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The spectra of 1-forms on simply connected compact irreducible Riemannian symmetric spaces II

By

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Introduction.

In the preceding paper [5], we established a method to calculate the spectra of 1-forms on simply connected compact irreducible Riemannian symmetric spaces. By use of it, we determined the spectra of 1-forms on [AIII] $SU(p+q)/S(U(p)\times U(q))$ and [G] $G_2/SU(2)\times SU(2)$. The purpose of this paper is to show the complete lists of the spectra of 1-forms on all simply connected irreducible Riemannain symmetric spaces except: (1°) Compact simple Lie groups; (2°) [AIII], [G]; (3°) [BDI, II] $SO(p+q)/SO(p)\times SO(q)$ ($q\geqq p$, $p=1, 2$). The spectra of 1-forms on [BDI, II] ($q\geqq p$, $p=1, 2$) can be seen in Ikeda-Taniguchi [4] and Tsukamoto [9]. See also Gallot-Meyer [3], Levy-Bruhl-Laperrière [6], [7] and Strese [8]. The spectra of 1-forms on compact simple Lie groups can be obtained by Theorem 2.1 and Corollary to Theorem 1.3 in [5], however they are not treated here.

In order to explain the contents of this paper, we review some fundamental notations. Let G/K be a simply connected compact irreducible Riemannian symmetric space with G simple. We denote by $\mathcal{D}(G)$ the set of equivalence classes of irreducible representations of G and $\mathcal{D}(G, K)$ the set of equivalence classes of spherical representaitons of the symmetric pair (G, K). Let $\mathfrak{g}=\mathfrak{k}+\mathfrak{m}$ be the canonical decomposition of the Lie algebra $\mathfrak{g}$ of G associated with G/K. We choose a Cartan subalgebra $\mathfrak{t}$ of $\mathfrak{g}$ containing a maximal abelian subspace of $\mathfrak{m}$. Let $\Pi=\{\alpha_1, \cdots, \alpha_n\}$ be the set of simple roots with respect to a suitable linear order in $\mathfrak{t}$. We denote by p the Satake involution of the set $I=\{i\,|\,\alpha_i\in\Pi,\ \alpha_i\notin\mathfrak{b}\}$, where $\mathfrak{b}=\mathfrak{t}\cap\mathfrak{k}$. Let $D(G)$ be the set of dominant integral forms on $\mathfrak{t}$ and let $D(G, K)$ be the subset of $D(G)$ consisting of all highest weights of $[\rho]\in\mathcal{D}(G, K)$. The set $D(G, K)$ is given by the additive semi-group generated by the following M_i's ($i\in I$, $p(i)\geqq i$):

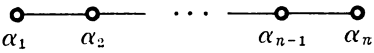
$$M_i=\begin{cases}2\Lambda_i & p(i)=i,\ (\alpha_i, \Pi\cap\mathfrak{b})=\{0\}\ ;\\ \Lambda_i & p(i)=i,\ (\alpha_i, \Pi\cap\mathfrak{b})\neq\{0\}\ ;\\ \Lambda_i+\Lambda_{p(i)} & p(i)>i\ ;\end{cases}$$

where $\{\Lambda_1, \cdots, \Lambda_n\}$ stands for the set of fundamental weights.

Let $\Lambda^1(G/K)$ be the space of complex continuous 1-forms on G/K. Under the canonical action of G, $\Lambda^1(G/K)$ can be regarded as a G-module. By definition the spectrum of $\Lambda^1(G/K)$ is the function $\mathcal{D}(G)\ni[\rho]\mapsto a([\rho])\in \boldsymbol{Z}$ determined by $a([\rho])=\dim_C \mathrm{Hom}_G(V^\rho, \Lambda^1(G/K))$, where we mean by $\rho: G\to GL(V^\rho)$ an irreducible representation of G. The spectrum describes how $\Lambda^1(G/K)$ decompose into a direct sum of irreducible G-submodules. Actually the number $a([\rho])$ indicates the number of irreducible factors isomorphic to V^ρ as G-modules in such a decomposiuion. The purpose of this paper is to determine the spectrum of $\Lambda^1(G/K)$ for each simply connected compact irreducible Riemannian symmetric space G/K with G simple. For this purpose we give in the following table the function $D(G)\ni\Lambda\mapsto a(\Lambda)\in \boldsymbol{Z}$ defined in [5]. We exhibit all dominant integral forms $\Lambda\in D(G)$ with $a(\Lambda)\neq 0$ and the numbers $a(\Lambda)$. As was announced in [5], if Λ denotes the highest weight of $[\rho]$, then two numbers $a([\rho])$ and $a(\Lambda)$ coincide, i.e., $a([\rho])=a(\Lambda)$. This equality can be verified by the examination of the subset $B(\Lambda)$ of Δ determined by Λ (see Proposition 3.4 and Proposition 3.6 in [5]). The details are omitted in this paper. Thus the spectra of $\Lambda^1(G/K)$ on all simply connected compact irreducible Riemannian symmetric spaces G/K with G simple can be obtained by the following tables.

