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Orbital decompositions of invariant distributions on a locally compact zero-dimensional space

Dedicated to Professor Hisaaki Yoshizawa on his 60th birthday

By

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Introduction.

Let G be a locally compact totally disconnected group and X a locally compact zero-dimensional space. Let G act on X continuously. We denote by $C_c^\infty(X)$ the space of compactly supported locally constant functions on X. We call an element of the algebraic dual $C_c^\infty(X)^*$ of $C_c^\infty(X)$, a *distribution* on X. A distribution T on X is called *invariant* if $T(f)=T(f^g)$ for all $g\in G$ and $f\in C_c^\infty(X)$, where $f^g(x)=f(g^{-1}x)$, $x\in X$. We denote by $C_c^\infty(X)^{*G}$ the space of all invariant distributions on X.

Assume that, for each G-orbit $\mathcal{O}$ in X, there is a non-zero invariant distribution on X supported by $\mathcal{O}$, called an *orbital distribution* associated with $\mathcal{O}$. In the present paper we treat a problem of expressing an invariant distribution on X as a "superposition" of orbital ones (Orbital decomposition problem).

Let $\Lambda=G\backslash X$ be the orbit space of X with respect to the group action. For each orbit $\lambda\in\Lambda$, we choose a suitably normalized orbital distribution T_λ associated with λ. We define a linear map Θ of $C_c^\infty(X)$ to the space V of all complex valued functions on Λ as follows:

$$\Theta(f)(\lambda)=T_\lambda(f) \qquad (\lambda\in\Lambda, f\in C_c^\infty(X)).$$

Then the dual Θ^* of Θ maps V^* into $C_c^\infty(X)^{*G}$.

Our results in this general setting are stated as follows.

Weak Decomposition Theorem (Theorem 1.8). *Assume that for each $x\in X$, the intersection of all closed open invariant subsets containing x consists of finitely many orbits (Property (W)). Then the space $C_c^\infty(X)^{*G}$ of all invariant distributions on X is exactly equal to the image of the dual Θ^* of Θ.*

Strong Decomposition Theorem (Theorem 3.7). *Assume that the saturation under G of each compact subset of X is closed in X (Property (S)). Then for each $f\in C_c^\infty(X)$, the function $\Theta(f)$ on Λ is locally constant and compactly supported with respect to the quotient topology on Λ. And the dual Θ^* establishes an isomorphism* *of* $C_c^\infty(\Lambda)^*$ *onto* $C_c^\infty(X)^{*G}$, *that is, an arbitrary invariant distribution* T *on* X *is expressed as* $T(f)=\int T_\lambda(f)d\tau(\lambda)$ $(f\in C_c^\infty(X))$. *Here* τ *is a distribution on* Λ *uniquely determined by* T.

We call a distribution m on X a *Radon measure* if for an arbitrary compact subset K of X, there is a positive constant M_K such that $|m(f)|\leq M_K\cdot\|f\|$ for all $f\in C_c^\infty(X)$ with $\mathrm{supp}(f)\subset K$. Here $\|f\|$ is the supremum norm of f. In the strong decomposition theorem, the space $\mathscr{M}(\Lambda)$ of the Radon measures on Λ corresponds bijectively to the space $\mathscr{M}(X)^G$ of invariant Randon measures on X.

Basing ourselves on the above two extreme cases, we inquire further into the problem for an intermediate case. Suppose the space X is decomposed as a disjoint union $Y\cup Z$ with a closed invariant subset Y and an open invariant subset Z. Then an invariant distribution on X may be decomposed into those on Y and on Z. This is treated in 1.3. Applying Weak Decomposition Theorem, we get there the two exact sequences for the spaces of invariant distributions and invariant Randon measures:

$$0\longleftarrow C_c^\infty(Z)^{*G}\longleftarrow C_c^\infty(X)^{*G}\longleftarrow C_c^\infty(Y)^{*G}\longleftarrow 0, \tag{1}$$

$$0\longleftarrow \mathscr{M}(Z)_b^G\longleftarrow \mathscr{M}(X)^G\longleftarrow \mathscr{M}(Y)^G\longleftarrow 0. \tag{2}$$

Here $\mathscr{M}(Z)_b^G$ is a subspace of $\mathscr{M}(Z)^G$ defined by a condition "finite at infinities of Z" (see Corollary 1.17). The sequence (2) splits canonically. So we have a canonical isomorphism of $\mathscr{M}(X)^G$ onto $\mathscr{M}(Y)^G\oplus\mathscr{M}(Z)_b^G$. We assume in addition that the G-space Z has Property (S). Then, by Strong Decomposition Theorem, we have an isomorphism of $C_c^\infty(Z)^{*G}$ onto $C_c^\infty(\mathscr{Z})^*$, and an isomorphism of $\mathscr{M}(X)^G$ onto $\mathscr{M}(Y)^G\oplus\mathscr{M}(\mathscr{Z})_b$, where $\mathscr{Z}$ is the orbit space of Z, and $\mathscr{M}(\mathscr{Z})_b$ is a subspace of $\mathscr{M}(\mathscr{Z})$ corresponding to $\mathscr{M}(Z)_b^G$.

By a successive use of the above argument, we get in §3 the following orbital decomposition theorem.

Theorem 3.9. *Assume that* X *has a finite filtration* $X=X_0\supset X_1\supset\cdots\supset X_r=\phi$ *by closed invariant subsets such that the* G*-space* $X_{i-1}-X_i$ $(1\leq i\leq r)$ *has Property* (S). *Then every invariant Radon measure* T *can be expressed as follows:*

$$T(f)=\sum_{i=1}^{r}\int_{\lambda\in\mathscr{Z}_i}T_\lambda(f)d\tau_i(\lambda)\qquad (f\in C_c^\infty(X)),$$

where τ_i *is a uniquely determined Radon measure on the orbit space* $\mathscr{Z}_i$ *corresponding to the* G*-space* $X_{i-1}-X_i$.

Now let k be a non-archimedean local field of characteristic zero. In §2, we prove that the assumption of Weak Decomposition Theorem is fulfilled in the case that $G=GL(n, k)$ or its connected semisimple algebraic subgroup defined over k, and $X=G$ with inner automorphisms.

After establishing Strong Decomposition Theorem in §3, we prove in §4 the existence of a filtration for $G=X=GL(n, k)$. So we get Theorem 4.16, an orbital decomposition in this case.

