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A note on Mergelyan's theorem

By

Cho-ichiro MATSUOKA

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0. Introduction

Let $X \subset \boldsymbol{C}$ be a compact plane set with connected complement, and let $C(X)$ be the Banach algebra of all complex valued continuous functions on X endowed with uniform norm. $M(X)$ denotes the linear space of all finite regular Borel measures supported on X; $M(X)$ is regarded as the dual Banach space of $C(X)$. We consider two closed subalgebras $A(X)$ and $R(X)$ of $C(X)$; namely $A(X)$ consists of all elements of $C(X)$ analytic on $Int(X)$, and $R(X)$ is a set of all functions of $C(X)$ uniformly approximable on X by functions in $\mathcal{O}(X)$, $\mathcal{O}(X)$ being the set of functions analytic in neighborhoods of X. Since the complement of X is connected, $R(X)$ coincides with the totality of uniform limits of polynomials. Clearly $A(X) \supseteq R(X)$, and actually in such a situation, $R(X)$ is identical with $A(X)$ by Mergelyan's theorem.

Much studies concerning this famous result have been made since Mergelyan established it in [5]. Roughly speaking, these works are classified into two groups. One group considers this theorem as a problem of the estimate on $\bar{\partial}$-operator and the continuous analytic capacity. This viewpoint succeeded in characterizing compact sets on which $R(X)=A(X)$ is valid, [5] [6] [7]. Another one considers $R(X)$ as a Dirichlet algebra via Walsh's theorem, then the decomposition theorem for orthogonal measures yields $A(X)=R(X)$ naturally, [2] [3].

Now, it is easily seen that $R(X)=A(X)$ holds if and only if the measures supported on ∂X and orthogonal to $R(X)$ are contained in the weak* closure of measures which are compactly supported on $Int(X)$ and annihilate all functions analytic on $Int(X)$. This is surely one of the most elementary criteria for $R(X)=A(X)$. Thus our purpose here is to give a direct proof based on the above criterion. In a sense our proofs succeed to E. Bishop's work [1]. But we shall need only conformal mappings and dispense with Walsh's theorem (though we give it a short and constructive proof in Th. 2.6.). Furthermore, the function theoretic knowledges required here are basic ones. So we often omit the explanations for statements. Details can be found in any current text book, e.g. Y. Kusunoki [4].

1. Preliminaries

At first, we note that by a short observation based on Runge's theorem, Mergelyan's theorem is valid for X if and only if it holds for each connected component of X. Therefore we shall assume throughout this paper that both X and its complement are connected. Then X admits an exhaustion $\{X_n: n \in \boldsymbol{N}\}$ from the exterior of X. Namely, each X_n is a simply connected domain bounded by an analytic Jordan curve. $\{X_n\}$ satisfies the condition: $X_n \supset \overline{X_{n+1}} \supset X$, $\bigcap_{n=1}^{\infty} X_n = X$. In the sequel $\{X_n\}$ always denotes such an exhaustion. (The idea of using an exhaustion is due to E. Bishop [1])

Proposition 1.1. *Suppose* $Int(X)$ *is nonvoid. Let* $Z_n: X_n \to D$ *(unit disk) be the biholomorphic mappings normalized at a point* $p \in Int(X)$ *(i.e.* $Z_n(p)=0$ *and* $Z_n'(p)>0$*). Then both* $\lim_n Z_n$ *and* $\lim_n Z_n^{-1}$ *exist, and these limit functions give the biholomorphic mapping between* D *and the connected component* U *of* $Int(X)$ *containing* p*, where* Z_n^{-1} *denotes the inverse mapping of* Z_n*. In particular,* $|Z_n| \to 1$ *uniformly on* $X-U$.

Let $\gamma \in \boldsymbol{C}$ be a Jordan arc and let $a, b \in \gamma$ $(a \neq b)$. Recall that a conformal mapping from $\boldsymbol{C} \cup \{\infty\} \backslash \{\gamma\}$ onto D is constructed via the map $z \mapsto \sqrt{\frac{z-b}{z-a}}$. From this, the following is easily obtained.

