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A necessary condition for the L^2-well posed Cauchy problem with variable coefficients

By

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§1. Introduction

In this article we shall derive a necessary condition for the L^2 well posed Cauchy problem of the first order system and the single higher order equation (Kowalewskian) with variable coefficients.

In the section 2, we consider the Cauchy problem of the first order system;

$$(1.1)\qquad \begin{cases} \dfrac{\partial u}{\partial t}=\sum_{j=1}^{p} A_j(x, t)\dfrac{\partial u}{\partial x_j}+B(x, t)u+f \\ u(x, 0)=0 \end{cases}$$

in which $A_j(x, t)$ and $B(x, t)$ are sufficiently smoothly varying $k\times k$ matrices defined on $R^p\times[0, T]$ and $u(x, t)={}^t(u_1(x, t), \dots, u_k(x, t))$. When A_j are constant matrices, K. Kasahara and M. Yamaguti proved in [1] that $\Sigma A_j\xi_j$ is diagonalizable for $\xi\in R^p-\{0\}$, if and only if the Cauchy problem (1.1) is uniquely solvable for any lower order B. Then (1.1) is also L^2-well posed. When A_j are function matrices, G. Strang in [4] proved that, if $A_j=A_j(x)$ are independent of variable t, $\Sigma A_j(x)\xi_j$ is necessarily diagonalizable for L^2-well posedness of (1.1), and T. Kano in [3] derived the same result if the multiplicity of characteristics of $\Sigma A_j(x, t)\xi_j$ is constant. Here we shall remove the restriction for the characteristic matrix $\Sigma A_j(x, t)\xi_j$ and the proof is seemed very simple.

We say that the Cauchy problem (1.1) is L^2-well posed in $[0, T]$, if for any $f(t)$ in $\mathscr{E}_t^1(L^2(R^p))$ and $\mathscr{E}_t^0(H^1(R^p))$, there exists the unique solution $u(t)$ in $\mathscr{E}_t^1(L^2(R^p))$ and in $\mathscr{E}_t^0(H^1(R^p))$ and satisfies

$$\|u(t)\| \leqslant c(T)\int_0^t \|f(s)\| ds \tag{1.2}$$

for $t \in [0, T]$, $T > 0$.

Theorem 1.1. *Suppose that the Cauchy problem is L^2-well posed in $[0, T]$. Then the matrix $A(x, t; \xi)$ is diagonalizable for any (x, t) in $R^p \times [0, T]$, ξ in $R^p - \{0\}$, where $A(x, t; \xi) = \Sigma A_j(x, t)\xi_j$.*

Remark. 1 It is known that the matrix $A(x, t; \xi)$ has only real eigen values, if (1.1) is well posed in C^∞ topology [5].

Next we consider the single higher order equation;

$$Pu = \left(\frac{\partial}{\partial t}\right)^m u + a_1\left(x, t; \frac{\partial}{\partial x}\right)\left(\frac{\partial}{\partial t}\right)^{m-1} u + \cdots + a_m\left(x, t; \frac{\partial}{\partial x}\right) u = f, \tag{1.3}$$

$$\left(\frac{\partial}{\partial t}\right)^j u(x, 0) = 0 \qquad (j = 0, 1, \dots, m-1),$$

in which $a_j\left(x, t; \frac{\partial}{\partial x}\right)$ are differential operators of order j. Denote by $P_m\left(x, t; \frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right)$ the principal part of P.

We say that Cauchy problem (1.3) is L^2-well posed in $[0, T]$, if for any $f(t)$ in $\mathscr{E}_t^1(L^2(R^p))$ and in $\mathscr{E}_t^0(H^1(R^p))$, there exists the unique solution $u(t)$ in $\mathscr{E}_t^{m-j}(H^j(R^p))$ $(j = 0, \dots, m)$ and satisfies

$$\sum_{j=1}^{m} \left\| \left(\frac{\partial}{\partial t}\right)^{j-1} u(t) \right\|_{m-j} \leqslant c(T)\int_0^t \|f(s)\| ds \tag{1.4}$$

for t in $[0, T]$.

