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A remark on the foliated cobordisms of codimension-one foliated 3-manifolds

by

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Introduction

In [6], Rosenberg and Thurston posed the following problem: Are the Reeb foliations of S^3 foliated cobordant to zero? And Mizutani [5] and Sergeraert [7] gave the affirmative answer.

The purpose of this note is to generalize their result.

Let M^3 be an oriented closed 3-manifold. Then the manifold M^3 has a spinnable structure (cf. Alexander [1]). By the wellknown method [3], we can construct a foliation on M^3 from this spinnable structure $\mathscr{S}$. Let this foliation denote $\mathscr{F}_{\mathscr{S}}$. Note that the Reeb foliations of S^3 are also constructed from a spinnable structure of S^3.

Our main theorem is as follows:

Theorem. *For any oriented closed 3-manifold M^3 with any spinnable structure $\mathscr{S}$, the foliated manifold $(M^3, \mathscr{F}_{\mathscr{S}})$ is foliated cobordant to zero.*

We shall work in the smooth category and all the foliations we shall consider, will be smooth and of codimension one.

§1. Reeb foliations and results of Sergeraert

We consider the Reeb foliation on S^3. Let T^2 be a torus which is a unique compact leaf of this foliation. The *holonomy* along T^2 is a homomorphism of groups, $\mathscr{H}: \pi_1(T^2) \to G$, where G is the set of germs at 0 of C^∞-diffeomorphisms of $\boldsymbol{R}$, $f: \boldsymbol{R} \to \boldsymbol{R}$, with $f(0)=0$. Let p_1, p_2 be the standard generators of $\pi_1(T^2)$. If we orient adequately a small segment transverse to T^2 serving to define $\mathscr{H}$, we may assume that the germs of diffeomorphisms $\mathscr{H}(p_1)$ and $\mathscr{H}(p_2)$ have their support respectively in $(-\varepsilon, 0]$ and $[0, \varepsilon)$. Furthermore $\mathscr{H}(p_1)$ and $\mathscr{H}(p_2)$ are contained in G_∞, where G_∞ is the set consisting of germs at 0 of C^∞-diffeomorphisms of $\boldsymbol{R}$ which are C^∞-tangent to identity at 0, and are fixed point free respectively on $(-\varepsilon, 0)$ and $(0, \varepsilon)$. The Reeb foliations are characterized by conjugates of $(\mathscr{H}(p_1), \mathscr{H}(p_2))$. Put $g_1=\mathscr{H}(p_1)$ and $g_2=\mathscr{H}(p_2)$. We denote by $\mathscr{F}(g_1, g_2)$ the associated Reeb foliation. In this section we shall recall the following theorem due to Sergeraert [7].

Theorem 1. *$\mathscr{F}(g_1, g_2)$ is foliated cobordant to zero.*

This foliation is represented by a homotopy class of a mapping f: $S^3 \to B\Gamma_1^\infty$, where $B\Gamma_1^\infty$ denotes the Haefliger classifying space. In our case the image of S^3 by f is contained in $B\bar{\Gamma}_1^\infty$, where $B\bar{\Gamma}_1^\infty$ denotes the homotopy fiber of the map ν: $B\Gamma_1^\infty \to BO_1$. We denote by $\pi_3(g_1, g_2)$ this homotopy class of $\pi_3(B\bar{\Gamma}_1^\infty)$ and $H_3(g_1, g_2)$ the homology class of $H_3(B\bar{\Gamma}_1^\infty)$ corresponding to $\pi_3(g_1, g_2)$ via the Hurewicz homomorphsm. This is the image of the fundamental class $[S^3]$ by the Hurewicz homomorphism $H_3(f)$.

Proposition 2. *The Hurewicz homomorphism $H_3: \pi_3(B\bar{\Gamma}_1^\infty) \to H_3(B\bar{\Gamma}_1^\infty)$ is an isomorphism.*

Proof. It is trivial from such a fact that $B\bar{\Gamma}_1^\infty$ is $2-$connected (Haefliger [2], Mather [4]).