In §5, the relationships between our Strong Decomposition Theorem and Choquet's integral representation theory, in case of Radon measures, are considered.

In the previous paper [8], we treated invariant distributions on $SL(2, k)$. An explicit splitting of the exact sequence (1) is given there for $G=X=SL(2, k)$, $Z=$ {regular semisimple elements}, and $Y=X-Z$. So an arbitrary invariant distribution on $SL(2, k)$ is "strongly" decomposed into orbital ones. Our Theorem 4.16 deals only with invariant Randon measures on $GL(n, k)$. The strong decomposition theorem available for all invariant distributions on $GL(n, k)$ is not yet known to the author.

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§ 1. Weak decomposition theorem

1.1. Group actions on topological spaces.

We call a topological space X an *l-space* if it is Hausdorff, locally compact, and zero-dimensional, that is, each point has a fundamental system of open compact neighbourhoods. We call a topological group G an *l-group* if it has a fundamental system of neighbourhoods of the unit element consisting of open compact subgroups. It can be shown that a topological group is an l-group if and only if it is an l-space. Throughout, when we say that "an l-group G acts on an l-space X", we always mean a continuous left action.

Definition 1.1. Let be an l-space. We denote by $C_c^\infty(X)$ the space of all locally constant complex-valued functions on X with compact support. We call an element of the algebraic dual $C_c^\infty(X)^*$ of $C_c^\infty(X)$ a *distribution* on X. Note that $X_c^\infty(X)$ and $C_c^\infty(X)^*$ are treated without any topology. If $f\in C_c^\infty(X)$ and $m\in C_c^\infty(X)^*$, then the value of m at f is denoted by $\int f(x)dm(x)$.

Definition 1.2. Let X be an l-space. A distribution m on X is called *positive* if $m(f)\geq 0$ for each non-negative $f\in C_c^\infty(X)$. A distribution on X is called a *complex Radon measure* on X if it can be expressed as a linear combination of positive ones.

Proposition 1.3. *Let X be an l-space. A distribution m on X is a complex Radon measure if and only if for an arbitrary compact subset K of X, there is a positive constant M_K such that $|m(f)|\leq M_K\cdot\|f\|$ if* supp $(f)\subset K$. *Here $\|f\|$ is the supremum norm of f.*

Definition 1.4. Let G be an l-group, X an l-space, and ρ: $G\times X\to X$ an action of G on X. The *saturation* of a subset A of X is the subset of X consisting of all elements $\rho(g, a)$ with $g\in G$ and $a\in A$, and is denoted by $G(A)$. When A is a one-point set $\{x\}$ the saturation of A is called the *orbit* of x and is denoted by $G(x)$. A subset B is called *invariant* if $G(B)=B$. A subset of X is called a *domain* if it is closed, open, and invariant. A domain is called a *tube* if it is the saturation of a compact subset of X.

If A and B are domains (resp. tubes), $A \cup B$, $A \cap B$, $A-B$, and $A \triangle B$ are domains (resp. tubes).

Definition 1.5. Let ρ be an action of an l-group G on an l-space X. A distribution m on X is called *invariant* if it satisfies $m(\rho(g)f)=m(f)$ ($f \in C_c^\infty(X)$ and $g \in G$), where $(\rho(g)f)(x)=f(\rho(g^{-1}, x))$. We denote by $C_c^\infty(X)^{*G}$ the space of all invariant distributions on X.

A distribution m on X is invariant if and only if m vanishes on the linear subspace $I_G(X)$ of $C_c^\infty(X)$ spanned by all elements $\rho(g)f-f$ with $f \in C_c^\infty(X)$ and $g \in G$.

Definition 1.6. Let an l-group G act on an l-space X. Let $x \in X$. If the following four conditions are fulfilled, we call the orbit $G(x)$ *admissible.*

(1) The orbit $G(x)$ is locally closed in X.

(2) The quotient space G/G_x with left translation and the space $G(x)$ are isomorphic as topological G-spaces, where G_x is the isotropy subgroup of x.

(3) The G-space $G(x)$ carries a G-invariant Radon measure dm.

(4) The integral $\int_{G(x)} f(y)dm(y)$ converges for all $f \in C_c^\infty(X)$.

In this case we have an invariant distribution on X by the integral in (4). Such a distribution is called the *orbital distribution* associated with the orbit $G(x)$.

1.2. The following result is fundamental.

Proposition 1.7. *Let an l-group G act on an l-space X transitively. Suppose the unique orbit X is admissible. Let m be its orbital distribution. Then* Ker $(m)=I_G(X)$.

Proof. See R. Howe [6] or S. Matsumoto [8].

Let an l-group G act on an l-space X. We name the following, *Property* (W).

Property (W): For each $x \in X$, the intersection of all domains containing x consists of finitely many orbits.

We state our weak orbital decomposition theorem.

Theorem 1.8. *Let an l-group G act on an l-space X. We assume that each orbit is admissible and the space X has Property* (W). *Let $f \in C_c^\infty(X)$. Then the following conditions for f are mutually equivalent*:

(1) $f \in I_G(X)$.

(2) *f is annihilated by all invariant distributions on X.*

(3) *f is annihilated by all orbital distributions on X.*

This subsection is devoted to prove that (3) implies (1). Implications (1)$\Rightarrow$(2)$\Rightarrow$(3) are clear.

First, let us sketch the proof. Given an element $f \in C_c^\infty(X)$ annihilated by all orbital distributions. We prove that $f \in I_G(X)$. For this purpose it is enough to show that f belongs to $I_G(X)$ "locally" in the sense that for each $x \in X$, there is a domain $D \ni x$ such that $\chi_D \cdot f \in I_G(X)$. Here χ_D denotes the characteristic function of D. Because the fact that $f \in I_G(X)$ follows easily from this by the partition of unity argument. On the other hand in order to prove that f belongs to $I_G(X)$ locally, we use Property (W) essentially.

Now we prepare some terminologies and lemmas for our proof. We call a subset of X a *complex* if it is closed and consists of finitely many orbits. Let $\mathcal{O}$ be an orbit. We call the subset $\bar{\mathcal{O}} - \mathcal{O}$ of X the *boundary* of $\mathcal{O}$ and denote it by $\partial\mathcal{O}$, where $\bar{\mathcal{O}}$ is the closure of $\mathcal{O}$. It should be noted that the boundary of an orbit is not the same as the topological boundary in general.

