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LARGE HIGHLY POWERFUL NUMBERS ARE CUBEFUL

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Let the **prodex** of n be the product of the exponents of the primes when n is written in standard form. M. V. Subbarao has called a number highly **powerful** if its prodex is larger than that of any smaller number. Assume that $n = \prod_{i=1}^{k} p_i^E$ is highly powerful. Then it is clear that p_i is the ith prime, the exponents $E = E(p_i)$ are nonincreasing, $E(p_k) \geqq 2$ and $E(p_{k-1}) \geqq 3$ (since $p_{k-1}^4 < p_{k-1}^2 p_k^2$). The theorem of the title asserts that if $p_k > N$, then $E(p_k) \geqq 3$. Further, we have developed an algorithm which finds all highly powerful numbers having $E(p_k) \neq 3$. The nineteen highly powerful numbers with $E(p_k) = 2$ are listed in Table 1.

Table 1
THE 19 HIGHLY POWERFUL NUMBERS WHICH ARE NOT CUBEFUL

2^2	$2^8 3^4 5^2$	$2^{11} 3^6 5^5 7^4 11^3 13^3 17^2$
$2^4 3^2$	$2^7 3^5 5^3 7^2$	$2^{10} 3^7 5^5 7^4 11^3 13^3 17^2$
$2^5 3^2$	$2^7 3^4 5^4 7^2$	$2^{11} 3^7 5^5 7^4 11^3 13^3 17^2$
$2^7 3^3 5^2$	$2^8 3^5 5^3 7^2$	$2^{11} 3^7 5^5 7^4 11^3 13^3 17^3 19^2$
$2^6 3^4 5^2$	$2^8 3^4 5^4 7^2$	$2^{11} 3^8 5^5 7^4 11^3 13^3 17^3 19^2$
$2^5 3^5 5^2$	$2^9 3^6 5^4 7^3 11^2$	
$2^7 3^4 5^2$	$2^{11} 3^7 5^4 7^3 11^3 13^2$	

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