Theorem 3 (*Sergeraert* [7]). *For any g_1, g_2 in G_∞, which have their support respectively in $(-\varepsilon, 0]$ and $[0, \varepsilon)$, and are fixed point free respectively on $(-\varepsilon, 0)$ and $(0, \varepsilon)$, $H_3(g_1, g_2)=0$.*

Let $\mathrm{Diff}_K^\infty(\boldsymbol{R})$ be the group of C^∞-diffeomorphisms of $\boldsymbol{R}$ with compact support, equipped with the discrete topology. Now we consider the Eilenberg-Maclane homology of $\mathrm{Diff}_K^\infty(\boldsymbol{R})$. If g_1 and g_2 in $\mathrm{Diff}_K^\infty(\boldsymbol{R})$ commute, then $(g_1, g_2)-(g_2, g_1)$ is a 2-cycle. Let denote this homology class by $H_2(g_1, g_2)$. In particular, if g_1 has the support in $(-\infty, 0]$ and g_2 in $[0, \infty)$, then g_1 and g_2 commute. Hence the homology class $H_2(g_1, g_2)$ is defined.

Theorem 4 (*Sergeraert* [7]). *If the supports of g_1 and g_2 are contained respectively in $(-\infty, 0]$ and $[0, \infty)$, then $H_2(g_1, g_2)=0$.*

Proof. Let D_1(resp. D_2) be the subgroup consisting of elements of $\mathrm{Diff}_K^\infty(\boldsymbol{R})$ whose supports are in $(-\infty, 0]$ (resp. $[0, \infty)$). There is a canonical inclusion $\iota: D_1 \times D_2 \to \mathrm{Diff}_K^\infty(\boldsymbol{R})$ defined by $\iota(g_1, g_2)=g_1 g_2$.

Therefore it is sufficient to prove that the homology class $H_2(g_1, g_2)$ is zero in $H_2(D_1\times D_2)$. From Künneth formula, $H_2(D_1\times D_2)\approx H_2(D_1)\oplus(H_1(D_1)\otimes H_1(D_2))\oplus H_2(D_2)$. This canonical isomorphism decomposes $H_2(g_1, g_2)$ into $H_2(g_1, e)\oplus(H_1(g_1)\otimes H_1(g_2))\oplus H_2(e, g_2)$, where e is a unit element. It is easy to see that the first and third parts are zero. On the other hand, $H_1(D_1)=0$ from the following result [7]: $\mathrm{Diff}^\infty_\infty([0, 1])$ is perfect, where $\mathrm{Diff}^\infty_\infty([0, 1])$ is the group of C^∞-diffeomorphisms of $[0, 1]$ which are C^∞-tangent to identity at 0 and 1. Hence $H_1(g_1)=H_1(g_2)=0$. This completes the proof.

Proof of Theorem 3. The germ g_1(resp. g_2) is the germ of an element $\hat{g}_1$(resp. $\hat{g}_2$) of D_1(resp. D_2). Furthermore we can assume that $\hat{g}_1(x)=x$, $\hat{g}_2(x)=x$ if $|x|\geqq 1$ and $\hat{g}_1$(resp. $\hat{g}_2$) is fixed point free on $(-1, 0)$ (resp. $(0, 1)$). Let $\mathscr{H}: \pi_1(T^2)\to\mathrm{Diff}^\infty_\infty([-1, 1])$ be the homomorphism which maps p_1 and p_2 to $\hat{g}_1$ and $\hat{g}_2$ respectiviely. We can construct a foliation on $T^2\times[-1, 1]$ whose global holonomy is $\mathscr{H}$. We define an equivalence relation $\sim$ on $T^2\times[-1, 1]$ as follows: for (θ_1, θ_2, t), $(\theta_1', \theta_2', t')\in T^2\times[-1, 1]$, $(\theta_1, \theta_2, t)\sim(\theta_1', \theta_2', t')$ if and only if $\theta_1=\theta_1'$ when $t=t'=1$, $\theta_2=\theta_2'$ when $t=t'=-1$, and $\theta_1=\theta_1'$, $\theta_2=\theta_2'$ and $t=t'$, otherwise. The foliation on $T^2\times[-1, 1]$ induces a Γ_1^∞-structure on S^3 under this quotient map, which is denoted by $\mathscr{F}(\hat{g}_1, \hat{g}_2)$. This Γ_1^∞-structure resembles the Reeb foliation $\mathscr{F}(g_1, g_2)$. Let $g: S^3\to B\bar{\Gamma}_1^\infty$ be a map representing the Γ_1^∞-structure $\mathscr{F}(\hat{g}_1, \hat{g}_2)$.

