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On the poles of the scattering matrix for two strictly convex obstacles: An addendum

By

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§ 1. Introduction

The purpose of this paper is to improve the second part of Theorem 1 of the previous paper [2]. Namely, we like to give a more precise information on the existence of the poles of the scattering matrix $\mathscr{S}(z)$. The result we want to show in this paper is

Theorem 1. *Suppose that* $\mathcal{O}$ *satisfies the same conditions as in Theorem* 1 *of* [2]. *Then there exists at least a pole of* $\mathscr{S}(z)$ *in* $\{z;\ |z-z_j|\leqq C(|j|+1)^{-1/2}\}$ *for all large* $|j|$.

As remarked in [2], in order to show Theorem 1 it suffices to prove

Theorem 2. *The operator* $U(\mu)$ *which is defined in Theorem* 2 *of* [2] *has at least a pole in* $\{\mu;\ |\mu-\mu_j|\leqq C(|j|+1)^{-1/2}\}$ *for all large* $|j|$.

The plan of the proof of Theorem 2 is as follows. First we shall construct an asymptotic solution $u(x, t; k)$ of the problem

$$(1.1)\qquad \begin{cases} \square u=0 & \text{in}\quad \Omega\times \boldsymbol{R} \\ u=m(x, t; k) & \text{on}\quad \Gamma\times \boldsymbol{R} \\ \operatorname{supp} u\subset\bar{\Omega}\times\{t;\ t>0\} & \end{cases}$$

for an oscillatory boundary data

$$(1.2)\qquad m(x, t; k)=e^{ik(\varphi_\infty(x)-t)}g(x)m(t)$$

following the process of [2], where φ_∞ is a phase function introduced in §3 of [2], and $g(x)\in C_0^\infty(\Gamma_1)$, $m(t)\in C_0(\boldsymbol{R})$. Then the Laplace transform $\hat{u}(x, \mu; k)$ of $u(x, t; k)$ becomes an approximation of $\hat{m}(\mu+ik)U(\mu)(e^{ik\varphi_\infty(\cdot)}g(\cdot))(x)$, and we estimate $\Delta_{C_j}\hat{u}(A(l_0), \mu; k_j)$ for $A(l_0)$ a point on the segment a_1a_2, $C_j=\{\mu;\ |\mu-\mu_j|=\eta\}$ $(\eta>0)$ and $k_j=-j\pi/d$, where $\Delta_C\hat{u}$ denotes the variation of arg $\hat{u}$ along the contour C.

It should be remarked that $z_j = ic_0 + j\pi/d$, $j=0, \pm 1, \pm 2, \dots$ are nothing but the pseudo-poles $\alpha_{m_0,\vec{m}}$, $\vec{m}=0$ of Bardos, Guillot and Ralston [1] (c.f. Definition 8). Our Theorem 1 shows that the pseudo-poles $\alpha_{m_0,\vec{m}}$ for $\vec{m}=0$ approximate the actual poles.

§ 2. On the Laplace transform of asymptotic solutions

Let $\varphi_\infty(x)$ be a phase function introduced in §3 of [2], and let $m(x, t; k)$ be an oscillatory function on $\Gamma_1 \times \boldsymbol{R}$ of the form

$$m(x, t; k) = e^{ik(\varphi_\infty(x)-t)} f(x, t) \tag{2.1}$$

where $f \in C_0^\infty(S_1(\delta_2) \times (0, d/2))$. Denote by $u(x, t; k)$ an asymptotic solution for an oscillatory data $m(x, t; k)$ which is constructed following the method of Proposition 7.2 of [2]. Then its Laplace transform

$$\hat{u}(x, \mu; k) = \int_{-\infty}^{\infty} e^{-\mu t} u(x, t; k) dt \tag{2.2}$$

converges for Re $\mu > -c_0$, and by virtue of Proposition 7.2, (i) and Proposition 8.3 of [2] have the following:

$$\begin{aligned} \hat{u}(x, \mu; k) &= \mathscr{P}(\mu)^{-1} F_0(x, \mu; k) + \tilde{F}_{0,0}(x, \mu; k) \\ &+ \sum_{r=1}^{N} k^{-r} \{ \mathscr{P}(\mu)^{-r-1} \tilde{F}_r(x, \mu; k) + \tilde{F}_{r,0}(x, \mu; k) \\ &+ \sum_{h=1}^{r} \sum_{l=0}^{\infty} (\lambda\tilde{\lambda} e^{-2\mu d})^l \sum_{j=0}^{r} \mathscr{P}(\mu)^{-(r-j+1)} \tilde{F}_{r,h,l,j}(x, \mu; k) \}. \end{aligned}$$

