_i > 0$, $i = 1, 2, \ldots, n$, such that

$$\liminf_{t\to\infty} \int_t^{t+\mu_i} a_i(s)\,\mathrm{d}s > 0, \qquad \liminf_{t\to\infty} \int_t^{t+\mu_i} \eta_i(s)\,\mathrm{d}s > 0.$$

On the permanence of system (1.3), we have the following result.

Theorem 4.3. *Suppose that assumptions* (D_1) *and* (D_2) *hold, assume further that there is a constant* $\lambda_i > 0$ *such that for each* $i = 1, 2, \dots, n$,

$$\liminf_{t\to\infty} \int_t^{t+\lambda_i} r_i(s)\left[1 - \sum_{j=1, j\neq i}^{n} \frac{a_i(s)x_{j0}(s)}{b_i(s) + c_i(s)x_{j0}(s)}\right] \mathrm{d}s > 0,$$

where $x_{i0}(t)$ *is some fixed positive solution of system*

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = r_i(t)x_i(t)\left[1 - \frac{a_i(t)x_i(t)}{b_i(t) + c_i(t)x_i(t)}\right].$$

Then, (a) *system* (1.3) *is permanent.*

(b) *If system* (1.3) *is* ω*-periodic and* $\int_0^\omega e_i(s)\,\mathrm{d}s > 0$, $i = 1, 2, \dots, n$, *then system* (1.3) *has at least a positive* ω*-periodic solution.*

Remark 4.4. In system(1.3), we note that the feedback controls are harmless to the permanence of system (1.3).

Example 4.4. Consider the permanence of any positive solution for system (1.4).

For system (1.4) we introduce the following assumptions:

($\mathbf{E}_1$) For each $i, j = 1, 2, \dots, n$ and $l = 1, 2, \dots, m$, α_{ij}, β_{ij}, γ_{ij}, ϑ_i, τ_{ij}, τ_i and η_i are positive constants; $r_i(t)$, $a_{ij}(t)$, $b_{ijl}(t)$, $d_i(t)$, $e_i(t)$, $f_i(t)$, $g_i(t)$ and $h_i(t)$ are nonnegative bounded continuous functions on $[t_0, \infty)$.

($\mathbf{E}_2$) For each $i, j = 1, 2, \dots, n$, $c_{ij}(t, s)$, $H_i(t, s)$ and $K_i(t, s)$ are continuous with respect to t on R and integrable with respect to s; also, $\sup\{\int_{-\tau_{ij}}^0 c_{ij}(t, s)\,\mathrm{d}s\} < \infty$, $\sup\{\int_{-\tau_i}^0 H_i(t, s)\,\mathrm{d}s\} < \infty$ and $\sup\{\int_{-\eta_i}^0 K_i(t, s)\,\mathrm{d}s\} < \infty$.

On the permanence of system (1.4), we have the following result.

Theorem 4.4. *Suppose that assumptions* (E_1) *and* (E_2) *hold, assume further that there exist positive constants* μ_i, υ_i, ρ_i *and* λ_i *such* *that for each* $i = 1, 2, \dots, n$,

$$\liminf_{t\to\infty} \int_t^{t+\mu_i} a_{ii}(s)\,\mathrm{d}s > 0, \quad \liminf_{t\to\infty} \int_t^{t+\upsilon_i} f_i(s)\,\mathrm{d}s > 0,$$
$$\liminf_{t\to\infty} \int_t^{t+\rho_i} g_i(s)\,\mathrm{d}s > 0$$

and

$$\liminf_{t\to\infty} \int_t^{t+\lambda_i} \Bigg[r_i(\theta) - d_i(\theta)u_{i0}(\theta) - \sum_{j=1, j\neq i}^{n} a_{ij}(\theta) x_{j0}^{\alpha_{ij}}(\theta) - \sum_{j=1, j\neq i}^{n} \sum_{l=1}^{m} b_{ijl}(\theta) x_{j0}^{\beta_{ij}}(\theta)(\theta - \tau_{ijl}) - \sum_{j=1, j\neq i}^{n} \int_{-\tau_{ij}}^{0} c_{ij}(\theta, s) x_{j0}^{\gamma_{ij}}(\theta + s)\,\mathrm{d}s - e_i(\theta) \int_{-\tau_i}^{0} H_i(\theta, s) u_{i0}(\theta + s)\,\mathrm{d}s \Bigg] \mathrm{d}\theta > 0,$$

where $x_{i0}(t)$ *and* $u_{i0}(t)$ *denote some fixed positive solutions of systems*

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = x_i(t)[r_i(t) - a_{ii}(t) x_i^{\alpha_{ii}}(t)]$$

and

$$\frac{\mathrm{d}u_i(t)}{\mathrm{d}t} = f_i(t) - g_i(t) u_i(t),$$

respectively. Then (a) *system* (1.4) *is permanent.*

(b) *If system* (1.4) *is* ω*-periodic, then system* (1.2) *has at least a positive* ω*-periodic solution.*

Remark 4.5. In system(1.4), we note that the feedback controls have influence on he permanence of system (1.4).

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Department of Applied Mathematics, Xi'an Jiaotong University, Xi'an 710049, China and College of Mathematics and Systems Science, Xinjiang University, Urumqi 830046, China
Email address: nielinfei@xju.edu.cn

Institute for Information and System Sciences, Research Center for Applied Mathematics, Xi'an Jiaotong University, Xi'an 710049, China
Email address: jgpeng@mail.xjtu.edu.cn

College of Mathematics and Systems Science, Xinjiang University, Urumqi 830046, China
Email address: zhidong@xju.edu.cn