On the other hand, Mather [4] constructed an isomorphism S: $H_2(\mathrm{Diff}^\infty_K(\boldsymbol{R}))\to H_3(B\bar{\Gamma}_1^\infty)$. We can see from the construction of this isomorphism that $S(H_2(\hat{g}_1, \hat{g}_2))=H_3(g)([S_3])$, where $[S^3]$ is the fundamental homology class of S^3.

Lemma 5. *Γ_1^∞-structures $\mathscr{F}(g_1, g_2)$ and $\mathscr{F}(\hat{g}_1, \hat{g}_2)$ are homotopic.*

Proof. See [7. Lemma 6. 9].

Therefore the map $g: S^3\to B\bar{\Gamma}_1^\infty$ is a map associated with the Reeb foliation $\mathscr{F}(g_1, g_2)$. Hence $H_3(g_1, g_2)=H_3(g)([S^3])=S(H_2(\hat{g}_1, \hat{g}_2))=0$ (from Theorem 4). This completes the proof of Theorem 3.

Proof of Theorem 1. From Proposition 2 and Theorem 3, we have $\pi_3(g_1, g_2)=0$, *i. e.*, $f: S^3\to B\bar{\Gamma}_1^\infty$ is homotopic to a constant map $f_0(p)=x_0$ for any $p\in S^3$, where x_0 is a base point of $B\bar{\Gamma}_1^\infty$. Choose a compact oriented 4-manifold V^4 such that $\partial V^4=S^3$ and its Euler number vanishes. Let $\partial V\times[0, 1]$ $(\subset V^4)$ be a collar neighborhood of ∂V, and $F: \partial V\times[0, 1]\to B\bar{\Gamma}_1^\infty$ denote a homotopy of f and f_0, *i. e.*, $F|_{\partial V\times\{0\}}=f$ and $F|_{\partial V\times\{1\}}=f_0$. Then we define a map $H: V^4\to B\bar{\Gamma}_1^\infty$ as follows:

$$H(p)=\begin{cases}F(p) & \text{for } p\in\partial V\times[0,1],\\ x_0 & \text{otherwise.}\end{cases}$$

Since the Euler number of V^4 vanishes, we can extend any vector field on S^3 transverse to $\mathscr{F}(g_1, g_2)$ to V^4 without singularities. From the theorem of Thurston [8, Theorem 2], there exists a C^∞-foliation $\mathscr{G}$ on V^4 such that $\mathscr{G}|_{\partial V}=\mathscr{F}(g_1, g_2)$. This completes the proof.

§2. Statement of results

A closed 3-manifold M is called *spinnable* if there exists a 1-submanifold X, which is a finite union of circles, called an axis, satisfying the following conditions: 1) the normal bundle of X is trivial, 2) let $X\times D^2$ be a tubular neighborhood of X, then $M\text{-}X\times intD^2$ is the total space of a fiber bundle ξ over a circle S^1, and 3) let p: $M\text{-}X\times intD^2\to S^1$ be the projection of ξ, then the diagram

$$\begin{array}{ccc} X\times S^1 & \xrightarrow{\iota} & M\text{-}X\times intD^2 \\ & {}_{p'}\searrow \quad \swarrow_{p} & \\ & S^1 & \end{array}$$

commutes, where ι denotes the inclusion and p' denotes the projection onto the second factor. The pair $\mathscr{S}=(X, \xi)$ is called a *spinnable strurture* on M. We can construct a foliation on M from a spinnable structure $\mathscr{S}=(X, \xi)$ as follows. Our problem is to extend the foliation of $M-X\times intD^2$, given naturally by p, to $X\times D^2$. Choose the polar coordinates on D^2, (θ, r), where θ is the polar angle mod. 1 and r is the radius, $0\leqq r\leqq 1$.