Lemma 1.2. *Let* $\{\gamma_n: n \in \boldsymbol{N}\}$ *be a sequence of Jordan arcs such that* $\gamma_n \subset \{|z|<M\}$, $diam\{\gamma_n\} > 1/M$. *Let* Z_n *be the conformal mappings from* $\boldsymbol{C} \cup \{\infty\} \backslash \{\gamma_n\}$ *onto* D *normalized at infinity. Then there exists a constant* m *depending only on* M *such that*

$$1/m \leqq Z_n'(\infty) = \lim_{z \to \infty} z\{Z_n(z)\} \leqq m.$$

Lemma 1.3. *Let* $\{\gamma_n\}$, $\{Z_n\}$ *be as above, and let* $q \in \boldsymbol{C}$ *be a point with* $dist(q, \gamma_n) \to 0$. *Then* $|Z_n(q)| \to 1$.

Proof. Assume that the assertion is false. Then there is a subsequence $\{Z_k\}$ of $\{Z_n\}$ such that $\lim_k Z_k(q) = w \in D$. We may assume that the limit $\lim_k Z_k^{-1} = Z^{-1}$ exists. Since Z^{-1} is univalent by Lemma 1.2, there are neighborhoods U of w and V of q such that $V \subset Z_k^{-1}(U)$ for all large $k \in \boldsymbol{N}$. This contradicts the assumption, $dist(q, \gamma_k) \to 0$.

Proposition 1.4. *Let* q *be an arbitrary point on* ∂X *and* ρ, ε $(0<\varepsilon<1)$ *be given positive numbers. Then there exists a function* $f \in \mathcal{O}(X)$ *such that* $\|f\|_X \leqq 1$, $f(q) > 1-\varepsilon$, *and* $|f(z)| \leqq \varepsilon$ *for all* $z \in X$ *with* $|z-q| \geqq \rho$.

Proof. Denote by $D(r, z_0)$ the disk with center z_0 and radius r. Let $\{\gamma_n: n \in \boldsymbol{N}\}$ be Jordan arcs in $\overline{D(\varepsilon\rho, q)} - X$ such that $\gamma_n \cap \partial\overline{D(\varepsilon\rho, q)} \neq \phi$, and $dist(q, \gamma_n) \to 0$. Since $\boldsymbol{C}-X$ is connected, these Jordan arcs always exist. Let Z_n be conformal mappings as in the preceding lemma, and set $f_n = \frac{|Z_n(q)|}{Z_n(q)} Z_n$. Then $\|f_n\|_X \leqq 1$ and $f_n(q) \to 1$. Further, by Schwarz lemma $|f_n(z)| \leqq \frac{\varepsilon\rho}{|z-q|}$ outside $D(\varepsilon\rho, q)$, so that $|f_n(z)| \leqq \varepsilon$ outside $D(\rho, q)$.

Proposition 1.5. *Let μ, ν be finite Borel measures compactly supported on $\boldsymbol{C}$. Then the function $z \mapsto \int \frac{1}{|w-z|} |d\mu(w)|$ is locally integrable with respect to area measure dxdy. Further denoting $\hat{\mu}(z) = \frac{1}{2\pi i} \int \frac{1}{w-z} d\mu(w)$, $\mu = \nu$ if and only if $\hat{\mu}(z) = \hat{\nu}(z)$ a.e. dxdy.*

2. Mergelyan's theorem

Proposition 2.1. *For a measure $\mu \in M(X)$ orthogonal to $R(X)$ (i.e. $\mu \in R(X)^{\perp}$), Cauchy transform $\hat{\mu}(z)$ of μ vanishes a.e. dxdy outside $Int(X)$.*

Proof. It suffices to show that $\hat{\mu}(z) = 0$ a.e. dxdy on ∂X. Let $q \in \partial X$ be a point such that $\int \frac{1}{|z-q|} |d\,\mu(z)| < +\infty$. These points exist a.e. dxdy on ∂X. Further let $\{f_n\}$ be a sequence from $\mathcal{O}(X)$ such that $\|f_n\| \leqq 1, f_n(q) > 1 - 1/n$, and $\|f_n\|_{X - D(1/n,q)} \leqq 1/n$ (Prop. 1.4). Using Lebesgue's dominated convergence theorem, the well-known method then yields $\hat{\mu}(z) = \frac{1}{2\pi i} \int \frac{1}{z-q} d\mu(z) = \lim_n \frac{1}{2\pi i} \int \frac{f_n(q) - f_n(z)}{z-q} d\mu(z) = 0$. Namely we obtain $\hat{\mu}(z) = 0$ a.e. dxdy on ∂X.