where

$$\mathscr{P}(\mu) = 1 - \lambda\tilde{\lambda} e^{-2\mu d}$$

and $\tilde{F}_r$, $\tilde{F}_{r,h,l,j}$ and $\tilde{F}_{r,0}$ are $C^\infty(\bar{\Omega})$-valued holomorphic function in $\mathscr{D} = \{\mu;\ \mathrm{Re}\,\mu > -c_0 - c_1\}$. Moreover they verify the following estimates for all $\mu \in \mathscr{D}_\varepsilon = \{\mu;\ \mathrm{Re}\,\mu \geq -c_0 - c_1 + \varepsilon\}$ $(\varepsilon > 0)$

$$\sum_{|\beta| \leq m} \sup_{x \in \Omega_R} |D_x^\beta \tilde{F}_r(x, \mu; k)| \leq C_{r,m,R,\varepsilon} k^m B_{m+2(N+N')}$$

$$\sum_{j=0}^{r} \sum_{|\beta| \leq m} \sup_{x \in \Omega_R} |D_x^\beta \tilde{F}_{r,h,l,j}(x, \mu; k)| \leq C_{r,m,R,\varepsilon} k^m \alpha^l l^{r-h} B_{m+2(N+N')},$$

$$B_m = \sum_{|\beta| \leq m} \sup_{\Gamma_1 \times \boldsymbol{R}} |D_{x,t}^\beta f(x, t)|,$$

which are derived from (7.12) and (7.13) of [2].

Let $\{\varphi_q\}_{q=0}^\infty$ be a sequence of phase functions defined for φ following the process in §2 of [2]. When $\varphi(x) = \varphi_\infty(x)$ it follows from Remark 2 of §3 of [2] that

$$\begin{cases} \varphi_{2q}(x) = \varphi_\infty(x) + 2qd \\ \varphi_{2q+1}(x) = \tilde{\varphi}_\infty(x) + (2q+1)d \end{cases} \tag{2.3}$$

for $q=0, 1, 2,\dots$, and if we use (2.3) in the definition of $\tilde{U}_r$, we see easily that the second term appeared in the definition of $\tilde{U}_r$ is identically zero, that is,

$$\tilde{U}_r=\{e^{ik(\varphi_{2q}-t)}\, z_{r,q},\ e^{ik(\varphi_{2q+1}-t)}\, \tilde{z}_{r,q}\}_{q=0}^{\infty}.$$

In this case the estimate (7.14) can be replaced by

$$|\tilde{U}_r|_{M_{2r,m}}\leq C_{m,r}k^m B_{m+2r},$$

which implies

$$\sum_{|\beta|\leq m}\sup_{x\in\Omega_R}|D_x^\beta\tilde{F}_{r,0}(x,\mu;k)|\leq C_{m,R,r,\varepsilon}k^m B_{m+2(N+N')}\qquad\text{for all}\quad \mu\in\mathscr{D}_\varepsilon.$$

Therefore if we set

$$\tilde{F}_{r,h}(x,\mu;k)=\sum_{l=0}^{\infty}\sum_{j=0}^{r}\mathscr{P}(\mu)^j(\lambda\tilde{\lambda}e^{-2\mu d})^l\tilde{F}_{r,h,l,j}$$

$$F_0(x,\mu;k)=\tilde{F}_0(x,\mu;k)+\mathscr{P}(\mu)\tilde{F}_{0,0}(x,\mu;k)$$

$$F_r(x,\mu;k)=\tilde{F}_r(x,\mu;k)+\sum_{h=0}^{r}\tilde{F}_{r,h}(x,\mu;k)+\mathscr{P}(\mu)^{r+1}\tilde{F}_{r,0}(x,\mu;k)$$
$$\text{for}\quad r=1, 2,\dots, N,$$

$F_r(x,\mu;k)$, $r=0, 1, 2,\dots, N$, are holomorphic in $\mathscr{D}$ and satisfies an estimate

$$\text{(2.4)}\qquad \sum_{|\beta|\leq m}\sup_{x\in\Omega_R}|D_x^\beta F_r(x,\mu;k)|\leq C_{m,R,r,\varepsilon}k^m B_{m+2(N+N')}\qquad\text{for all}\quad \mu\in\mathscr{D}_\varepsilon.$$