Then for Cauchy problem (1.1) we have.

Theorem 1.2. *Suppose that Cauchy problem (1.3) is L^2-well posed.*

Then the roots of $P_m(x, t; \lambda, \xi)=0$ *with respect to* λ *are real and distinct for any* (x, t) *in* $R^p\times[0, T]$, ξ *in* $R^p-\{0\}$.

Remark. 2 When the multiplicity of the roots of $P_m(x, t; \lambda, \xi)$ is independent of (x, t) and ξ, the same result was obtained by T. Kano [2].

§2. Systems

In this section we shall pove Theorem 1.1. Now we introduce a notation;

$$[u]^2_{m,\mu}=\sum_{j+|\alpha|\leqslant m}\int_0^T\left\|e^{-\mu t}\left(\frac{\partial}{\partial t}\right)^j\left(\frac{\partial}{\partial x}\right)^\alpha u(t)\right\|^2 dt, \tag{2.1}$$

where $\|\cdot\|$ is the norm of $L^2(R^p)$, and m is any non negative integer.

Then we have easily from (1.2)

Proposition 2.1. *Suppose that Cauchy problem* (1.1) *is* L^2*-well posed in* $[0, T]$. *Then for any* $f(t)$ *in* $\mathscr{D}(R^p\times(0, T))$ *there exists the solution* u *in* $C^\infty(R^p\times(0, T))$ *of* (1.1) *and satisfies for any non negative integer* s *and any positive number* μ,

$$\mu[u]_{s,\mu}\leqslant const.[f]_{s,\mu} \tag{2.2}$$

where const. is independent of μ.

The proof of Theorem. 1.1 Suppose that for some (x_0, t_0) in $R^p\times[0, T]$ and ξ_0 in $R^p-\{0\}$, $|\xi|=1$, $A(x_0, t_0; \xi_0)$ is not diagonalizable. We may sappose $(x_0, t_0)=(0, 0)$ without loss of generality. Then we have a non singular matrix N_0 such that for $A_0=A(0, 0; \xi_0)$,

$$N_0^{-1}A_0N_0=D=\begin{pmatrix} D_1 & 0 \\ 0 & D_2\end{pmatrix}$$

where $D_1=\begin{pmatrix}\lambda_0 & 1 & & 0\\ & \ddots & \ddots & \\ & & \ddots & 1\\ 0 & & & \lambda_0\end{pmatrix}$ is a $l\times l$ Jordan's matrix, $l\geqslant 2$ and D_2 is a $(m-l)\times(m-l)$ matrix. Let us recall that λ_0 is real (Remark 1.).

We consider the following Canchy problem,

$$(2.3)\qquad \begin{cases} \left(\dfrac{\partial}{\partial t}-A-B\right)u=f, \\ u(0)=0, \end{cases}$$

where $A=\Sigma A_j\dfrac{\partial}{\partial x_j}$, $f=N_0^{-1\,t}(0, g, 0, \dots, 0)$ and g is a scalar function in $C_0^\infty(R^p\times(0, T))$. We define $\alpha(t)$ as a function in $C^\infty(0, T)$ which is equal to one in a neighourbood of 0 and zero at $t=T$. We apply $\alpha(t)$ to (2.3),

$$(2.4)\qquad \begin{cases} \left(\dfrac{\partial}{\partial t}-A\right)(\alpha u)-\alpha Bu-\alpha' u=\alpha f \\ \alpha u(0)=0 \end{cases}$$

By virtue of (2.2), we have, for $\mu>0$

$$(2.5)\qquad \mu[\alpha u]_{0,\mu}\leqslant \text{const.}[f]_{0,\mu}\leqslant \text{const,}[g]_{0,\mu}.$$