At first we construct a foliation on the anullus $A=\{(\theta, r)\in D^2;\ 1/2\leqq r\leqq 1\}$ choosing a C^∞-vector field v on A such that $v=\frac{\partial}{\partial r}$ for $3/4\leqq r\leqq 1$ and $v=\frac{\partial}{\partial\theta}$ for $r=1/2$ (see Fig. 1).

A

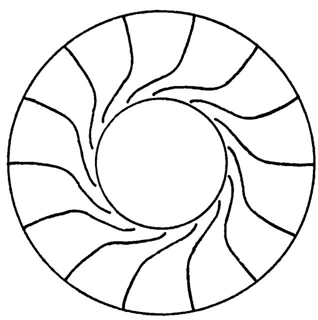

Fig. 1.

Defining each on $X\times A$ to be a product of a orbit of the flow v and a connected component of X, we can extend the foliation of $M-X\times intD^2$ to $X\times A$ naturally. Note that $X\times\partial_0 A$ is a union of tori, where $\partial_0 A=\{(\theta, r)\in A; r=1/2\}$. The place where we do not construct a foliation is $X\times D(1/2)$, which is a finite union of solid tori, where $D(1/2)=\{(\theta, r)\in D^2; 0\leqq r\leqq 1/2\}$. Therefore we put the Reeb component into each solid torus. We denote this foliation by $\mathscr{F}_{\mathscr{S}}$.

Remark 1. In the above construction, there is an ambiguity for an orientation of the Reeb component (see Mizutani [5] for definition).

Remark 2. When the number of connected components of X is greater than one, we can construct another foliation on M, which is different from $\mathscr{F}_{\mathscr{S}}$ on $X\times A$. Choose a C^∞-vector field v' on the anullus A such that $v'=\frac{\partial}{\partial r}$ for $3/4\leqq r\leqq 1$ and $v'=-\frac{\partial}{\partial\theta}$ for $r=1/2$. We define a foliation on $X\times A$ by putting foliations induced from the vector fields v and v' on $X_1\times A$ and $X_2\times A$ respectively, where X_1 and X_2 are connected components of X such that $X_1\cup X_2=X$. We denote this foliation by $\mathscr{F}_{\mathscr{S}}'$.

Theorem 6. *For any closed oriented 3-manifold M^3 with any spinnable strurture $\mathscr{S}$, $(M, \mathscr{F}_{\mathscr{S}})$ is foliated cobordant to zero.*

Theorem 7. *For any closed oriented 3-manifold M^3 with any spinnable strurture $\mathscr{S}$, $(M, \mathscr{F}_{\mathscr{S}}')$ is foliated cobordant to zero.*