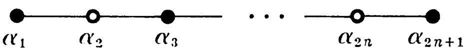
Throughout this paper M_0 means an arbitrary element of $D(G, K)$. Making use of the generators M_i's stated as above, M_0 can be expressed by $M_0=\sum m_i M_i$, where $m_i\in \boldsymbol{Z}$, $m_i\geqq 0$. We also mean by $m(M_0)$ $(M_0=\sum m_i M_i\in D(G, K))$ the number defined by the following equality: $m(M_0)=\#\{m_i \mid m_i>0\}$.

[AI] $SU(n+1)/SO(n+1)$ $(n\geqq 1)$



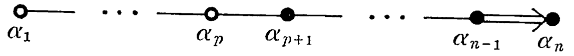
	Λ	$a(\Lambda)$
(I)	M_0	$m(M_0)$
(II)	$\Lambda_{i-1}+\Lambda_i+\Lambda_j+\Lambda_{j+1}+M_0$ $(1\leqq i\leqq j\leqq n)$	1

[AII] $SU(2(n+1))/Sp(n+1)$ $(n\geqq 1)$

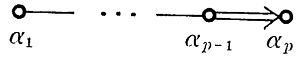


	Λ	$a(\Lambda)$
(I)	M_0	$m(M_0)$
(II)	$\Lambda_{2i-1}+\Lambda_{2j+1}+M_0$ $(1\leqq i\leqq j\leqq n)$	1

[BI] $SO(2n+1)/SO(p)\times SO(2n-p+1)$ $(3\leqq p\leqq n)$



$(n\geqq p+1)$



$(n=p)$

	Λ	$a(\Lambda)$
(I)	M_0	$m(M_0)$
(II)	$\Lambda_{i-1}+\Lambda_i+\Lambda_j+\Lambda_{j+1}+M_0$ $(1\leqq i<j\leqq p-2)$	2
(III)	$\Lambda_{i-1}+\Lambda_{i+1}+M_0$ $(1\leqq i\leqq p-2)$ $(M_0,\ \alpha_i{}^*)=\begin{cases}0\\+\end{cases}$	 1 2
(IV)	$\Lambda_{i-1}+\Lambda_i+\Lambda_{p-1}+M_0$ $(1\leqq i\leqq p-2)$ $(M_0,\ \alpha_p)^*=\begin{cases}0\\+\end{cases}$	 1 2
(V)	$\Lambda_{i-1}+\Lambda_i+\Lambda^{(1)}+M_0$ $(1\leqq i\leqq p-1)$	1 (*)
(VI)	$\Lambda_{p-2}+M_0$ $(M_0,\ \alpha_{p-1}{}^*)$ $(M_0,\ \alpha_p{}^*)$ 0 0 + 0 0 + + +	 0 1 1 2
(VII)	$\Lambda_{p-1}+\Lambda^{(1)}+M_0$	1 (*)

Remark (*).