Corollary 2.2. *If $\mu \in R(X)^{\perp}$ satisfies $\hat{\mu}(z) = 0$ a.e. dxdy on $Int(X)$, then $\mu = 0$. In particular, if $Int(X)$ is empty, $R(X)^{\perp} = \{0\}$, namely $R(X) = C(X)$.*

In the sequel we shall assume that $Int(X)$ *is nonvoid.*

Corollary 2.3. *Let $\{\mu_n\}$ be a bounded sequence from $R(X)^{\perp}$ such that any neighborhood of ∂X contains the closed support $\overline{\text{supp}}\{\mu_n\}$ of μ_n for sufficiently large $n \in \boldsymbol{N}$. Then $\{\mu_n\}$ is a weak* convergent sequence if and only if $\{\hat{\mu}_n(z) \colon n \in \boldsymbol{N}\}$ converges locally uniformly on $Int(X)$.*

Proof. Since $\{\mu_n\} \in M(X)$ $(= C(X)^*)$ is a bounded sequence, $\{\mu_n\}$ has a cluster point ν in $M(X)$. Clearly $\overline{\text{supp}}\{\nu\} \subseteq \partial X$ and $\nu \in R(X)^{\perp}$. For a fixed $z \in Int(X)$, $\frac{1}{w-z}$ is continuous in a neighborhood of ∂X. This implies together with the assumption for $\overline{\text{supp}}\{\mu_n\}$, that $\{\hat{\mu}_n(z)\}$ converges locally uniformly on $Int(X)$ to $\hat{\nu}(z)$ whenever $\{\mu_n\}$ is a weak* Cauchy sequence. Conversely if $\{\hat{\mu}_n(z)\}$ is convergent on $Int(X)$, then we have $\lim_n \hat{\mu}_n(z) = \nu(z)$, $z \in Int(X)$, where ν is an arbitrary cluster point of $\{\mu_n\}$. Therefore ν is unique by Corollary 2.2.

Lemma 2.4. *Let U be a connected component of $Int(X)$ and let $\mu \in R(X)^{\perp}$ with $\overline{\text{supp}}\{\mu\} \subseteq \partial X$. Then there exists a measure $\mu_U \in A(\overline{U})^{\perp}$ with $\overline{\text{supp}}\{\mu_U\} \subseteq \partial U$ such that $\|\mu_U\| \leqq \|\mu\|$ and $\hat{\mu}_U = \hat{\mu}$ on U, $\hat{\mu}_U = 0$ a.e. dxdy outside U.*

Proof. Let $Z: U \to D$ and $Z_n: X_n \to D$ be biholomorphic mappings normalized at $p \in U$, and set $C_\rho = Z^{-1}\{|Z| = \rho\}$ $(0 < \rho < 1)$. We consider the measure $\hat{\mu}(z)dz$ on C_ρ. At first we show that $\|\hat{\mu}(z)dz\| \leqq \|\mu\|$. Setting $Z_n(z)^* = 1/\overline{Z_n(z)}$, $\mathcal{O}(X)$ contains the following function for a fixed $z \in C_\rho$:
$\left(\frac{Z_n(w)}{Z_n(w) - Z_n(z)} - \frac{Z_n(w)}{Z_n(w) - Z_n(z)^*}\right)\frac{Z_n'(z)}{Z_n(w)} - \frac{1}{w-z}$. This implies that

$$\hat{\mu}(z) = \frac{1}{2\pi i}\int_{\partial X}\left(\frac{Z_n(w)}{Z_n(w) - Z_n(z)} - \frac{Z_n(w)}{Z_n(w) - Z_n(z)^*}\right)\frac{Z_n'(z)}{Z_n(w)}\,d\mu(w).$$

