# Special issue in honor of Ralf Fröberg
# on the occasion of his 65th birthday

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In June 2009 co-authors, colleagues, students and former students of Ralf Fröberg gathered in Stockholm for the conference *Topics in Algebra* that was held to celebrate Ralf's birthday. The speakers at the conference were: L. Avramov, J. Backelin, V. Barucci, G. Björck, A. Björner, M. Boij, W. Bruns, M. D'Anna, S. Di Rocco, E. Emtander, A. Geramita, C. Gottlieb, J. Herzog, F. Hreinsdóttir, A. Iarrobino, S. Lundqvist, C. Löfwall, V. Micale, M. Passare, J.-E. Roos, E. Sköldberg, J. Snellman, A. Torstensson, N. V. Trung, V. Welker, E. Wulcan and S. Zarzuela.

As the name indicates the conference covered many topics in algebra, from commutative and homological algebra to topics in group theory, algebraic geometry, combinatorics and computational algebra. Many of the participants have written articles for these proceedings that reflect the variety of mathematics presented at the conference.

Among Ralf's mathematical interests are Poincaré series, Koszul algebras, Stanley-Reisner rings, numerical semigroups, analytically irreducible one-dimensional rings and computational algebra. To name only a few of his achievements in these areas:

• His paper in 1975 on the rationality of the Poincaré series of a new class of rings. There it was shown that a quotient ring of a polynomial ring, $R/I$ is Koszul if $I$ is generated by monomials of degree 2. This led to the very important result that if an ideal $J$ has a quadratic Gröbner basis then $R/J$ is Koszul (e.g. using Anick's spectral sequence).

• His work on Koszul algebras and Veronese subrings (joint with J. Backelin) which is a systematic account of Koszul algebras where it is shown that the property is preserved under various constructions such as taking tensor products, Segre products, Veronese subrings and dual rings.

• The Fröberg conjecture (1985): If $I$ is a homogeneous ideal generated by generic forms $F_1, \ldots, F_r$ in $R = k[x_1, \ldots, x_n]$ of degrees $\deg F_i = d_i$, then the Hilbert series $H_{R/I}(t)$ of $R/I$ is given by

$$H_{R/I}(t) = \left\lceil \frac{\prod_{i=1}^{r}(1 - t^{d_i})}{(1 - t)^n} \right\rceil$$

The conjecture has been shown to be true for many special cases (by e.g. Stanley, Fröberg, Anick, Hochster and Laksov). In joint work with J. Hollman (1993) Ralf showed how to use computer calculations to prove several new cases.

• Work on numerical semigroups, which started with joint work with Häggkvist and Gottlieb (1987) where among other results it was shown that for every fixed odd number $n$, the number of symmetric numerical semigroups $S$ with $g(S) = n$ is at least $2^{[n/8]}$ ($g(S)$ is the Frobenius number), a numerical semigroup minimally generated by three pairwise relatively prime elements is not symmetric and if $S$ has type $t$ then $(t + 1)n(S) \geq g(S) + 1$, where $n(S)$ is the number of elements of $S$ which are less than $g(S)$. This work has been cited and used by many mathematicians. Ralf's interest comes mainly from value semigroups of analytically irreducible, residually rational curves.

• Work on Stanley-Reisner rings (1990) giving a characterization of simplicial complexes whose associated Stanley-Reisner rings have 2-linear resolutions.

• The very readable book *An Introduction to Gröbner bases* in 1997.

• In the last 15 years Ralf has produced much work on one-dimensional rings, often with co-authors V. Barucci and M. D'Anna, on topics as diverse as the Poincaré series of modules of derivations on affine monomial curves, algorithms for plane curve singularities, and work on associated graded rings. With Roos, he found an example of an affine monomial curve having irrational Poincaré-Betti series.

A group of graduate students at Stockholm University, E. Emtander, Q. Xantcha, S. Lundqvist and E. Knutsson helped with practical matters and PhD. J. Jonsson at KTH was also of great assistance.

As as former student of Ralf I am not only grateful for his mathematical advice at the time but also for his friendship. This is an experience I share with many former graduate students in Mathematics at Stockholm University.

Reykjavík May 2010                                    Freyja Hreinsdóttir