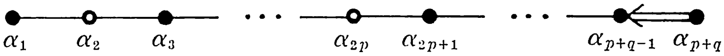
$$\Lambda^{(1)}=\begin{cases}\Lambda_{p+1} & (n\geqq p+2)\\ 2\Lambda_{p+1} & (n=p+1)\\ 2\Lambda_p & (n=p)\end{cases}$$

[CI] $Sp(n)/U(n)$ $(n\geqq 1)$

	Λ	$a(\Lambda)$
(I)	M_0	$2m(M_0)$
(II)	$\Lambda_{i-1}+\Lambda_i+\Lambda_j+\Lambda_{j+1}+M_0$ $(1\leqq i\leqq j\leqq n-1)$	2

[CII] $Sp(p+q)/Sp(p)\times Sp(q)$ $(1\leqq p\leqq q)$

(i) $q\geqq p+1$:



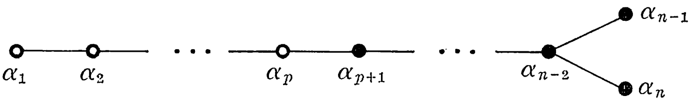
	Λ	$a(\Lambda)$
(I)	M_0	$m(M_0)$
(II)	$\Lambda_{2i-1}+\Lambda_{2j+1}+M_0$ $(1\leqq i\leqq j\leqq p-1)$	2
(III)	$\Lambda_{2i-1}+\Lambda_{2p+1}+M_0$ $(1\leqq i\leqq p)$	1
(IV)	$2\Lambda_{2i-1}+M_0$ $(1\leqq i\leqq p)$	1

(ii) $q=p$:

	Λ	$a(\Lambda)$
(I)	M_0	$m(M_0)$
(II)	$\Lambda_{2i-1}+\Lambda_{2j+1}+M_0$ $(1\leqq i\leqq j\leqq p-1)$	2
(III)	$2\Lambda_{2i-1}+M_0$ $(1\leqq i\leqq p)$	1

[DI] $SO(2n)/SO(p)\times SO(2n-p)$ $(3\leqq p\leqq n)$

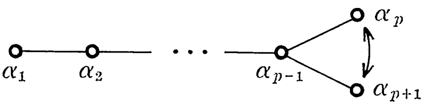
(i) $n\geqq p+2$:



	Λ	$a(\Lambda)$
(I)	M_0	$m(M_0)$
(II)	$\Lambda_{i-1}+\Lambda_i+\Lambda_j+\Lambda_{j+1}+M_0$ $(1\leqq i<j\leqq p-2)$	2
(III)	$\Lambda_{i-1}+\Lambda_{i+1}+M_0$ $(1\leqq i\leqq p-2)$ $(M_0, \alpha_i^*)=\begin{cases}0\\+\end{cases}$	 1 2
(IV)	$\Lambda_{i-1}+\Lambda_i+\Lambda_{p-1}+M_0$ $(1\leqq i\leqq p-2)$ $(M_0, \alpha_p^*)=\begin{cases}0\\+\end{cases}$	 1 2
(V)	$\Lambda_{i-1}+\Lambda_i+\Lambda^{(2)}+M_0$ $(1\leqq i\leqq p-1)$	1
(VI)	$\Lambda_{p-2}+M_0$ (M_0, α_{p-1}^*) (M_0, α_p^*) 0 0 + 0 0 + + +	 0 1 1 2
(VII)	$\Lambda_{p-1}+\Lambda^{(2)}+M_0$	1(**)

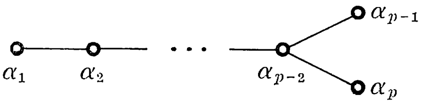
Remark (**). $\Lambda^{(2)}=\begin{cases}\Lambda_{p+1} & (n\geqq p+3)\\ \Lambda_{p+1}+\Lambda_{p+2} & (n=p+2)\end{cases}$

(ii) $n=p+1$:



	Λ	$a(\Lambda)$
(I)	M_0	$m(M_0)$
(II)	$\Lambda_{i-1}+\Lambda_i+\Lambda_j+\Lambda_{j+1}+M_0 \quad (1\leqq i<j\leqq p-2)$	2
(III)	$\Lambda_{i-1}+\Lambda_{i+1}+M_0 \quad (1\leqq i\leqq p-2)$	
	$(M_0, \alpha_i^*)=0$	1
	$(M_0, \alpha_i^*)=+$	2
(IV)	$\Lambda_{i-1}+\Lambda_i+\Lambda_{p-1}+M_0 \quad (1\leqq i\leqq p-2)$	
	$(M_0, \alpha_p^*)=0$	1
	$(M_0, \alpha_p^*)=+$	2
(V)	$\Lambda_{i-1}+\Lambda_i+2\Lambda_p+M_0 \quad (1\leqq i\leqq p-1)$	1
	$\Lambda_{i-1}+\Lambda_i+2\Lambda_{p+1}+M_0 \quad (1\leqq i\leqq p-1)$	1
(VI)	$\Lambda_{p-2}+M_0$	
	(M_0, α_{p-1}^*) (M_0, α_p^*)	
	0 0	0
	+ 0	1
	0 +	1
	+ +	2
(VII)	$\Lambda_{p-1}+2\Lambda_p+M_0$	1
	$\Lambda_{p-1}+2\Lambda_{p+1}+M_0$	1