Lemma 1.9. *Under the same situation in Theorem* 1.8, *for an arbitrary orbit, the closure and the boundary of it are complexes.*

Proof. The conclusions follow from local closedness of the orbit and Property (W) on X. Q.E.D.

Let $\mathcal{O}$ be an orbit. We call the number of orbits contained in the closure of $\mathcal{O}$ the *degree* of $\mathcal{O}$, which we denote by $\deg(\mathcal{O})$. For a complex K, we define the *degree* $\deg(K)$ of K as the highest degree of orbits in K. For a complex K and a natural number r, the union of all orbits $\mathcal{O}$ in K with $\deg(\mathcal{O}) \leq r$ is called the *r-skeleton* of K. Skeletons are complexes. In fact, if $\mathcal{O}$ is an orbit and L is a complex contained in the boundary of $\mathcal{O}$, then $\deg(L) < \deg(\mathcal{O})$.

Lemma 1.10. *Let $\mathcal{O}$ be an orbit and F be a closed invariant subset of X disjoint from $\mathcal{O}$. Assume that an element $f \in C_c^\infty(X)$ vanishes on the boundary $\partial\mathcal{O}$ and is annihilated by the orbital distribution associated with $\mathcal{O}$. Then there exists an element $h \in I_G(X)$ which vanishes on F and coincides with f on $\mathcal{O}$.*

Proof. Regard $\mathcal{O}$ itself as a G-space. Since f vanishes on $\partial\mathcal{O}$, the restriction $f|_{\mathcal{O}}$ of f to the orbit $\mathcal{O}$ belongs to $C_c^\infty(\mathcal{O})$, and is annihilated by the orbital distribution on $\mathcal{O}$. So we conclude $f|_{\mathcal{O}} \in I_G(\mathcal{O})$ from Proposition 1.7. In other words, $f|_{\mathcal{O}}$ can be expressed as a linear combination $\sum_{1 \leq i \leq r} c_i(\rho_{\mathcal{O}}(g_i)f_i - f_i)$, where $c_i \in \boldsymbol{C}$, $g_i \in G$, $f_i \in C_c^\infty(\mathcal{O})$, and $\rho_{\mathcal{O}}$ is the G-action on the orbit $\mathcal{O}$. We may assume here each f_i is the characteristic function of a compact open subset A_i of $\mathcal{O}$. For each A_i, we find a compact open subset $\tilde{A}_i$ of X such that $\tilde{A}_i \cap \mathcal{O} = A_i$ and $\tilde{A}_i \cap F = \phi$. Obviously $h = \sum_{1 \leq i \leq r} c_i(\rho(g_i)\chi_{\tilde{A}_i} - \chi_{\tilde{A}_i})$ has the required properties, where ρ is the G-action on X. Q.E.D.

Proposition 1.11. *Let $K \subset X$ be a complex and f be an element of $C_c^\infty(X)$ annihilated by all orbital distributions supported in K. Then there exists an element $h \in I_G(X)$ which coincides with f on K.*

Proof. We prove the proposition by induction on the degree of K. We assume that the proposition is valid for any complex with degree $< \deg(K)$. Let $\mathcal{O}_1, \mathcal{O}_2, \ldots, \mathcal{O}_r$ be the orbits in K with degree $d = \deg(K)$. Then the $(d-1)$-skeleton of K is equal to $K - (\bigcup_i \mathcal{O}_i)$. By virtue of the assumption of induction, we find an element $f_0 \in$

$I_G(X)$ such that $f-f_0$ vanishes on the skeleton. In particular, $f-f_0$ vanishes on each boundary $\partial\mathcal{O}_i$ of $\mathcal{O}_i$, so by Lemma 1.10 there is an element $f_i\in I_G(X)$ which vanishes on the closed invariant subset $K-\mathcal{O}_i$ of X and coincides with $f-f_0$ on $\mathcal{O}_i$. Clearly $f=\sum_{0\leq i\leq r} f_i$ on K. Q.E.D.

Proof of implication (3)⇒(1) *in Theorem* 1.8. Let f be an element of $C_c^\infty(X)$ annihilated by all oribital distributions and $\mathcal{D}$ be the totality of all domains D such that $\chi_D f\in I_G(X)$. $\mathcal{D}$ is closed under the operation of finite union. In fact, if D_1, $D_2\in\mathcal{D}$, then $\chi_{D_1\cup D_2}\cdot f=\chi_{D_1-D_2}\cdot(\chi_{D_1}\cdot f)+\chi_{D_2}\cdot f\in I_G(X)$. Here we use the fact the space $I_G(X)$ is closed under the multiplication by the characteristic function of a domain. Next we show that $\mathcal{D}$ is an open covering of X. Let $x\in X$. Then the intersection K of all domains containing x is a complex, by Property (W). By Proposition 1.11, we find an element $h\in I_G(X)$ such that $f=h$ on K. Put $U=\{y\in X; f(y)=h(y)\}$ and $M=\operatorname{supp}(f)\cup\operatorname{supp}(h)$. Then U is an open neighbourhood of K and M is compact. So it is easily verified that there exists a domain $B\ni x$ such that $M\cap B\subset U$. Thus $\chi_B\cdot f=\chi_B\cdot h\in I_G(X)$. These two properties of $\mathcal{D}$ prove our assertion. Q.E.D.

1.3. Open invariant subsets with Property (W).

Let Y be a closed subset of an l-space X and Z be the complement of Y in X. We define the mappings i_Z of $C_c^\infty(Z)$ to $C_c^\infty(X)$ and p_Y of $C_c^\infty(X)$ to $C_c^\infty(Y)$ as follows: $i_Z(f)$ is the extension of f by zero outside Y, and $p_Y(f)$ is the restriction of f to Y.

Proposition 1.12. *The following sequence is exact*:

$$0 \longrightarrow C_c^\infty(Z) \xrightarrow{i_Z} C_c^\infty(X) \xrightarrow{p_Y} C_c^\infty(Y) \longrightarrow 0.$$

Proof. See [2].

We consider an action on X of an l-group G. We assume that Y is invariant with respect to the group action. Then the l-group G acts on l-spaces Y and Z naturally.