Let n be a positive integer. We decompose (2.4),

$$\left(\frac{\partial}{\partial t}-inA_0\right)(\alpha u)-(A-inA_0)(\alpha u)-\alpha Bu-\alpha' u=\alpha f$$

We put $v=N_0^{-1}\alpha u$. Then we can write

$$(2.6)\qquad \left(\frac{\partial}{\partial t}-inD\right)v+\varphi=N_0^{-1}\alpha f$$

where $\varphi=N_0^{-1}\{(inA_0-A)\alpha u-\alpha Bu-\alpha' u\}$.
Here we note that φ satisfies

$$(2.7)\qquad [\varphi]_{0,\mu}\leqq \text{const.}\left\{\sum_j\left[t\frac{\partial u}{\partial x_j}\right]_{0,\mu}+\sum_{i,j}\left[x_i\frac{\partial u}{\partial x_j}\right]_{0,\mu} + \sum_j\left[\left(\frac{\partial}{\partial x_j}-in\xi_j^0\right)u\right]_{0,\mu}+[u]_{0,\mu}\right\}$$

where $\xi_0=(\xi_1^0, \xi_2^0, \dots, \xi_p^0)$.
Now we define $\tilde{u}$ by

$$\tilde{u}=\int_0^T e^{-(2\mu+in\lambda_0)t}u(t)dt.$$

Then it holds

$$\|\tilde{u}\| \leqslant \text{const.} \frac{1}{\sqrt{\mu}}[u]_{0,\mu}.$$

Multiplying (2.6) by $e^{-(2\mu+in\lambda_0)t}$ and integrating with respect to t in the interval $[0, T]$, we have the following relation,

$$(2.8)\qquad \begin{cases} 2\mu\tilde{v}_1 - in\tilde{v}_2 + \tilde{\varphi}_1 = 0, \\ 2\mu\tilde{v}_2 - in\tilde{v}_3 + \tilde{\varphi}_2 = \widetilde{\alpha g}, \\ 2\mu\tilde{v}_j - in\tilde{v}_{j+1} + \tilde{\varphi}_j = 0, \qquad (j=3, \dots, l-1) \\ 2\mu\tilde{v}_l + \tilde{\varphi}_l = 0, \end{cases}$$

in which $\tilde{v}_j$ and $\tilde{\varphi}_j$ is the j-th component of $\tilde{v}$ and $\tilde{\varphi}$ respectively. Hence from (2.8) we have

$$\|\widetilde{\alpha g}\| \leqslant \frac{2\mu}{n}\|\tilde{\varphi}_1\| + \frac{4\mu^2}{n}\|\tilde{v}_1\| + \sum_{j=2}^{l}\left(\frac{n}{2\mu}\right)^{j-2}\|\tilde{\varphi}_j\|.$$

If $n \geqslant \mu$, we have

$$(2.9)\qquad \|\widetilde{\alpha g}\| \leqslant \text{const.}\left\{\left(\frac{n}{\mu}\right)^{l-2}\|\tilde{\varphi}\| + \frac{\mu^2}{n}\|\tilde{v}\|\right\}$$

$$\leqslant \text{const.}\frac{1}{\sqrt{\mu}}\left\{\left(\frac{n}{\mu}\right)^{l-2}[\varphi]_{0,\mu} + \frac{\mu^2}{n}[u]_{0,\mu}\right\},$$

where const. is independent of μ and n.

Now we choose $g(x, t)$ as follows,

$$(2.10)\qquad g(x, t) = \beta(n^{\frac{1}{2}}x)n^{\frac{p}{4}}\gamma(\mu t)\mu e^{in\lambda_0 t + in\xi_0 x}$$

where $\gamma(t)$ in $C_0^\infty(0, T)$ and $\beta(x)$ in $C_0^\infty(R^p)$ satisfies

$$\int_0^T e^{-t}\gamma(t)dt = 1 \qquad \text{and} \qquad \int|\beta(x)|^2 dx = 1$$

respectively.