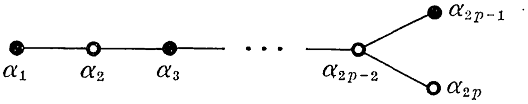
(iii) $n=p$:



	Λ	$a(\Lambda)$
(I)	M_0	$m(M_0)$
(II)	$\Lambda_{i-1}+\Lambda_i+\Lambda_j+\Lambda_{j+1}+M_0 \quad (1\leqq i<j\leqq p-3)$	2
(III)	$\Lambda_{i-1}+\Lambda_{i+1}+M_0 \quad (1\leqq i\leqq p-3)$ $(M_0, \alpha_i^*)=\begin{cases}0\\+\end{cases}$	 1 2
(IV)	$\Lambda_{i-1}+\Lambda_i+\Lambda_{p-2}+\Lambda_{p-1}+\Lambda_p+M_0 \quad (1\leqq i\leqq p-3)$	2
(V)	$\Lambda_{i-1}+\Lambda_i+\Lambda_{p-1}+\Lambda_p+M_0 \quad (1\leqq i\leqq p-2)$	2
(VI)	$\Lambda_{p-2}+M_0$ (M_0, α_{p-1}^*) (M_0, α_p^*) 0 0 + 0 0 + + +	 0 1 1 2
(VII)	$\Lambda_{p-3}+\Lambda_{p-1}+\Lambda_p+M_0$ $(M_0, \alpha_{p-2}^*)=\begin{cases}0\\+\end{cases}$	 1 2

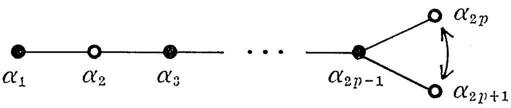
[DIII] $SO(2n)/U(n)$ $(n\geqq 3)$

(i) $n=2p$:



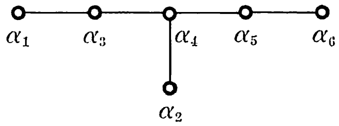
	Λ	$a(\Lambda)$
(I)	M_0	$2m(M_0)$
(II)	$\Lambda_{2i-1}+\Lambda_{2j+1}+M_0$ $(1\leqq i\leqq j\leqq p-2)$	2
(III)	$\Lambda_{2i-1}+\Lambda_{2p-1}+\Lambda_{2p}+M_0$ $(1\leqq i\leqq p-1)$	2

(ii) $n=2p+1$:



	Λ	$a(\Lambda)$
(I)	M_0	$2m(M_0)$
(II)	$\Lambda_{2i-1}+\Lambda_{2j+1}+M_0$ $(1\leqq i\leqq j\leqq p-1)$	2
(III)	$\Lambda_{2i-1}+2\Lambda_{2p}+M_0$ $(1\leqq i\leqq p)$	1
	$\Lambda_{2i-1}+2\Lambda_{2p+1}+M_0$ $(1\leqq i\leqq p)$	1

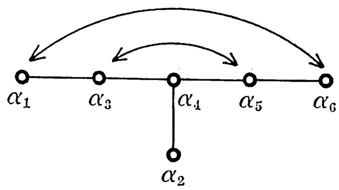
[EI] $E_6/Sp(4)$



	Λ	$a(\Lambda)$
(I)	M_0	$m(M_0)$
(II)	$\Lambda_3+2\Lambda_1+M_0$ $\Lambda_4+2\Lambda_2+M_0$ $\Lambda_1+\Lambda_4+2\Lambda_3+M_0$ $\Lambda_2+\Lambda_3+\Lambda_5+2\Lambda_4+M_0$ $\Lambda_4+\Lambda_6+2\Lambda_5+M_0$ $\Lambda_5+2\Lambda_6+M_0$	1
(III)	$\lvert\alpha\rvert+M_0 \quad (\alpha\in\Delta,\ \alpha>0,\ \alpha\notin\Pi)$	1 (***)