§3. Proof of Theorem 6

Let $S^1\times[0, 2]$ be an anullus with natural coordinates (θ, t). We define a foliation on the anullus $S^1\times[0, 2]$ by choosing a C^∞-vector field u such that $u=\frac{\partial}{\partial t}$ for $0\leqq t\leqq 1/2$ and $u=-\frac{\partial}{\partial\theta}$ for $1\leqq t\leqq 2$. And we can lift this foliation to $\{M-X\times intD^2\}\times[0, 2]$ via the map $p\times identity$, where p denotes the projection of ξ. From definition of spinnable structure, we see that θ in the above coordinates is identified with the polar angle in the polar coordinates of D^2 in § 2. We denote by $\mathscr{F}_1$ the foliation on $\{M-X\times intD^2\}\times[0, 2]$. $\mathscr{F}_1$ restricted to $\{M-X\times intD^2\}\times\{0\}$ is $\mathscr{F}_{\mathscr{S}}$ restricted to $M-X\times intD^2$ and $\mathscr{F}_1$ restricted to $\{M-X\times intD^2\}\times[1, 2]$ is a product foliation such that each leaf is defined by $\{M-X\times intD^2\}\times\{t\}$, $t\in[1, 2]$. Furthermore we investigate the foliation on a boundary of $\{M-X\times intD^2\}\times[0, 2]$, $X\times S^1\times[0, 2]$. $\mathscr{F}_1$ restricted to $X\times S^1\times[0, 2]$ is the foliation lifted from the above foliation on the anullus $S^1\times[0, 2]$, that is, $\mathscr{F}_1$ restricted to $X\times S^1\times\{0\}$ is a foliation such that each leaf is defined by {a connected component of $X\} \times \{\theta\}$, $\theta \in S^1$ and $\mathscr{F}_1$ restricted to $X \times S^1 \times [1, 2]$ is a product foliation such that each leaf is defined by a connected component of $X \times S^1 \times \{t\}$, $t \in [1, 2]$, which is a torus. Let $f_1 : \{M - X \times intD^2\} \times [0, 2] \to B\bar{\Gamma}_1^\infty$ be a map representing the Γ_1^∞-structure $\mathscr{F}_1$. Since $\mathscr{F}_1$ restricted to $\{M - X \times intD^2\} \times [1, 2]$ is the product foliation, we may assume that f_1 restricted to $\{M - X \times intD^2\} \times [3/2, 2]$ is a constant map, *i.e.*, $f_1(p) = x_0$ for any p in $\{M - X \times intD^2\} \times [3/2, 2]$, where x_0 denotes a base point of $B\bar{\Gamma}_1^\infty$. Without loss of generality, we may assume the number of connected components of the axis X is equal to one, *i.e.*, X is a circle. Put $Y = X \times S^1 \times [0, 2] \cup X \times D^2 / \sim$, where $\sim$ is an equivalence relation which identifies $X \times S^1 \times \{0\}$ with $X \times \partial D^2$. This is a solid torus. Note that Y has a foliation $\mathscr{F}_2$ as follows: $\mathscr{F}_2$ on $X \times S^1 \times [0, 2]$ is defined by $\mathscr{F}_1$ restricted to $X \times S^1 \times [0, 2]$ and $\mathscr{F}_2$ on $X \times D^2$ is defined by $\mathscr{F}_{\mathscr{S}}$ restricted to $X \times D^2$. Let $f_2 : Y \to B\bar{\Gamma}_1^\infty$ be a map representing the Γ_1^∞-structure $\mathscr{F}_2$ such that f_2 restricted to $X \times S^1 \times [0, 2]$ is equal to f_1 restricted to $X \times S^1 \times [0, 2]$.

Now we shall prove Theorem 6 assuming that f_2 is homotopic to the constant map $f_0 (f_0(p) = x_0$ for any p in $Y)$, relative to $X \times S^1 \times [3/2, 2]$. Choose an oriented 4-manifold V_1 such that $\partial V_1 = M$ and the Euler number of V_1 vanishes. (This is possible.) Let $F_s (0 \leqq s \leqq 1)$ be a homotopy relative to $X \times S^1 \times [3/2, 2]$ from f_2 to f_0, *i.e.*, $F_0 = f_2$ and $F_1 = f_0$. Put $V = V_1 \cup M \times [0, 2] / \sim$, where $\sim$ is an equivalence relation which identifies ∂V_1 with $M \times \{2\}$. And let $N = Y \times [0, 1]$ be a one-sided tubular neighborhood of Y in $M \times [0, 2]$ such that $Y \times \{0\}$ corresponds to Y (see Fig. 2).

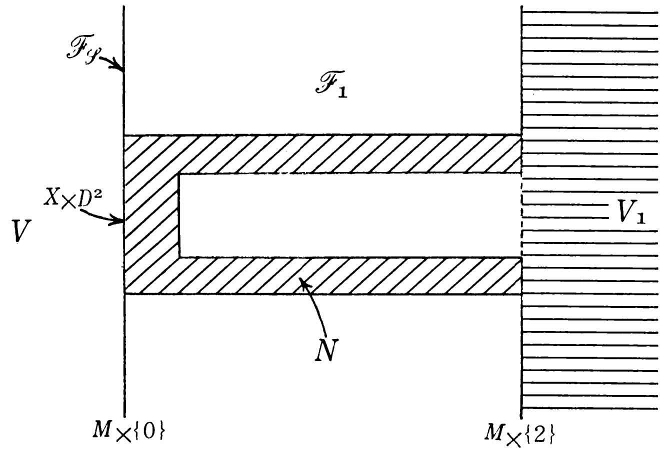

Fig. 2.