Proposition 1.13 *Assume that all orbits contained in Z are admissible in the G-space X, and the space Z has Property (W) with respect to the G-action. Then we have the following commutative diagram whose rows and columns are all exact. Here ξ, η, and ζ are inclusions.*

$$\begin{array}{ccccccccc}
 & & 0 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & I_G(Z) & \longrightarrow & I_G(X) & \longrightarrow & I_G(Y) & \longrightarrow & 0 \\
 & & \downarrow{\scriptstyle \zeta} & & \downarrow{\scriptstyle \xi} & & \downarrow{\scriptstyle \eta} & & \\
0 & \longrightarrow & C_c^\infty(Z) & \xrightarrow{i_Z} & C_c^\infty(X) & \xrightarrow{p_Y} & C_c^\infty(Y) & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & C_c^\infty(Z)/I_G(Z) & \longrightarrow & C_c^\infty(X)/I_G(X) & \longrightarrow & C_c^\infty(Y)/I_G(Y) & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & & 0. & &
\end{array}$$

The essential part of Proposition 1.13 is exactness at the term $I_G(X)$ in the top column of the diagram. Proposition 1.13 can be reduced to the following

Proposition 1.14. *Let the situation be as in the previous proposition. Identify the space $C_c^\infty(Z)$ as a linear subspace of $C_c^\infty(X)$ by zero extension outside Z, then $C_c^\infty(Z) \cap I_G(X) = I_G(Z)$.*

We prepare an elementary lemma for the proof.

Lemma 1.15. *Let $\mathcal{O}$ be an orbit in Z. Then it is admissible in Z. The orbital distribution on X associated with $\mathcal{O}$ is mapped by i_Z^* again to the orbital distribution on Z associated with $\mathcal{O}$.*

Proof of Proposition 1.14. Let $f \in C_c^\infty(Z) \cap I_G(X)$. Then f is annihilated by all orbital distributions supported in Z. By virtue of the weak decomposition theorem (Theorem 1.8), we conclude $f \in I_G(Z)$. Q. E. D.

Corollary 1.16. *By dualizing the diagram in Proposition* 1.13, *we have an exact sequence*

$$0 \longleftarrow \operatorname{Ker}(\zeta^*) \longleftarrow \operatorname{Ker}(\xi^*) \longleftarrow \operatorname{Ker}(\eta^*) \longleftarrow 0.$$

This sequence can be written also as the following one

$$0 \longleftarrow C_c^\infty(Z)^{*G} \longleftarrow C_c^\infty(X)^{*G} \longleftarrow C_c^\infty(Y)^{*G} \longleftarrow 0.$$

Remark. In Proposition 1.13 the assumption "admissible in X" on orbits can not be replaced by "admissible in Z". In fact, we have the following simple example. Let k be a non-archimedean local field. The unit group $G = k^*$ of k acts on $X = k$ by multiplication. Then the G-space X consists of two orbits $Y = \{0\}$ and $Z = k^*$. The former is closed and the latter open. Clearly the following sequence is not exact:

$$0 \longrightarrow I_G(Z) \longrightarrow I_G(X) \longrightarrow I_G(Y) \longrightarrow 0.$$

We denote by $\mathcal{M}(Z)_b$ the subspace of $\mathcal{M}(Z)$ consisting of all complex Radon measures m such that $|m|(A)$ is finite for all closed open subset A of Z, relatively compact in X. Here $|m|$ is the absolute value of m.

Corollary 1.17. The following sequence is exact and canonically splits:

$$0 \longleftarrow \mathscr{M}(Z)^G \cap \mathscr{M}(Z)_b \longleftarrow \mathscr{M}(X)^G \longleftarrow \mathscr{M}(Y)^G \longleftarrow 0.$$

1.4. Let an l-group G act on an l-space X. Assume that X consists of finitely many orbits, and all orbits are admissible. Then the orbital decomposition theorem has the following simple form.

Proposition 1.18. *The orbital distributions $m_1, m_2, \ldots, m_r$ on X associated with all orbits in X form a basis of $C_c^\infty(X)^{*G}$.*

Proof. We define a linear map $\mathscr{T}$ of $C_c^\infty(X)$ to $\boldsymbol{C}^r$ as follows: $\mathscr{T}(f) = (m_i(f))_{1 \leq i \leq r}$ $(f \in C_c^\infty(X))$. Then the sequence

$$0 \longrightarrow I_G(X) \longrightarrow C_c^\infty(X) \xrightarrow{\mathscr{T}} \boldsymbol{C}^r$$

is exact. So $C_c^\infty(X)^{*G} = \mathrm{Im}\,(T^*)$. Cleraly $\mathrm{Im}\,(T^*)$ is the linear span of m_i's.
We show that (m_i) is linearly independent. Suppose

$$(*) \qquad \sum_i c_i m_i = 0$$

is a non-trivial relation. Let q be the maximum of degrees of orbits corresponding to non-zero coefficients c_i. Then we may regard $(*)$ as a linear relation of orbital distributions on the q-skeleton X^q. Using the exact sequence

$$0 \longleftarrow C_c^\infty(X^q - X^{q-1})^{*G} \longleftarrow C_c^\infty(X^q)^{*G} \longleftarrow C_c^\infty(X^{q-1})^{*G} \longleftarrow 0,$$

we get a non-trivial linear relation of orbital distributions on the G-space $X^q - X^{q-1}$. But this is a contradiction, for the G-space $X^q - X^{q-1}$ consists of finitely many closed open orbits. Q.E.D.

§2. Property (W) in semisimple groups

Let k be a non-archimedean local field of characteristic zero, and $\bar{k}$ its algebraic closure. Let $\boldsymbol{G}$ be $GL(n, \bar{k})$ or its connected semisimple algebraic subgroup defined over k, and $G = \boldsymbol{G}(k)$ the group of k-rational points of $\boldsymbol{G}$. Then G is an l-group with its natural topology. Denote by X the underlying l-space of G. We define a G-action on X as follows: $G \times X \ni (g, x) \mapsto gxg^{-1} \in X$. This section is devoted to prove the following