Remark (***). $\lvert\alpha\rvert=\sum_{i=1}^{6}\lvert m_i\rvert\Lambda_i \ \left(\alpha=\sum_{i=1}^{6}m_i\Lambda_i\right)$

[EII] $E_6/SU(2)\cdot SU(6)$

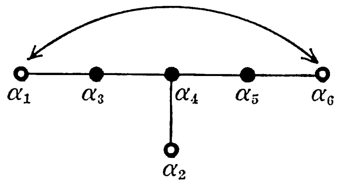


	Λ	$a(\Lambda)$
(I)	M_0	$m(M_0)$
(II)	$\Lambda_1+\Lambda_3+M_0$ $\Lambda_1+\Lambda_3+\Lambda_2+M_0$ $\Lambda_1+\Lambda_3+\Lambda_4+M_0$ $\Lambda_1+\Lambda_3+\Lambda_2+\Lambda_4+M_0$	1

(II)	$\Lambda_5+\Lambda_6+M_0$				1
	$\Lambda_5+\Lambda_6+\Lambda_2+M_0$				
	$\Lambda_5+\Lambda_6+\Lambda_4+M_0$				
	$\Lambda_5+\Lambda_6+\Lambda_2+\Lambda_4+M_0$				
	$\Lambda_1+2\Lambda_5+M_0$				1
	$\Lambda_1+2\Lambda_5+\Lambda_2+M_0$				
	$\Lambda_1+2\Lambda_5+\Lambda_4+M_0$				
	$\Lambda_1+2\Lambda_5+\Lambda_2+\Lambda_4+M_0$				
	$2\Lambda_3+\Lambda_6+M_0$				1
	$2\Lambda_3+\Lambda_6+\Lambda_2+M_0$				
	$2\Lambda_3+\Lambda_6+\Lambda_4+M_0$				
	$2\Lambda_3+\Lambda_6+\Lambda_2+\Lambda_4+M_0$				
	$2\Lambda_1+\Lambda_5+M_0$				1
	$2\Lambda_1+\Lambda_5+\Lambda_2+M_0$				
	$2\Lambda_1+\Lambda_5+\Lambda_4+M_0$				
	$2\Lambda_1+\Lambda_5+\Lambda_2+\Lambda_4+M_0$				
	$\Lambda_3+2\Lambda_6+M_0$				1
	$\Lambda_3+2\Lambda_6+\Lambda_2+M_0$				
	$\Lambda_3+2\Lambda_6+\Lambda_4+M_0$				
	$\Lambda_3+2\Lambda_6+\Lambda_2+\Lambda_4+M_0$				
(III)	Λ_2+M_0				
		$(M_0,\ \alpha_1^*)$	$(M_0,\ \alpha_3^*)$	$(M_0,\ \alpha_4^*)$	
		0	0	0	1
		+	0	0	2
		0	+	0	2
		0	0	+	2
		+	+	0	3
		+	0	+	3
		0	+	+	3
		+	+	+	4

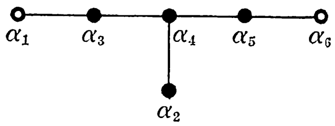
(IV)	Λ_4+M_0			
	(M_0, α_1^*)	(M_0, α_2^*)	(M_0, α_3^*)	
	0	0	0	1
	+	0	0	2
	0	+	0	2
	0	0	+	2
	+	+	0	3
	+	0	+	3
	0	+	+	3
	+	+	+	4
(V)	$\Lambda_2+\Lambda_4+M_0$			
	(M_0, α_1^*)	(M_0, α_3^*)		
	0	0		2
	+	0		3
	0	+		3
	+	+		4