Evidently we have

$$\text{(2.5)}\qquad \hat{u}(x,\mu;k)=\mathscr{P}(\mu)^{-1}\{F_0(x,\mu;k)+(k\mathscr{P}(\mu))^{-1}F_1(x,\mu;k)+\cdots$$
$$\cdots+(k\mathscr{P}(\mu))^{-N}F_N(x,\mu;k)\}.$$

Concerning the boundary value of $\hat{u}$ it follows from (ii) of Proposition 7.2 and Propostion 8.3 that

$$\text{(2.6)}\qquad \hat{u}(x,\mu;k)=\begin{cases} e^{ik\varphi_\infty(x)}\hat{f}(x,\mu+ik)+k^{-N}\mathscr{P}(\mu)^{-N-1}G_{N,1}(x,\mu;k) & \text{on}\quad \Gamma_1\\ k^{-N}\mathscr{P}(\mu)^{-N-1}G_{N,2}(x,\mu;k) & \text{on}\quad \Gamma_2,\end{cases}$$

where $G_{N,j}$, $j=1, 2$, are $C^\infty(\Gamma_j)$ valued holomorphic functions in $\mathscr{D}$ satisfying

$$\text{(2.7)}\qquad |G_{N,j}(\cdot,\mu;k)|_m(\Gamma_j)\leq C_{N,m,\varepsilon}k^m B_{m+2(N+N')}\qquad\text{for all}\quad \mu\in\mathscr{D}_\varepsilon.$$

Thus we have

Lemma 2.1. *Let $\varphi_\infty(x)$ be a real valued C^∞ function introduced in §3 of* [2], *and let $f(x,t)\in C_0^\infty(S_1(\delta_2)\times(0, d/2))$. Then there exists a $C^\infty(\bar{\Omega})$ valued function $\hat{u}(x,\mu;k)$ defined in $\mathscr{D}=\{\mu;\ \mathrm{Re}\,\mu>-c_0-c_1\}$ which has the form* (2.5) *and satisfies* (2.4), (2.6) *and* (2.7), *and*

$$\text{(2.8)}\qquad (\mu^2-\Delta)\hat{u}(x,\mu;k)=0\qquad\text{in}\quad\Omega$$

for all $\mu\in\mathscr{D}-\{\mu_j;\ j=0,\pm1,\pm2,\dots\}$ and $k\in\boldsymbol{R}$.

§3. An explicit representation of $F_0(x, \mu; k)$ on the segment $a_1 a_2$

Denote by v_0 the solution of the transport equation

$$\begin{cases} Tv_0=0 & \text{in} \quad \omega\times R \\ v_0=f & \text{on} \quad S(\delta_2)\times R \end{cases}$$

in the sense of Definition 6.2 of [2] where $f=\{f_q, \tilde{f}_q\}_{q=0}^{\infty}$, $f_0=f$, $f_q=0$ for all $q\geq 1$ and $\tilde{f}_q=0$ for all $q\geq 0$. Then Proposition 5.6 shows that v_0 is decomposed as

$$v_0=w_0+z_0, \quad w_0\in K(0), \quad z_0\in M_1(0).$$

Set

$$A(l)=a_1+l(a_2-a_1)/|a_2-a_1|, \quad 0\leq l\leq d.$$

About the functions and the constant appearing in Proposition 5.6 we have from (2.3)

$$a(A(l))=(\det[I+l\mathscr{K}_\infty(0)])^{-1/2}/\lambda$$

$$\tilde{a}(A(l))=(\det[I+(d-l)\tilde{\mathscr{K}}_\infty(0)])^{-1/2}/\tilde{\lambda}$$

$$j_\infty(A(l))=l, \quad \tilde{j}_\infty(A(l))=d-l,$$

$$A_0=a_1, \quad b_0=1, \quad d_{\infty,0}=0.$$

Then we have for all $q\geq 0$

$$(3.1)\quad \begin{cases} w_q(A(l), t)=(\lambda\tilde{\lambda})^q(\det[I+l\mathscr{K}_\infty(0)])^{-1/2}f(a_1, t-2qd-l) \\ \tilde{w}_q(A(l), t)=(\lambda\tilde{\lambda})^q\lambda(\det[I+(d-l)\tilde{\mathscr{K}}_\infty(0)])^{-1/2}f(a_1, t-(2q+2)d-l).^{1)} \end{cases}$$