Then we can obtain easily,

Lemma 2.1. *Let* $g(x, t)$ *be defined in* (2.10). *Then* $g(x, t)$ *satisfies* $(\mu \leqslant n)$

1) $\|\widetilde{\alpha g}\| = 1$,

2) $[g]_{s,\mu} \leqslant \text{const.}\sqrt{\mu}\, n^s \qquad (s=0, 1, 2, \dots\dots)$,

3) $\left[t\frac{\partial g}{\partial x_j}\right]_{0,\mu} \leqslant \text{const.}\frac{n}{\sqrt{\mu}}$,

4) $\left[x_i\frac{\partial g}{\partial x_j}\right]_{0,\mu} + \left[\left(\frac{\partial}{\partial x_j} - in\xi_j^0\right)g\right]_{0,\mu} \leqslant \text{const.}\sqrt{\mu n}$,

where const. is independent of μ and n. Moreover, taking account of Proposition 2.1, we have

Lemma 2.2. Suppose that (1.1) is L^2-well posed and u is a solution of (2.2) for g. Then u satisfies

1) $\sum_{i,j}\left[x_i\frac{\partial u}{\partial x_j}\right]_{0,\mu} \leqslant \text{const.}\left\{\frac{1}{\mu}\sum_{i,j}\left[x_i\frac{\partial g}{\partial x_j}\right]_{0,\mu} + \frac{1}{\mu^2}[g]_{1,\mu}\right\}$

2) $\sum_j\left[t\frac{\partial u}{\partial x_j}\right]_{0,\mu} \leqslant \text{const.}\left\{\frac{1}{\mu}\sum_j\left[t\frac{\partial g}{\partial x_j}\right]_{0,\mu} + \frac{1}{\mu^2}[g]_{1,\mu}\right\}$

3) $\sum_j\left[\left(\frac{\partial}{\partial x_j} - in\xi_j^0\right)u\right]_{0,\mu} \leqslant \text{const.}\left\{\frac{1}{\mu}\sum\left[\left(\frac{\partial}{\partial x_j} - in\xi_j^0\right)g\right]_{0,\mu} + \frac{1}{\mu^2}[g]_{1,\mu}\right\}$

where const. is independent of μ and n.

Proof. 1) applying $x_i\frac{\partial}{\partial x_j}$ to (2.3), we have

$$\begin{cases} \left(\frac{\partial}{\partial t} - A - B\right)\left(x_i\frac{\partial u}{\partial x_j}\right) = x_i\frac{\partial f}{\partial x_j} + \left[A - B,\ x_i\frac{\partial}{\partial x_j}\right]u \\ \left(x_i\frac{\partial u}{\partial x_j}\right)(x, 0) = 0 \end{cases}$$

Hence, by virtue of Proposition 2.1, we obtain

$$\mu\Big[x_i\frac{\partial u}{\partial x_j}\Big]_{0,\mu}\leqslant \text{const.}\Big\{\Big[x_i\frac{\partial f}{\partial x_j}\Big]_{0,\mu}+[u]_{1,\mu}\Big\}$$

$$\leqslant \text{const.}\Big\{\Big[x_i\frac{\partial g}{\partial x_j}\Big]_{0,\mu}+\frac{1}{\mu}[g]_{1,\mu}\Big\}.$$

We can prove 2) and 3) as same as 1).