Theorem 2.1. (1) *Each orbit in X is admissible,*
(2) *X has Property (W).*

Proof. For the proof of (1), see [9]. We shall prove here (2). Take $x \in X$, and let F be the intersection of all domains containing x. We prove that F consists of finitely many G-orbits by contradiction. Suppose F consists of infinitely many G-orbits. Let S be a complete system of representatives of these G-orbits. We define the continuous map P of G to k^n as follows: The image $P(y)$ of y is the coefficients of the characteristic polynomial of the $n\times n$ matrix y, that is, $P(y)=(p_1, p_2,\dots, p_n)$, $\det(y-tI_n)=(-t)^n+p_1t^{n-1}+\dots+p_{n-1}t+p_n$ (t: indeterminate). Since the inverse image of an arbitrary closed open subset of k^n is a domain, F is, *a fortiori* S is, contained in a fibre of P. Each fibre of P is, as a subset of $GL(n, \bar{k})$, covered by a finite number of $GL(n, \bar{k})$-orbits. So there is an infinite subset S_1 of S contained in a $GL(n, \bar{k})$-orbit C_1 in $GL(n, \bar{k})$. Applying Richardson's theorem (Theorem 3.1 in [10]) to the reductive pair $(GL(n, \bar{k}), \boldsymbol{G})$, we know that $C_1\cap \boldsymbol{G}$ is a finite union of $\boldsymbol{G}$-orbits. Thus we find an infinite subset S_2 of S_1 contained in a $\boldsymbol{G}$-orbit C_2. Since S_2 is contained in the subset $C_2(k)$ of k-rational points in C_2, $C_2(k)$ consists of infinitely many $\boldsymbol{G}(k)$-orbits. This contradicts to 6.4. Corollaire in [3]. Q. E. D.

§ 3. Strong decomposition theorem

3.1. In this section we treat the orbital decomposition under a stronger assumption. In this case, the invariant distributions on an l-space correspond completely to the distributions on the orbit space.

Let X be an l-space, and G an l-group acting on X. We name the following, *Property* (S).

Property (S): The saturation of each compact subset of X is closed in X. If X has Property (S), in particular, each orbit is closed.

Proposition 3.1. *Let an l-group act on an l-space X. If B is a tube and D is a domain in X, then $B\cap D$ is a tube.*

Proof. If B is the saturation of a compact set A, then $B\cap D$ is the saturation of the compact set $A\cap D$. Q. E. D.

Corollary 3.2. *The collection of tubes forms an ideal, in the sense of Halmos* [4], *in the Boorean algebra of domains.*

Corollary 3.3. *Let an l-group G act on an l-space X. Assume that the space X has Property* (S). *Then an orbit $\mathcal{O}$ is exactly equal to the intersection of all tubes containing it. In particular, Property* (S) *implies Property* (W).

Proof. Let $x\in X$ be outside of the orbit $\mathcal{O}$. Since the orbit is closed, we find a compact open neighbourhood A of x not intersecting $\mathcal{O}$. Then for an arbitrary tube B containing $\mathcal{O}$ the difference $B-G(A)$ is a tube by Proposition 3.1. This contains $\mathcal{O}$ and does not contain the point x. Q. E. D.

Proposition 3.4. *Let an l-group G act on an l-space X. Assume that X has Property* (S) *and is σ-compact. Then there exists an open subset N of X satisfying the following two conditions*:

(1) *For each orbit $\mathcal{O}$ in X, $N\cap\mathcal{O}\neq\phi$.*

(2) *For each tube B in X, $N\cap B$ is compact.*

Proof. Since X is σ-compact, we find a sequence (B_i) of tubes which covers the whole space X. We may assume that the covering is disjoint. Let A_i be a compact open subset of X such that $G(A_i)=B_i$. Then $N=\bigcup_i A_i$ has the required properties. Q. E. D.

Now we define the orbit space as follows. Let the notation be as in Proposition 3.4. Assume that each orbit is admissible. We denote by $\mathscr{X}$ the totality of all orbital distributions m on X such that $m(\chi_{B\cap N})=1$ for a tube B containing the orbit corresponding to m. Then there exists a canonical bijection between the set $\mathscr{X}$ and the set of all orbits. We also have a canonical surjection p of X to $\mathscr{X}$. We equip the space $\mathscr{X}$ the quotient topology induced by p. We call this topological space $\mathscr{X}$ the *orbit space*.

We fix the notation G, X, N, $\mathscr{X}$, and p in the rest of this subsection. For an invariant subset C of X, we denote by $\mathscr{X}(C)$ the subset of $\mathscr{X}$ consisting of all orbital distributions corresponding to orbits in C. Then, for a tube B, the subset $\mathscr{X}(B)$ is open and closed by definition of the topology on $\mathscr{X}$. Furthermore we have

Proposition 3.5. *Let $m\in\mathscr{X}$, and $\mathfrak{L}$ be the totality of all tubes containing $p^{-1}(m)$. Then $(\mathscr{X}(B))_{B\in\mathfrak{L}}$ is a fundamental system of open compact neighbourhoods of m.*

Proof. First we show that $\mathscr{X}(B)$ is compact for a tube B. Let $(S_i)_{i\in I}$ be an open covering of $\mathscr{X}(B)$. Then $(p^{-1}(S_i))_{i\in I}$ is an open covering of a compact set $B\cap N$. So we may find a finite subcovering $(p^{-1}(S_j))_{j\in J}$. Clearly $(S_j)_{j\in J}$ is a covering of $\mathscr{X}(B)$.

Let U be an arbitrary open neighbourhood of m. We must find a tube $C\in\mathfrak{L}$ such that $\mathscr{X}(C)\subset U$. We fix a tube $B_0\in\mathfrak{L}$. Then open sets $U\cap\mathscr{X}(B_0)$ and $\mathscr{X}(B)^c$ $(B\in\mathfrak{L})$ cover the compact set $\mathscr{X}(B_0)$, for the intersection of all $\mathscr{X}(B)$ $(B\in\mathfrak{L})$ is equal to $p^{-1}(m)$ by Corollary 3.3. So we find a finite number of tubes $B_1, B_2,\dots, B_t\in\mathfrak{L}$ such that $\mathscr{X}(B_0)\subset(U\cap\mathscr{X}(B_0))\cup\bigcup_{1\le i\le t}\mathscr{X}(B_i)^c$ or equivalently $\mathscr{X}(\bigcap_{0\le i\le t}B_i)\subset U\cap\mathscr{X}(B_0)\subset U$. Q. E. D.

For an element $f\in C_c^\infty(X)$, we define a function $\mathscr{T}_f$ on the orbit space $\mathscr{X}$ as follows: $\mathscr{T}_f(m)=m(f)$ $(m\in\mathscr{X})$.