Substituting $x=A(l)$ and (2.3) into (5.9) of [2] we have

$$(3.2)\quad v_q(A(l), t)=w_q(A(l), t), \quad \tilde{v}_q(A(l), t)=\tilde{w}_q(A(l), t) \quad \text{for all} \quad q.^{2)}$$

Recall that $\mathscr{P}(\mu)^{-1}F_0(x, \mu; k)$ is

$$\int_{-\infty}^{\infty} e^{-\mu t}\sum_{q=0}^{\infty}\{e^{ik(\varphi_{2q}(x)-t)}v_{0,q}(x, t)-e^{ik(\varphi_{2q+1}(x)-t)}\tilde{v}_{0,q}(x, t)\}dt,$$

where we set $v_0=\{v_{0,q}, \tilde{v}_{0,q}\}_{q=0}^{\infty}$. Then it follows from (3.1) and (3.2)

$$(3.3)\quad F_0(A(l), \mu; k)=\Big[\int_{-\infty}^{\infty} e^{-\mu t}\{e^{ik(\varphi_\infty(x)-t)}w_0(x, t) - e^{ik(\tilde{\varphi}_\infty(x)-t)}\tilde{w}_0(x, t)\}dt\Big]_{x=A(l)}.$$

Note that

$$\varphi_\infty(A(l))=l, \quad \tilde{\varphi}_\infty(A(l))=d-l.$$

1, 2) Since we adopt now Definition 6.2, w_q, $\tilde{w}_q$ and v_q, $\tilde{v}_q$ correspond to w_{2q}, w_{2q+1} and v_{2q}, v_{2q+1} in Proposition 5.6 of [2] respectively.

Then we have from (3.1)

$$w_0(A(l), t) = R(l)f(a_1, t-l)$$

$$\tilde{w}_0(A(l), t) = \tilde{R}(l)f(a_1, t-(2d-l))\lambda$$

where we set

$$R(l) = (\det[I + l\mathscr{K}_\infty(0)])^{-1/2} \tag{3.4}$$

$$\tilde{R}(l) = (\det[I + (d-l)\tilde{\mathscr{K}}_\infty(0)])^{-1/2}. \tag{3.5}$$

Substituting these relations into (3.3) we have

$$\begin{aligned} &F_0(A(l), \mu; k) \\ &= \int_{-\infty}^{\infty} e^{-\mu t}\{e^{ik(l-t)} R(l)f(a_1, t-l) - e^{ik(d-l+d-t)} \tilde{R}(l)f(a_1, t-(2d-l))\lambda\}dt \\ &= e^{-\mu l} R(l)\hat{f}(a_1, \mu+ik) - e^{-(2d-l)\mu} \tilde{R}(l)\hat{f}(a_1, \mu+ik)\lambda, \end{aligned}$$

where

$$\hat{f}(x, \mu) = \int_{-\infty}^{\infty} e^{-\mu t} f(x, t)dt.$$

Thus we have

Lemma 3.1. *For all $\mu \in \boldsymbol{C}$, $k \in \boldsymbol{R}$, $l \in (0, d)$ it holds that*

$$F_0(A(l), \mu; k) = e^{-\mu l} R(l)\{1 - e^{-2(d-l)\mu} \lambda\tilde{R}(l)/R(l)\}\hat{f}(a_1, \mu+ik),$$

where $R(l)$ and $\tilde{R}(l)$ are given by (3.4) *and* (3.5) *respectively.*

§4. Existence of the poles of $U(\mu)$

Lemma 4.1. *There exists $l_0 \in (0, d)$, and positive constants ε_0, η_0 such that*

$$|e^{-\mu l_0} R(l_0)(1 - e^{-2(d-l_0)\mu} \tilde{R}(l_0)R(l_0)^{-1}\lambda)| \geq 2\varepsilon_0 \tag{4.1}$$

for all $\mu \in \{\mu;\ |\mathrm{Re}\,\mu - (-c_0)| \leq \eta_0\}$.