Taking account of Lemma 2.2, from (2.7) we have

$$[\varphi]_{0,\mu}\leqslant \text{const.}\frac{1}{\sqrt{\mu}}\Big\{\sum_j\Big[t\frac{\partial g}{\partial x_j}\Big]_{0,\mu}+\sum_{i,j}\Big[x_i\frac{\partial g}{\partial x_j}\Big]_{0,\mu}$$

$$+\sum_j\Big[\Big(\frac{\partial}{\partial x_j}-in\xi_j^0\Big)g\Big]_{0,\mu}+\frac{1}{\mu}[g]_{1,\mu}\Big\}.$$

Moreover by virtue of Lemma 2.1, we obtain

$$[\varphi]_{0,\mu}\leqslant \text{const.}\Big\{\frac{n}{\mu\sqrt{\mu}}+\sqrt{\frac{n}{\mu}}\Big\},$$

which and (2.9) imply

$$1\leqslant \text{const.}\Big\{\frac{n^{l-1}}{\mu^l}+\frac{n^{l-\frac{3}{2}}}{\mu^{l-1}}+\frac{\mu}{n}\Big\} \tag{2.11}$$

We put $\mu=n^\delta$, $1>\delta>1-\frac{1}{2(l-1)}$. Then $l\geqslant 2$ contradicts to (2.11), if n is sufficiently large. This proves Theorem 1.1.

§3. Single higher order eqtations.

In this section we shall prove Theorem 1.2. In principle this proof is as same as that of Theorem 1.1.

At first, we introduce an operator Λ_μ; $(\tau=\mu+i\sigma)$,

$$\Lambda_\mu u=\int e^{(\tau t+ix\xi)}(\tau+|\xi|)\hat{u}(\xi,\tau)d\xi d\sigma$$

where $\hat{u}(\xi,\tau)$ is the Fourier-Laplace transform of $u(x,t)$, that is,

$$\hat{u}(\xi,\tau)=\int_0^\infty e^{-\tau t}dt\int e^{-ix\xi}u(x,t)dx.$$

Then it follows easily that

Lemma 3.1. *It holds that for u in $C_0^\infty(R^p\times(0,\infty))$*

1) $[u]_{s,\mu}\leqslant c_1[\Lambda_\mu^s u]_{0,\mu}\leqslant c_2[u]_{s,\mu}$ *for any non negative integer s, where c_1 and c_2 are in dependent of μ.*

2) *for $s=1, 2, 3, \dots$, and for $e^{-\mu t}u(t)$ in $L^2(R^p\times(0,\infty))$*

$$\left(\left(\frac{\partial}{\partial t}\right)^j \Lambda_\mu^{-s}u\right)(x, 0)=0, \qquad (j=0, 1, \dots, s-1).$$

Now we consider the following Cauchy problem,

$$(3.1)\qquad \begin{cases} Pu=\left(\dfrac{\partial}{\partial t}\right)^m u+\displaystyle\sum_{\substack{|\alpha|+j\leqslant m\\ j\leqslant m-1}} a_{\alpha j}(x, t)\left(\frac{\partial}{\partial x}\right)^\alpha\left(\frac{\partial}{\partial t}\right)^j u=f \\ \left(\dfrac{\partial}{\partial t}\right)^i u(x, 0)=0, \qquad (j=0, 1, \dots, m-1). \end{cases}$$

Denote the principal part of P by $P_m\left(x, t; \dfrac{\partial}{\partial t}, \dfrac{\partial}{\partial x}\right)$. As same as Proposition 2.1, it follows from (1.4) that,

Proposition 3.1. *Suppose that Cauchy problem of* (3.1) *is L^2-well posed in $[0, T]$. Then for $\mu>0$ it holds that,*

$$(3.2)\qquad \mu[u]_{m-1,\mu}\leqslant \text{const.}\,[f]_{0,\mu}$$

where const. is independent of μ.