Proposition 3.6. *For an arbitrary $f\in C_c^\infty(X)$, the function $\mathscr{T}_f$ on $\mathscr{X}$ is locally constant and compactly supported, that is, $\mathscr{T}_f\in C_c^\infty(\mathscr{X})$.*

Proof. If the support of f contained in a compact open subset A of X, then the support of $\mathscr{T}_f$ is contained in a compact set $\mathscr{X}(G(A))$. Next we show that the function $\mathscr{T}_f$ is locally constant. Let $m\in\mathscr{X}$, and B be a tube which contains the orbit $\mathcal{O}$ corresponding to m. Let h be the characteristic function of the compact open subset $B\cap N$ of X. Then $\mathscr{T}_h$ is equal to the characteristic function of the compact open subset $\mathscr{X}(B)$ of $\mathscr{X}$, in particular it is locally constant. The function $\mathscr{T}_{f-m(f)h}=\mathscr{T}_f-m(f)\mathscr{T}_h$ vanishes at m. So we may assume that f itself vanishes at m without loss of generality. By Proposition 1.11, we find an element $f_1\in I_G(X)$ which coincides with f on the closed orbit $\mathcal{O}$. By using a similar argument in the proof of implication

(3)⇒(1) in Theorem 1.8, we conclude that there is a tube B_1 containing $\mathcal{O}$, on which f and f_1 coincide with each other. Thus $\mathcal{T}_f$ vanishes on a neighbourhood $\mathscr{X}(B_1)$ of m, because so does $\mathcal{T}_{f_1}$. Q. E. D.

We state our strong decomposition theorem.

Theorem 3.7. *The following sequence is exact:*

$$0 \longrightarrow I_G(X) \longrightarrow C_c^\infty(X) \xrightarrow{\mathcal{T}} C_c^\infty(\mathscr{X}) \longrightarrow 0.$$

The dual $\mathcal{T}^*$ of $\mathcal{T}$ gives an isomorphism of $C_c^\infty(\mathscr{X})^*$ to $C_c^\infty(X)^{*G}$. Furthermore the isomorphism $\mathcal{T}^*$ maps Radon measures on $\mathscr{X}$ to invariant Radon measures on X, and preserves lattice structures with respect to positive measures.

Proof. Exactness of the sequence at the term $C_c^\infty(X)$ is neither more nor less than weak decomposition theorem. We show that the linear map $f \mapsto \mathcal{T}_f$ is onto. By Proposition 3.5 and Corollary 3.2 we know that an arbitrary element of $C_c^\infty(\mathscr{X})$ can be expressed as a linear combination of characteristic functions of subsets of the form $\mathscr{X}(B)$ (B: tube). On the other hand, we have already seen in the proof of Proposition 3.6 that the image of $\mathcal{T}$ contains all such functions.

For the latter half of the theorem, we notice that isomorphisms $\mathcal{T}^*$ and $\mathcal{T}^{*-1}$ preserve positivity of distributions. The statement follows from it without difficulties. Q. E. D.

3.2. In this subsection we consider a generalization of our Strong Decomposition Theorem.

Definition 3.8. Let an l-group G act on an l-space X. We call a descending sequence $(X_i)_{0\le i\le r}$ of closed invariant subsets of X a *filtration* of X if $X_0=X$ and $X_r=\phi$. And we call the filtration $(X_i)_{0\le i\le r}$ an *S-filtration*, if each difference $X_{i-1}-X_i$ $(1\le i\le r)$ has Property (S) with respect to the canonical G-action induced from that on X.

By a successive use of Corollary 1.17, we have the next

Theorem 3.9. *Let an l-group G act on an l-space X. Assume that all G-orbits in X are admissible, and that X has an S-filtration $(X_i)_{0\le i\le r}$. Then every invariant Radon measure T can be expressed as follows:*

$$T(f)=\sum_{1=1}^{r}\int_{\lambda\in\mathscr{X}_i} T_\lambda(f)\,d\tau_i(\lambda) \qquad (f\in C_c^\infty(X)),$$

where τ_i is a uniquely determined Radon measure on the orbit space $\mathscr{X}_i$ corresponding to the G-space $X_{i-1}-X_i$.

§4. Filtration in GL (n, k)

4.1. In this section we construct an S-filtration for the following special G-space X: $X=G=GL(n, k)$, where k is a non-archimedean local field of characteristic zero. The action of G on X is given by inner automorphisms.

Throughout we fix the above notations.

Theorem 4.1. *There is an S-filtration of X.*

In order to prove Theorem 4.1, it is sufficient to verify the following two propositions.

Let $x \in X$. We denote by $d(x)$ the dimension of the orbit of x. We may have $d(x)=n^2-\dim_k C_k(x)$, where $C_k(x)$ is the k-vector space of all $n \times n$ matrices commuting with x. Also we denote by $v(x)$ the number of the distinct eigenvalues of x in the algebraic closure $\bar{k}$ of k.

Proposition 4.2. *Let Y be a locally closed invariant subset of X on which two functions d and v are constant. Then Property (S) is satisfied on the G-space Y, that is, every compact subset of Y has the closed G-saturation.*

Proposition 4.3. *There is a filtration* $(X_i)_{0 \le i \le r}$ *of X such that the functions d and v are constant on each* $X_{i-1}-X_i$.

4.2. Proof of Proposition 4.2.

We prepare some results on linear algebra. Denote by $M_n(k)$ the set of all $n \times n$ matrices with entries in k, and $k[t]$ the polynomial ring in one indeterminate t.

Let $a \in M_n(k)$. Then there exist two elements q_1 and $q_2 \in GL(n, k[t])$, and monic polynomials $e_1(t), e_2(t), \ldots, e_n(t)$ in $k[t]$ such that $q_1(tI_n-a)q_2=\mathrm{diag}(e_1(t), e_2(t), \ldots, e_n(t))$ and $e_i(t) \mid e_{i+1}(t)$ $(i=1, 2, \ldots, n-1)$. Here I_n is the $n \times n$ unit matrix, and we read $e_i(t) \mid e_{i+1}(t)$ as "the polynomial $e_i(t)$ divides the polynomial $e_{i+1}(t)$". The sequence $(e_1(t), e_2(t), \ldots, e_n(t))$ is uniquely determined by a. We call the sequence the *invariants* of a and denote it by $e(a)$. The next theorem gives fundamental properties of elementary divisors.

Theorem 4.4. (1) *Let* $a \in M_n(k)$ *and* $e(a)=(e_1(t), e_2(t), \ldots, e_n(t))$. *Then, for each i,* $e_1(t)e_2(t)\cdot \ldots \cdot e_i(t)$ *is equal to the greatest common divisor of i-th minor determinants of the matrix* tI_n-a, *in particular* $e_1(t)e_2(t)\cdot \ldots \cdot e_n(t)$ *is equal to the characteristic polynomial of a.*

(2) *Two matrices in* $M_n(k)$ *are conjugate if and only if their invariants coincide with each other.*

Corollary 4.5. *Let L be an extension field of k. Then, two matrices in* $M_n(k)$ *are conjugate in* $M_n(k)$ *if and only if they are conjugate in* $M_n(L)$.