Here recall that the convergence of $Z_n \to Z$ on C_ρ and $|Z_n| \to 1$ on ∂X are both uniform (Prop. 1.1). Therefore we have

$$\hat{\mu}(z) = \frac{1}{2\pi i}\int_{\partial X}\left(\frac{Z_n(w)}{Z_n(w) - Z(z)} - \frac{Z_n(w)}{Z_n(w) - Z(z)^*}\right)\frac{Z'(z)}{Z_n(w)}\,d\mu(w) + o(1).$$

Further setting $\arg Z(z) = \Theta$ and $\arg Z_n(w) = \Phi_n$, the above yields

$$\hat{\mu}(z) = \frac{1}{2\pi i}\int\left(\frac{1}{1 - \rho\exp i(\Theta - \Phi_n)} - \frac{\rho}{\rho - \exp i(\Theta - \Phi_n)}\right)\frac{Z'(z)}{\exp i\Phi_n}\,d\mu(w) + o(1).$$

From this, it follows that

$$\int_{C_\rho}|\hat{\mu}(z)||dz| \leqq \int|d\mu(w)|\frac{1}{2\pi}\int_0^{2\pi}\frac{1-\rho^2}{1 - 2\rho\cos(\Theta - \Phi_n) + \rho^2}\,\rho\, d\Theta + o(1).$$

$$= \rho\|\mu\| + o(1).$$

Letting $n \to \infty$, we have $\|\hat{\mu}(z)\,dz\| \leqq \rho\|\mu\| \leqq \|\mu\|$. Clearly these measures $\hat{\mu}(z)dz|C_\rho$, are orthogonal to $\mathcal{O}(U)$, so that, to $A(\overline{U})$ and satisfy conditions of Corollary 2.3. Therefore as $\rho \to 1$, $\{\hat{\mu}(z)\,dz \mid C_\rho\}$ is weak* convergent to some measure μ_U. It is easily seen that $\|\mu_U\| \leqq \|\mu\|$, $\mu_U \in A(\overline{U})^\perp$ and $\hat{\mu}_U = \hat{\mu}$ on U and $\hat{\mu}_U = 0$ a.e. dxdy off U.

Theorem 2.5. (*Mergelyan*) $R(X) = A(X)$.

Proof. For the theorem it suffices to show that $A(X)|_{\partial X}^\perp = R(X)|_{\partial X}^\perp$, namely that every measure supported on ∂X and orthogonal to $R(X)$ is also orthogonal to $A(X)$. Let $\mu \in R(X)^\perp$ with $\operatorname{supp}\{\mu\} \subseteq \partial X$ and $Int(X) = \bigcup_{j=1}^{M} U_j$ be the decomposition of $Int(X)$ into connected components. Denote by $\mu_j = \mu_{U_j}$ the measures as in Lemma 2.4. Note that the measures $\sum_{j=1}^{m} \mu_j$, $m \leqq M$, are orthogonal to $A(X)$ and their Cauchy transforms coincide with $\hat{\mu}(z)$ on $\bigcup_{j=1}^{m} U_j$ and vanish a.e. dxdy outside $\bigcup_{j=1}^{m} U_j$. Therefore, by Corollary 2.3, we have only to show that these measures are uniformly bounded. So we may assume that $M = \infty$. Pick up $p_j \in U_j$ and fix them throughout. Following the idea of E. Bishop [1], consider biholomorphic mappings Z_n^j: $X_n \to D$ normalized at $p_j \in U_j$. Changing the exhaustion if necessary, we may assume that each sequence $\{Z_n^j\colon\ n \in \boldsymbol{N}\}$ converges locally uniformly on $Int(X)$ as n tends to infinity. For a fixed positive integer m $(m < M)$, consider functions $F_n = Z_n^1 \cdot Z_n^2 \cdots Z_n^m$ $(n \in \boldsymbol{N})$. From these we have the following double sequence; $\left\{\left(\prod_{j=m+1}^{m+k}\frac{F_n(p_j) - F_n}{1 - \overline{F_n(p_j)}\cdot F_n}\cdot\frac{|F_n(p_j)|}{F_n(p_j)}\right)^k\right\}_{k,n}$.