We apply $\Lambda_\mu^{-(m-1)}$ to (3.1). We can write,

$$(3.3)\qquad \begin{cases} P(\Lambda_\mu^{-(m-1)}u)=\Lambda_\mu^{-(m-1)}f+Qu \\ \left(\left(\dfrac{\partial}{\partial t}\right)^i \Lambda_\mu^{-(m-1)}u\right)(x, 0)=0 \ (j=0, 1, \dots, m-1), \end{cases}$$

where Q is a pseudo differential operator of order 0. Applying (3.2) to (3.3), we obtain,

$$(3.4)\qquad \mu[\Lambda_\mu^{-(m-1)}u]_{m-1,\mu}\leqslant \text{const.}\,\{[\Lambda_\mu^{-(m-1)}f]_{0,\mu}+[Qu]_{0,\mu}\}$$

By virtue of 1) of Lemma 3.1, we have

$$[\Lambda_\mu^{-(m-1)}u]_{m-1,\mu}\geqslant \text{const.}\,[u]_{0,\mu}.$$

Moreover, since Q is of order 0, it follows

$$[Qu]_{0,\mu} \leqslant \text{const.}\,[u]_{0,\mu}$$

where const. is independent of μ. Hence we have from (3.4), for μ (sufficiently large),

$$\mu[u]_{0,\mu} \leqslant \text{const.}\,[\Lambda_\mu^{-(m-1)}f]_{0,\mu}, \tag{3.5}$$

The proof of Theorem 1.2. Suppose that there exists a point $\xi_0 = (\xi_1^0, \dots, \xi_p^0)$ in R^p, $|\xi_0| = 1$ such that

$$P_m(0, 0; \lambda, \xi_0) = \prod_{j=0}^{l} (\lambda - \lambda_j)^{\nu_j}, \qquad (\nu_0 \geqslant 2), \tag{3.6}$$

where $\sum_{j=0}^{l} \nu_j = m$, $\lambda_j \neq \lambda_i (i \neq j)$, and λ_j is real.

Multiplying (3.1) by $\alpha(t)e^{-(2\mu + i\lambda_0 nt)}$ and integrating it over $[0, T]$, we can write

$$\widetilde{\alpha f} = P_m(0, 0; (2\mu + i\lambda_0 n), in\,\xi_0)\widetilde{\alpha u} + \tilde{\varphi} + \widetilde{Q_1 u} \tag{3.7}$$

where Q_1 is a differential operator of order $m-1$ and

$$\varphi = \left\{P_m\left(x, t; \frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right) - P_m(0, 0; (2\mu + in\,\lambda_0), in\,\xi_0)\right\}\alpha u.$$

Here we note that it holds, when $\mu \leqslant n$,

$$[Qu]_{0,\mu} \leqslant \text{const.}\,[u]_{m-1,\mu} \tag{3.8}$$

and

$$[\varphi]_{0,\mu} \leqslant \text{const.}\left\{\sum_{i,j}\left[x_i \frac{\partial u}{\partial x_j}\right]_{m-1,\mu} + \sum_j \left[t\frac{\partial u}{\partial x_j}\right]_{m-1,\mu} \right.$$
$$+ \sum_j \sum_{k=0}^{m-1} n^{m-k-1}\left[\left(\frac{\partial}{\partial x_j} - in\,\xi_j^0\right)u\right]_{k,\mu}$$
$$\left. + [u]_{m-1,\mu}\right\} \tag{3.9}$$

For, we can decompose

$$P_m\left(x, t; \frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right) - P_m(0, 0, (2\mu + in\lambda_0), in\xi_0)$$

$$=\sum_{\substack{|\alpha|+j\leqslant m\\ j\leqslant m-1,}}\Big\{a_{\alpha j}(x, t)-a_{\alpha j}(0, 0))\Big(\frac{\partial}{\partial x}\Big)^{\alpha}\Big(\frac{\partial}{\partial t}\Big)^{j}$$

$$+a_{\alpha j}(0, 0)\Big(\Big(\frac{\partial}{\partial x}\Big)^{\alpha}-(in\xi_0)^{\alpha}\Big)\Big(\frac{\partial}{\partial t}\Big)^{j}\Big\}$$