Then we can define a map H: $V\times B\bar{\Gamma}_1^\infty$ as follows:

$$H(p)=\begin{cases} f_1(p) & \text{for } p\in\{M-X\times intD^2\}\times[0,\ 2],\\ F_s(q) & \text{for } p=(q,\ s)\in N=Y\times[0,\ 1],\\ x_0 & \text{otherwise.}\end{cases}$$

Hence by Thurston's Theorem [8, Theorem 2], we can extend the foliation $\mathscr{F}_{\mathscr{S}}$ on M to V as in the proof of Theorem 1 in § 1.

Construction of a homotopy of f_2 and f_0

We will construct a Γ_1^∞-structure on Y which is homotopic to the Γ_1^∞-structure $\mathscr{F}_2$ by the same way as in § 1. Let a torus T^2 be an isolated compact leaf of $\mathscr{F}_2$ and a homomorphism $\mathscr{H}$: $\pi_1(T^2)\rightarrow G$ the holonomy. Let p_1, p_2 be the standard generators of $\pi_1(T^2)$ which is mapped to the germs of diffeomorphisms having their support respectively in $(-\varepsilon,\ 0]$ and $[0,\ \varepsilon)$, by the map $\mathscr{H}$. Furthermore $\mathscr{H}(p_1)$ and $\mathscr{H}(p_2)$ are C^∞-tangent to identity at 0 and are fixed point free respectively on $(-\varepsilon,\ 0)$ and $(0,\ \varepsilon)$. As in § 1, the germ $\mathscr{H}(p_1)$ (resp. $\mathscr{H}(p_2)$) is represented by an element $\hat{g}_1$(resp. $\hat{g}_2$) of D_1(resp. D_2) such that $\hat{g}_1(x)=x$, $\hat{g}_2(x)=x$ if $|x|\geqq 1$ and $\hat{g}_1$(resp. $\hat{g}_2$) is fixed point free on $(-1,\ 0)$ (resp. $(0,\ 1)$). Let $\hat{\mathscr{H}}$: $\pi_1(T^2)\rightarrow \mathrm{Diff}_\infty^\infty([-1,\ 2])$ be the homomorphism which maps p_1 and p_2 to $\hat{g}_1$ and $\hat{g}_2$ respectively. Therefore we can construct a foliation on $T^2\times[-1,\ 2]$ whose global holonomy is $\hat{\mathscr{H}}$. We define an equivalence relation $\approx$ on $T^2\times[-1,\ 2]$ as follows: for $(\theta_1,\ \theta_2,\ t)$, $(\theta_1',\ \theta_2',\ t')\in T^2\times[-1,\ 2]$, $(\theta_1,\ \theta_2,\ t)\approx(\theta_1',\ \theta_2',\ t')$ if and only if $\theta_2=\theta_2'$ when $t=t'=-1$ and $\theta_1=\theta_1'$, $\theta_2=\theta_2'$, and $t=t'$ otherwise. Then the quotient space $T^2\times[-1,\ 2]/\approx$ is homeomorphic to Y. The foliation on $T^2\times[-1,\ 2]$ induces a Γ_1^∞-structure on Y under this quotient map, which is denoted by $\mathscr{F}_2'(\hat{g}_1,\ \hat{g}_2)$. This Γ_1^∞-structure resembles the Γ_1^∞-structure $\mathscr{F}_2$ on Y.