For a fixed k, this converges to 1 locally uniformly on $\bigcup_{j=1}^{m} U_j$ as $n\to\infty$. Further Schwarz lemma applied to each U_j yields that, as $k\to\infty$ this converges to 0 locally uniformly on $\bigcup_{j=m+1}^{M} U_j$, independently of the index n. Therefore we can pick up a subsequence $\{g_n\}$ from the above sequence such that $\|g_n\|\leqq 1$, $g_n\to 1$ on $\bigcup_{j=1}^{m} U_j$ and $g_n\to 0$ on $\bigcup_{j=m+1}^{M} U_j$. Clearly $g_n\in\mathcal{O}(X)$. Here consider a bounded sequence $\{g_n\mu\}$ from $R(X)^{\perp}$. By an equality: $0=\frac{1}{2\pi i}\int\frac{g_n(w)-g_n(z)}{w-z}\,d\mu(w)$, we have that $g_n(z)\hat{\mu}(z)=\widehat{g_n\mu}(z)$ a.e. dxdy. By Corollary 2.3, $\{g_n\mu\}$ is actually a weak* Cauchy sequence. Denote its limit by μ^m. Then it is easily seen that $\hat{\mu}^m=\sum_{j=1}^{m}\hat{\mu}_j$ a.e. dxdy and $\|\mu^m\|\leqq\|\mu\|$, so that $\mu^m=\sum_{j=1}^{m}\mu_j$. This completes the proof.

Theorem 2.6. (*Walsh*) *$\mathcal{R}e\, R(X)|_{\partial X}$ is uniformly dense in $C_R(\partial X)$, the set of all real valued continuous functions on ∂X.*

Proof. Let $q\in\partial X$ and $g\in C_R(\partial X)$ be arbitrary. At first, we verify the equalities: $g(q)=\sup\{u(q)\colon u\in\mathcal{R}e\,\mathcal{O}(X),\ u\leqq g \text{ on } \partial X\}$ and $g(q)=\inf\{v(q)\colon v\in\mathcal{R}e\,\mathcal{O}(X),\ g\leqq v \text{ on } \partial X\}$. It is easily seen that we have only to prove the former equality in case that $g\geqq 0$ and $g(q)=1$. For any ε, $0<\varepsilon<1$, choose $\rho>0$ so that $g>1-\varepsilon$ on $D(\rho,q)\cap\partial X$. By Prop. 1.4, there exists an $f\in\mathcal{O}(X)$ such that $\|f\|_X\leqq 1$, $f(q)>1-\varepsilon$, $\|f\|_{X-D(\rho,q)}\leqq\varepsilon$. Clearly $\mathcal{R}e(f-\varepsilon)\leqq g$ on ∂X, and $\mathcal{R}e(f-(q)>(1-\varepsilon)-\varepsilon$. Since ε is arbitrary, this yields a desired equality.

Now, ∂X is compact. So there are finitely many functions $v_1\cdots v_n$, $u_1\cdots u_n$ of $Re\,\mathcal{O}(X)$ such that $g+\varepsilon>v_1^{\wedge}\cdots{}^{\wedge}v_n>g>u_1^{\vee}\cdots{}^{\vee}u_n>g-\varepsilon$ on ∂X. Here we may assume that g is continuously defined on the whole plane. So we can find a neighborhood V of X such that the above inequalities are still valid on ∂V. Let $U(V,g)$ be the harmonic function on V with boundary value $g|_{\partial V}$. Then we have $v_1^{\wedge}\cdots{}^{\wedge}v_n>U(V,g)>u_1^{\vee}\cdots{}^{\wedge}u_n$ on V, so that $\|U(V,g)-g\|_{\partial X}\leqq\varepsilon$. Since V contains X_n for a large $n\in N$, and X_n is simply connected, $U(V,g)$ is contained in $Re\,\mathcal{O}(X)$.