Definition 4.6. Two sequences (a_i) and (b_i) in $M_n(k)$ are called *conjugate* if a_i and b_i are conjugate in $M_n(k)$ for all i.

Definition 4.7. Let $a \in M_n(k)$ and L be an extension field of k. If L is a (not necessarily minimal) splitting field of the characteristic polynomial of a, we say that *a splits* in L.

Definition 4.8. Let $a, a_i \in M_n(k)$ $(i=1, 2, \ldots)$, and $e(a)=(e_1(t), e_2(t), \ldots, e_n(t))$,

$e(a_i)=(e_1^{(i)}(t), e_2^{(i)}(t),\ldots, e_n^{(i)}(t))$. Suppose the sequence (a_i) converges to a. We say (a_i) *converges with invariants* to a if $e_j^{(i)}(t)\to e_j(t)$ $(i\to\infty)$ for all j. Here $e_j^{(i)}(t)\to e_j(t)$ means that the coefficients of $e_j^{(i)}(t)$ converge to the corresponding ones of $e_j(t)$.

Proposition 4.9. *Let (a_i) be a sequence in $M_n(k)$ convergent to some element $a\in M_n(k)$. Assume that $v(a_i)=v(a)$ for every i, and that all a_i split in k. Then there exists a subsequence (a_{i_p}) of (a_i) such that there exists a sequence (c_p) in $M_n(k)$ which is conjugate to (a_{i_p}), and converges with invariants to an element in $M_n(k)$.*

Proof. Since the types of Jordan canonical forms are finite, we can choose a subsequence (a_{i_p}) of (a_i) for which the Jordan canonical form c_p of a_{i_p} is of the same type for $i=1, 2,\ldots$. Put $s_1^{(p)},\ldots, s_v^{(p)}$ the distinct eigenvalues of a_{i_p}. By extracting a suitable subsequence, we may assume that each sequence $(s_j^{(p)})_{1\leq p}$ is convergent, without loss of generality. Then the limits $s_1, s_2,\ldots, s_v$ of the sequences are distinct, for $v(a)=v$. Therefore the sequence (c_{i_p}) converges with invariants. Q.E.D.

Theorem 4.10 (*Closure relation in $M_n(k)$*). *Let $a, b\in M_n(k)$ splitting in k, $e(a)=(e_1(t),\ldots, e_n(t))$, $e(b)=(e'_1(t),\ldots, e'_n(t))$. Then a belongs to the closure of the conjugacy class of b if and only if $e_{j+1}(t)e_{j+2}(t)\cdot\ldots\cdot e_n(t)\mid e'_{j+1}(t)e'_{j+2}(t)\cdot\ldots\cdot e'_n(t)$ for all j.*

Proposition 4.11. *Let (a_i), (b_i) be sequences mutually conjugate in $M_n(k)$. Assume that (a_i) converges to an element $a\in M_n(k)$, and that (b_i) converges with invariants to an element $b\in M_n(k)$, and that the limits a, b split in k. Then $\overline{\mathcal{O}(a)}\subset\overline{\mathcal{O}(b)}$. Here $\mathcal{O}(a)$ denotes the conjugacy class of a, and $\overline{\mathcal{O}(a)}$ denotes the closure of it.*

Proof. Let $e(a)=(e_1(t),\ldots, e_n(t))$, $e(b)=(e'_1(t),\ldots, e'_n(t))$, and $e(a_i)=e(b_i)=(e_1^{(i)}(t),\ldots, e_n^{(i)}(t))$. We put $d_j(t)=e_1(t)e_2(t)\cdot\ldots\cdot e_j(t)$, $d'_j(t)=e'_1(t)e'_2(t)\cdot\ldots\cdot e'_j(t)$ $(1\leq j\leq n)$. By the above theorem, it is sufficient to show $d'_j(t)\mid d_j(t)$. Let $\psi(t)$ be a j-th minor determinant of tI_n-a. Then there is a sequence $(\psi_i(t))$ convergent to $\psi(t)$ such that each $\psi_i(t)$ is a j-th minor determinant of tI_n-a_i. Since $d_j^{(i)}(t)$ divides $\psi_i(t)$ for each i, the limit $d'_j(t)$ divides $\psi(t)$. So we conclude $d'_j(t)\mid d_j(t)$. Q.E.D.

Proposition 4.12. *Let k be a non-archimedean local field of characteristic zero as stated above, and $\bar{k}$ be its algebraic closure. Then there are only finitely many intermediate fields of k and $\bar{k}$, of given finite degree over k.*

Proof. It is enough to show that there are only finitely many totally ramified extensions of k in $\bar{k}$ of degree equal to a given number r. It is well known that such an extension is obtained by adding to k a root of an Eisenstein equation of degree r over k. Put $\mathfrak{L}=\{(a_1, a_2,\ldots, a_r);\ |a|<1$ for each i and $|a_r|=|\pi|\}$, where π is a primitive element in k. For an element $\alpha=(a_1, a_2,\ldots, a_r)\in\mathfrak{L}$, we denote by f_α the polynomial map of $\bar{k}$ to $\bar{k}$: $f_\alpha(x)=x^r+a_1x^{r-1}+\ldots+a_r$ $(x\in\bar{k})$. Let $\alpha\in\mathfrak{L}$. If $\beta\in\mathfrak{L}$ is sufficiently close to α, then each element of $f_{\bar{\beta}}^{-1}(0)$ is close to some element of $f_{\bar{\alpha}}^{-1}(0)$. So, by Krasner's lemma (see e.g. [7]), the totality of all $\beta\in\mathfrak{L}$ satisfying the following is a neighbourhood of α: the simple extension of k by any element $f_{\bar{\beta}}^{-1}(0)$ is of the form

$k(a)$ for some element a of $f_\alpha^{-1}(0)$. By virtue of compactness of $\mathfrak{L}$, there exists a finite subset $\mathfrak{M}$ of $\mathfrak{L}$ such that every totally ramified extension of degree r is of the form $k(a)$ with $a \in \cup f_\alpha^{-1}(0)$ $(\alpha \in \mathfrak{M})$. Q.E.D.