Therefore it follows from (3.6) and (3.7) that

$$\|\widetilde{\alpha f}\|\leqslant\text{const.}\{\mu^{\nu_0}n^{m-\nu_0}\|\widetilde{\alpha u}\|+\|\tilde{\varphi}\|+\|\widetilde{Q_1 u}\|\} \tag{3.10}$$

$$\leqslant\frac{\text{const.}}{\sqrt{\mu}}\{\mu^{\nu_0}n^{m-\nu_0}[\alpha u]_{0,\mu}+[\varphi]_{0,\mu}+[Q_1 u]_{0,\mu}\}$$

As same as Lemma 2.2, making use of (3.2) or (3.5), we can derive

$$\mu\sum_{i,j}\Big[x_i\frac{\partial u}{\partial x_j}\Big]_{m-1,\mu}\leqslant\text{const.}\Big\{\sum_{i,j}\Big[x_i\frac{\partial f}{\partial x_j}\Big]_{0,\mu}+\frac{1}{\mu}[f]_{1,\mu}\Big\},$$

$$\mu\sum_{j}\Big[\Big(\frac{\partial}{\partial x_j}-in\,\xi_j^0\Big)u\Big]_{k,\mu}\leqslant\text{const.}\Big\{\sum_{j}\Big[\Lambda_{\mu}^{-(m-1-k)}\Big(\frac{\partial}{\partial x_j}-in\,\xi_j^0\Big)f\Big]_{0,\mu}$$

$$+\frac{1}{\mu}[\Lambda^{-(m-1-k)}f]_{1,\mu}\Big\},$$

for $k=0, 1, 2, \ldots, m-1$, and

$$\mu\sum_{j}\Big[t\frac{\partial u}{\partial x_j}\Big]_{m-1,\mu}\leqslant\text{const.}\Big\{\sum_{j}\Big[t\frac{\partial f}{\partial x_j}\Big]_{0,\mu}+\frac{1}{\mu}[f]_{1,\mu}\Big\}.$$

Hence from (3.8), (3.9) and (3.10), we have

$$\|\widetilde{\alpha f}\|\leqslant\text{const.}\frac{1}{\sqrt{\mu}}\Big\{\mu^{\nu_0-1}n^{m-\nu_0}[\Lambda_{\mu}^{-(m-1)}f]_{0,\mu}+\frac{1}{\mu}[f]_{0,\mu} \tag{3.11}$$

$$+\frac{1}{\mu^2}[f]_{1,\mu}+\frac{1}{\mu}\sum_{j}\sum_{k=0}^{m-1}\Big[\Lambda_{\mu}^{-(m-1-k)}\Big(\frac{\partial}{\partial x_j}-in\,\xi_j^0\Big)f\Big]_{0,\mu}n^{m-k-1}$$

$$+\sum_{k=0}^{m-1}\frac{n^{m-k-1}}{\mu^2}[\Lambda_{\mu}^{-(m-k-1)}f]_{1,\mu}+\frac{1}{\mu}\sum_{j}\Big[t\frac{\partial f}{\partial x_j}\Big]_{0,\mu}\Big\}.$$

Here we choose $f(x, t)$ as same as g defined in (2.9). Then making use of Lemma 2.1 and noting that

$$[\Lambda_{\mu}^{-k}f]_{0,\mu}\leqslant\text{const.}\sqrt{\mu}\,n^{-k},\qquad (k=0, 1, \ldots),$$

and

$$[\Lambda_{\mu}^{-k}f]_{1,\mu} \leqq \text{const.}\sqrt{\mu}\, n^{-k+1}, \qquad (k=0, 1, 2, \dots),$$

we obtain from (3.11)

$$1 \leqslant \text{const.}\left\{\left(\frac{\mu}{n}\right)^{\nu_0 - 1} + \frac{\sqrt{n}}{\mu} + \frac{n}{\mu^2}\right\}$$

which is not valid, if we put $\mu = n^{\delta}$, $1 > \delta > \frac{1}{2}$, and $\nu_0 \geqslant 2$ and if n is sufficiently large. This proves Theorem 1.2.

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