Let $H(X)$ be the subspace of $C(X)$ consists of all functions harmonic on $Int(X)$. Then from the above, the following is easily seen.

Corollary 2.7. *$H(X)|_{\partial X}=C(\partial X)$, and $\mathcal{R}e\,\mathcal{O}(X)$ is uniformly dense in $\mathcal{R}eH(X)$.*

3. REMARK. (Relation with the Choquet boundary.)

Reflecting our proof of Mergelyan's theorem, we see that it based on three propositions: Prop. 1.4, Theorem 2.5 and Lemma 2.4. Prop. 1.4 assures us that ∂X is the Choquet boundary with respect to $R(X)$, and Walsh's theorem is a direct consequence of this fact. Theorem 2.5 is essentially a lemma concerning Gleason parts. In other words, part theory for $Int(X)$ reduces to the asymptotic behaviour of conformal mappings as mentioned in Prop. 1.1. On the other hand, Lemma 2.4 corresponds to F. and M. Riesz theorem on the unit circle. Indeed, let W be a bounded (not necessarily simply connected) plane domain such that ∂W is the Choquet boundary relative to $R(\overline{W})$, and let $\{W_n\}$ (*resp.* $\{X_n\}$) be an exhaustion of W from the interior (*resp.* exterior). Further denote by ν_p^n, λ_p^n the harmonic measures on ∂W_n and ∂X_n with center $p \in W_1$, respectively. By the same argument as in Th. 2.6, we have $\|U(X_n, g) - g\|_{\partial W} \to 0$ for all $g \in C(\boldsymbol{C})$. So we can approximate each $\mu \in R(\overline{W})^\perp$ with $\operatorname{supp}\{\mu\} \subseteq \partial W$, vaguely from exterior; one may take measures $\mu^n \in R(\overline{X}_n)^\perp$ on ∂X_n such that $\int g \mathrm{d}\, \mu^n = \int U(X_n, g) \mathrm{d}\, \mu$ for all $g \in C(\boldsymbol{C})$. Clearly $\mu^n \in R(\overline{X}_n)^\perp$ and $\|\mu^n\| \leqq \|\mu\|$. By F. and M. Riesz theorem on ∂X_n we have an equality $d\mu^n = h^n \mathrm{d}\, \lambda_p^n + N^n \mathrm{d}\, \lambda_p^n$, where h^n belongs to Hardy class $H_p^1(\lambda_p^n)$ and $N^n \mathrm{d}\, \lambda_p^n$ is a Schottky differential. If N^n vanishes identically for all $n \in \boldsymbol{N}$, μ can also be exhausted from interior; one may take measures $h^n \mathrm{d}\, \nu_p^n$ on ∂W_n. Indeed from an equality $\frac{h^n(p) - h^n(z)}{p - z} = \int \frac{h^n(w) - h^n(z)}{w - z} \mathrm{d}\, \nu_p^n(w)$ $(z \in W)$, it follows that $\lim_n \widehat{h^n \mathrm{d}\, \nu_p^n}(z) = \lim_n \left(\frac{h^n(p) - h^n(z)}{2\pi i(p - z)} + h^n(z) \widehat{\nu_p^n}(z) \right) = \lim_n \left(\frac{h^n(p) - h^n(z)}{2\pi i(p - z)} + h^n(z) \widehat{\lambda_p^n}(z) \right) = \lim_n \widehat{h^n \mathrm{d}\, \lambda_p^n}(z) = \hat{\mu}(z)$. Namely $w^*\text{-}\lim_n h^n \mathrm{d}\, \nu_p^n = w^*\text{-}\lim_n h^n \mathrm{d}\, \lambda_p^n = \mu$. Therefore we have $\mu \in A(\overline{W})^\perp$ whenever N^n is identically zero for all $n \in \boldsymbol{N}$. Thus from our point of view, the validity of Mergelyan's theorem mainly depends on the following two facts: (1) *∂X is the the Choquet boundary for $R(X)$.* (2) *X admits a simply connected exhaustion from exterior.*

DEPARTMENT OF MATHEMATICS
FACULTY OF SCIENCE
KYOTO UNIVERSITY.

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