Corollary 4.13. *There is a finite extension field K of k such that every element of $M_n(k)$ splits in K.*

Proof of Proposition 4.2. Let A be a compact subset of Y. We must show that the G-saturation $G(A)$ is closed in Y. Let (b_i) be a sequence in $G(A)$ convergent to $b \in Y$. We select a sequence (a_i) in A which is conjugate to (b_i). We may assume that the sequence (a_i) converges to an element $a \in A$. By virtue of Corollary 4.5, it is enough to show that a is conjugate to b in $M_n(K)$, for a finite extension K of k as stated in Corollary 4.13. By Proposition 4.9, we find a subsequence (a_{i_p}) of (a_i) and a conjugate sequence (c_p) of (a_{i_p}) in $M_n(K)$ such that (c_p) converges with invariants to an element $c \in M_n(K)$. Then we have $\overline{\mathcal{O}_K(a)} \subset \overline{\mathcal{O}_K(c)}$ by Proposition 4.11, where the notation $\mathcal{O}_K(a)$ means the conjugacy class of a in $M_n(K)$. On the other hand we have $\dim_K \mathcal{O}_K(a) = \dim_k \mathcal{O}_k(a) = \dim_k \mathcal{O}_k(a_{i_1}) = \dim_K \mathcal{O}_K(a_{i_1}) = \dim_K \mathcal{O}_K(c_1) = \dim_K \mathcal{O}_K(c)$. Hence we conclude $\mathcal{O}_K(a) = \mathcal{O}_K(c)$. By the same reason $\mathcal{O}_K(b) = \mathcal{O}_K(c)$. So we have $\mathcal{O}_K(a) = \mathcal{O}_K(b)$. Q.E.D.

4.3. Proof of Proposition 4.3.

We prepare a few lemmas for the proof.

Lemma 4.14. *Let S be a topological space, L be a topological field, and $(T_s)_{s \in S}$ be a family of matrices of a fixed size whose entries depend continuously on the parameter s. Then the function on S defined by $s \mapsto \operatorname{rank}(T_s)$ is lower semicontinuous, and $s \mapsto \operatorname{nullity}(T_s)$ is upper semicontinuous.*

Lemma 4.15. *Let k be a non-archimedean local field of characteristic zero. Then the function on $M_n(k)$ defined by $a \mapsto v(a)$ is lower semicontinuous.*

Proof. It is enough to show the following. If (a_i) is a sequence in $M_n(k)$ convergent to a and $v(a_i)$ are all equal to a constant v. Then $v(a) \leq v$.

We prove this. Let $r_1^{(i)}, r_2^{(i)}, \dots, r_v^{(i)}$ be the distinct eigenvalues of a_i in the algebraic closure $\bar{k}$ of k. By virtue of Corollary 4.13, we may assume all $r_j^{(i)}$ belong to a finite extension K of k. Since the sequences $(r_j^{(i)})_{1 \leq i}$ are bounded, we may also assume that they are all convergent, without loss of generality. Now the conclusion follows easily. Q.E.D.

Proof of Proposition 4.3. By the above lemmas, we find two filtrations $(Y_i)_{0 \leq i \leq r}$ and $(Z_j)_{0 \leq j \leq s}$ of X such that the function d is constant on each $Y_i - Y_{i+1}$ and the function v is constant on each $Z_j - Z_{j+1}$. We put $X_{ij} = Z_j \cap (Y_i \cup Z_{j+1}) = (Z_j \cap Y_i) \cup Z_{j+1}$. Then $(X_{ij})_{0 \leq i \leq r}$ is a descending sequence of closed invariant subsets from Z_j to Z_{j+1} and both d and v are constant on $X_{ij} - X_{i+1,j} = (Y_i - Y_{i+1}) \cap (Z_j - Z_{j+1})$. Now Proposition 4.3 is clear. Q.E.D.

4.4. Let $G=GL(n, k)$ act on $X=G$ by inner automorphisms, where k is a non-archimedean local field of characteristic zero. Then all G-orbits are admissible (see Theorem 2.1). By virtue of Theorem 4.1, we have an S-filtration $(X_i)_{0\le i\le r}$ of X. On the other hand we prove the orbital decomposition theorem for a space with an S-filtration (Theorem 3.9). So we have

Theorem 4.16. *Every invariant Radon measure T on GL(n, k) can be expressed as follows:*

$$T(f)=\sum_{i=1}^{r}\int_{\lambda\in\mathscr{X}_i} T_\lambda(f)\,d\tau_i(\lambda) \qquad (f\in C_c^\infty(GL(n.\ k))),$$

where τ_i is a uniquely determined Radon measure on the orbit space $\mathscr{X}_i$ corresponding to $X_{i-1}-X_i$.

§5. Relationships between Strong Decomposition Theorem and Choquet's integral representation theory

In this section we discuss relationships between our strong decomposition theorem and Choquet's integral representation theory [1].

Let an l-group G act on an l-space X. We assume that X has Property (S) and all orbits are admissible. For simplicity, we assume further that the space X has a countable base, and there is a compact open subset N of X such that $G(N)=X$. Using this open set N, we define the orbit space $\mathscr{X}$ as in §3. Then the orbit space $\mathscr{X}$ is a compact l-space. We denote by E the space of invariant Radon measures on X with vague topology. We put $E^+=\{m\in E;\ m \text{ is positive}\}$ and $E_1=\{m\in E^+;\int_N dm=1\}$.

Theorem 5.1. (1) *The topological space E^+ is metrizable and separable.*

(2) *The subset E_1 of E is compact, convex, and furthermore is a simplex in the sense of Choquet* (28.1 *in* [1]).

(3) *The canonical injection of $\mathscr{X}$ to E_1 is a homeomorphism onto the set $\mathscr{E}(E_1)$ of extreme points in E_1. In particular, $\mathscr{E}(E_1)$ is closed in the convex set E_1.*

(4) *For all $\Psi\in E_1$ there is unique $\Phi\in\mathscr{M}^1(\mathscr{E}(E_1))$ such that*

$$\Psi(f)=\int_{\mathscr{E}(E_1)} m(f)\,d\Phi(m) \qquad (f\in C_c^\infty(X)).$$

Here $\mathscr{M}^1(\mathscr{E}(E_1))$ denotes the set of all probability Radon measures on $\mathscr{E}(E_1)$.

The last assertion in Theorem 5.1 corresponds to the assertion on Radon measures in our strong decomposition theorem.

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