On the other hand, we can define a quotient map

$$\begin{array}{ccc} q:\ T^2\times[-1,2]/\approx & \longrightarrow & T^2\times[-1,2]/\sim \\ \| & & \| \\ Y & & S^3 \end{array}$$

where the relation $\sim$ is a relation which adds to the relation $\approx$ a following condition: $(\theta_1,\ \theta_2,\ t)\sim(\theta_1',\ \theta_2',\ t')$ if $\theta_1=\theta_1'$ when $t=t'=2$. Let $\mathscr{F}'(\hat{g}_1,\ \hat{g}_2)$ denote the Γ_1^∞-structure on S^3 as in § 1. The map q carries the Γ_1^∞-structure $\mathscr{F}_2'(\hat{g}_1,\ \hat{g}_2)$ on Y to the Γ_1^∞-structure $\mathscr{F}'(\hat{g}_1,\ \hat{g}_2)$ on S^3. If $f: S^3\rightarrow B\bar{\Gamma}_1^\infty$ is a map representing the Γ_1^∞-structure $\mathscr{F}'(\hat{g}_1,\ \hat{g}_2)$, then the composition map $f\circ q$ represents the Γ_1^∞-structure $\mathscr{F}_2'(\hat{g}_1,\ \hat{g}_2)$ on Y. We can assume $f\circ q(p)=x_0$ for any p in $T^2\times[3/2,\ 2]$. Using the same method in the proof of Lemma 5, we can see that f_2 is homotopic to $f\circ q$

relative to $X\times S^1\times[3/2, 2]$. By the arguement in § 1, we see that f is homotopic to the constant map f_0. Since $B\bar{\Gamma}_1^\infty$ is 2-connected, f_2 is homotopic to the constant map f_0 relative to $X\times S^1\times[3/2, 2]$.

Corollary 8. *The Γ_1^∞-structure $\mathscr{F}_{\mathscr{S}}$ on M is homotopic to a trivial one.*

§ 4. Proof of Theorem 7

It is sufficient to prove for the case of the foliation constructed using the vector field v' (see Remark 2 in § 2). In this case, the foliation restricted to $B=X\times S^1\times[0, 1]\cup X\times D^2$ is as follows.

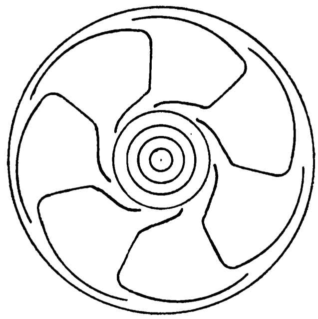

Fig. 3.

Put $C=B\cup D^2\times S^1/\sim$, where $\sim$ is an equivalence relation which identifies $X\times S^1\times\{1\}$ with $\partial D^2\times S^1$. Note that C is homeomorphic to a 3-sphere. We put an oriented Reeb component on the solid torus as follows. Let α be a C^∞-function $\alpha:[0, 1)\to\boldsymbol{R}$, such that $\alpha(0)=0$, $\alpha'(t)>0$ for all $t\in(0, 1)$, $\alpha^{(k)}(0)=0$, $\lim_{t\to1}\alpha^{(k)}(t)=\infty$ for all k. Express a point p of $D^2\times S^1$ as $p=(t, x, \theta)$, $(t, x)\in D^2, \theta\in S^1$, t is the radius $(0\leqq t\leqq1)$ and x is the polar angle mod. 1. Define a foliation on $D^2\times S^1$ as follows: for two points $p=(t, x, \theta)$, $p'=(t', x', \theta')$ of $D^2\times S^1$, $L_p=L_{p'}$ if and only if $t=t'=1$ or $\alpha(t)-\theta\equiv\alpha(t')-\theta'(\text{mod. }1)$, where L_p is the leaf that contains p. We denote this foliation on the 3-sphere C by $\mathscr{F}_3$

Proposition 9. *$(C, \mathscr{F}_3)$ is foliated cobordant to zero.*

Proof. This foliation $\mathscr{F}_3$ and a Reeb foliation are concordant because $\mathscr{F}_3$ is obatined from the Reeb foliation by perturbing along a transversal simple curve. From Theorem 1, the Reeb foliation is foliated cobordant to zero. Hence $(C, \mathscr{F}_3)$ is so.

We consider the foliation on $X\times S^1\times[1, 2]\cup D^2\times S^1/\sim$, where $\sim$ is an equivalence relation which identifies $X \times S^1 \times \{1\}$ with $\partial D^2 \times S^1$. This is a special case of the foliation $\mathscr{F}_2$ on Y in §3. Therefore by the same method as in the proof of Theorem 6, we can prove Theorem 7.

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