Proof. Since $e^{4(d-l)c_0}$ is a holomorphic function of $l \in \boldsymbol{C}$, and $\tilde{R}(l)^2 R(l)^{-2}\lambda^2$ is rational but not holomorphic in the whole plane, they are not identical in $\boldsymbol{C}$. Therefore it does not holds that

$$e^{4(d-l)c_0} = \tilde{R}(l)^2 R(l)^{-2}\lambda^2 \qquad \text{for all} \quad l \in (0, d).$$

This assures that there exists $l_0 \in (0, d)$ such that

$$e^{4(d-l_0)c_0} \neq \tilde{R}(l_0)^2 R(l_0)^{-2}\lambda^2.$$

This implies that

$$1 - |e^{-2(d-l_0)\mu} \tilde{R}(l_0)R(l_0)^{-1}\lambda| \neq 0$$

holds for all $\mathrm{Re}\,\mu=-c_0$. Set the absolute value of the left hand side $=4\varepsilon_1$, $\varepsilon_1>0$. Since

$$\frac{d}{d\mu}(e^{-2\mu(d-l_0)}\tilde{R}(l_0)R(l_0)^{-1}\lambda)$$

is uniformly bounded in $\{\mu;\ \mathrm{Re}\,\mu\geq -c_0-\eta\}$ for η fixed, it holds that

$$|1-|e^{-2(d-l_0)\mu}\tilde{R}(l_0)R(l_0)^{-1}\lambda||\geq 2\varepsilon_1 \qquad \text{if} \quad |\mathrm{Re}\,\mu-(-c_0)|\leq \eta_0$$

for η_0 sufficiently small. Note that $R(l_0)>R(d)>0$. Set $\varepsilon_0=R(l_0)e^{l_0(c_0-\eta_0)}\varepsilon_1$. Then we have for $|\mathrm{Re}\,\mu-(-c_0)|\leq\eta_0$

$$|e^{-\mu l_0}R(l_0)(1-e^{-2(d-l_0)\mu}\tilde{R}(l_0)R(l_0)^{-1}\lambda)|$$
$$\geq e^{l_0(c_0-\eta_0)}R(l_0)|1-|e^{-2(d-l_0)\mu}\tilde{R}(l_0)R(l_0)^{-1}\lambda||\geq 2\varepsilon_0,$$

which is the desired estimate. Q.E.D.

Let $m(t)$ be a function of $C_0^\infty(0, d/2)$ such that

$$m(t)\geq 0, \quad \int_{-\infty}^{\infty} m(t)dt=1.$$

Then, since $\hat{m}(-c_0)=\int_{-\infty}^{\infty}e^{c_0t}\,m(t)dt>1$, we have for some $\eta_1>0$

$$|\hat{m}(\mu)|\geq 1 \quad \text{for all} \quad |\mu-(-c_0)|\leq\eta_1. \tag{4.2}$$

Let $g(x)$ be a function in $C_0^\infty(S_1(\delta_2))$ verifying

$$g(a_1)=1. \tag{4.3}$$

Set

$$m(x, t;\ k)=e^{ik(\varphi_\infty(x)-t)}\,g(x)m(t)$$

and denote by $\hat{u}(x, \mu;\ k)$ the one in Lemma 2.1 for this $m(x, t;\ k)$. Set

$$\eta=\min\{\eta_0, \eta_1, \pi/4d\},$$
$$D_j=\{\mu;\ |\mu-\mu_j|\leq\eta\}, \quad C_j=\{\mu;\ |\mu-\mu_j|=\eta\}.$$

Recall that $U(\mu)$ exists on C_j for large $|j|$ by virtue of Theorem 2 of [2]. Then from relations (2.6) and (2.8) it follows that

$$\begin{aligned} &U(\mu)(e^{ik\varphi_\infty(\cdot)}g(\cdot)\hat{m}(\mu+ik))(x) \\ &=\hat{u}(x, \mu;\ k)-U(\mu)(k^{-N}\mathscr{P}(\mu)^{-N-1}G_N(\cdot, \mu;\ k))(x) \qquad \text{for all} \quad \mu\in C_j, \end{aligned} \tag{4.4}$$

where G_N is a function on Γ defined by

$$G_N(x, \mu;\ k)=G_{N,j}(x, \mu;\ k) \qquad \text{on} \quad \Gamma_j,\ j=1, 2.$$

Suppose that $U(\mu)$ is holomorphic in D_j. Then $U(\mu)(e^{ik\varphi_\infty(\cdot)}g(\cdot))$ is holomorphic in D_j. Therefore we have

(4.5) $$\frac{1}{2\pi}\Delta_{C_j}(U(\mu)(e^{ik\varphi_\infty(\cdot)}g(\cdot)\hat{m}(\mu+ik))(x)\geq 0 \qquad \text{for all} \quad x\in\Omega.$$

Set

$$k_j=-jd/\pi.$$

Then for $\mu\in D_j$ we have

$$\mu+ik_j=-c_0+\tau e^{i\theta}, \quad 0\leq\tau\leq\eta \quad \text{and} \quad 0\leq\theta<2\pi.$$

Therefore (4.2) implies

(4.6) $$|\hat{m}(\mu+ik_j)|\geq 1 \qquad \text{for all} \quad \mu\in D_j.$$

By using (2.7) and the estimate of $U(\mu)$ of Theorem 2 of [2] we have

(4.7) $$|U(\mu)(G_N(\cdot,\mu;k_j))(A(l_0))|\leq C|j|^7 \qquad \text{for all} \quad \mu\in C_j.$$

Note that

(4.8) $$|\mathscr{P}(\mu)|\geq\alpha_0>0 \qquad \text{for all} \quad \mu\in C_j.$$

Since

$$F_0(A(l_0),\mu;k)=e^{-\mu l_0}R(l_0)(1-e^{-2(d-l_0)\mu}\tilde{R}(l_0)R(l_0)^{-1}\lambda)\hat{m}(\mu+ik_j)$$

follows from Lemma 3.1 and (4.3), the estimates (4.1) and (4.6) imply

(4.9) $$\frac{1}{2\pi}\Delta_{C_j}F_0(A(l_0),\mu;k_j)=0.$$

On the other hand we have from (4.1), (4.6) and (4.8)

$$|\mathscr{P}(\mu)^{-1}F_0(A(l_0),\mu;k_j)|\geq\alpha_0^{-1}2\varepsilon_0, \qquad \text{for all} \quad \mu\in C_j$$

and by using (2.4), (4.7) and (4.8) we have for all $\mu\in C_j$

$$|\mathscr{P}(\mu)^{-1}\sum_{r=1}^{N}(k_j\mathscr{P}(\mu))^{-r}F_r(A(l_0),\mu;k_j)$$
$$-U(\mu)(k_j^{-N}\mathscr{P}(\mu)^{-N-1}G_N(\cdot,\mu,;k_j))(A(l_0))|$$
$$\leq C\alpha_0^{-1}\{\sum_{r=1}^{N}|\alpha_0k_j|^{-r}+|k_j|^{-N}|k_j|^7\}$$

where C is a constant independent of j. Therefore for large $|j|$

$$|\mathscr{P}(\mu)^{-1}F_0(A(l_0),\mu;k_j)|>|\mathscr{P}(\mu)^{-1}\sum_{r=1}^{N}(k_j\mathscr{P}(\mu))^{-r}F_r(A(l_0),\mu;k_j)|$$
$$+|U(\mu)(k_j^{-N}\mathscr{P}(\mu)^{-N-1}G_N(\cdot,\mu;k_j))(A(l_0))|$$

holds for all $\mu\in C_j$. This shows that

$$\frac{1}{2\pi}\Delta_{C_j}\{\hat{u}(A(l_0),\mu;k_j)-U(\mu)(k_j^{-N}\mathscr{P}(\mu)^{-N-1}G_N(\cdot,\mu;k_j))(A(l_0))\}$$
$$=\frac{1}{2\pi}\Delta_{C_j}\mathscr{P}(\mu)^{-1}F_0(A(l_0),\mu;k_j).$$

Taking account of (4.9) we have

$$\frac{1}{2\pi}\Delta_{C_j}\mathscr{P}(\mu)^{-1}F_0(A(l_0),\ \mu;\ k_j)=\frac{1}{2\pi}\Delta_{C_j}\mathscr{P}(\mu)^{-1}+\frac{1}{2\pi}\Delta_{C_j}F_0(A(l_0),\ \mu;\ k_j)=-1.$$

Then it is proved that the variation of the argument along C_j of the right hand side of (4.4) at $x=A(l_0)$ is equal to -2π for large $|j|$. This contradicts with (4.5). Thus $U(\mu)$ is not holomorphic in D_j for large $|j|$, which prove Theorem 2.

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