[EIII] $E_6/Spin(10)\cdot SO(2)$



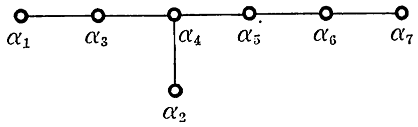
	Λ	$a(\Lambda)$
(I)	M_0	
	(M_0, α_1^*) (M_0, α_2^*)	
	0 0	0
	+ 0	2
	0 +	2
	+ +	4
(II)	Λ_4+M_0	2
(III)	$\Lambda_1+\Lambda_3+M_0$	1
	$\Lambda_5+\Lambda_6+M_0$	
	$2\Lambda_1+\Lambda_5+M_0$	
	$\Lambda_3+2\Lambda_6+M_0$	

[EIV] E_6/F_4



	Λ	$a(\Lambda)$
(I)	M_0	$m(M_0)$
(II)	Λ_2+M_0	1
	Λ_3+M_0	1
	Λ_5+M_0	1

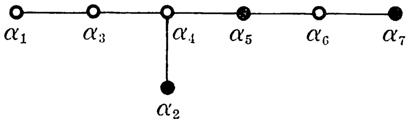
[EV] $E_7/SU(8)$



	Λ	$a(\Lambda)$
(I)	M_0	$m(M_0)$
(II)	$\Lambda_3+2\Lambda_1+M_0$ $\Lambda_4+2\Lambda_2+M_0$ $\Lambda_1+\Lambda_4+2\Lambda_3+M_0$ $\Lambda_2+\Lambda_3+\Lambda_5+2\Lambda_4+M_0$ $\Lambda_4+\Lambda_6+2\Lambda_5+M_0$ $\Lambda_5+\Lambda_7+2\Lambda_6+M_0$ $\Lambda_6+2\Lambda_7+M_0$	1
(III)	$\lvert\alpha\rvert+M_0 \quad (\alpha\in\Delta,\ \alpha>0,\ \alpha\notin\Pi)$	1 (**)

Remark (**). $\lvert\alpha\rvert=\sum_{i=1}^{7}\lvert m_i\rvert\Lambda_i \ \left(\alpha=\sum_{i=1}^{7}m_i\Lambda_i\right)$

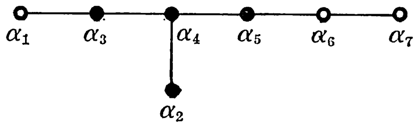
[EVI] $E_7/SO(12)\cdot SU(2)$



	Λ	$\alpha(\Lambda)$
(I)	M_0	$m(M_0)$
(II)	$\Lambda_2+\Lambda_5+M_0$	
	$\Lambda_1+\Lambda_2+\Lambda_5+M_0$	1
	$\Lambda_3+\Lambda_2+\Lambda_5+M_0$	
	$\Lambda_1+\Lambda_3+\Lambda_2+\Lambda_5+M_0$	
	$\Lambda_2+\Lambda_7+M_0$	
	$\Lambda_1+\Lambda_2+\Lambda_7+M_0$	1
	$\Lambda_3+\Lambda_2+\Lambda_7+M_0$	
	$\Lambda_1+\Lambda_3+\Lambda_2+\Lambda_7+M_0$	
	$\Lambda_5+\Lambda_7+M_0$	
	$\Lambda_1+\Lambda_5+\Lambda_7+M_0$	1
	$\Lambda_3+\Lambda_5+\Lambda_7+M_0$	
	$\Lambda_1+\Lambda_3+\Lambda_5+\Lambda_7+M_0$	
(III)	Λ_1+M_0	
	$(M_0, \alpha_3{}^*)$ $(M_0, \alpha_4{}^*)$ $(M_0, \alpha_6{}^*)$	
	0 0 0	1
	+ 0 0	2
	0 + 0	2
	0 0 +	2
	+ + 0	3
	+ 0 +	3
	0 + +	3
	+ + +	4

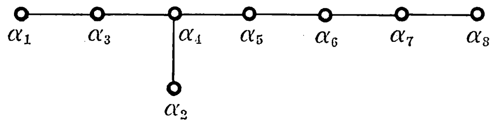
(IV)	Λ_3+M_0	
	(M_0, α_1^*) (M_0, α_4^*) (M_0, α_6^*)	
	0 0 0	1
	+ 0 0	2
	0 + 0	2
	0 0 +	2
	+ + 0	3
	+ 0 +	3
	0 + +	3
	+ + +	4
(V)	$\Lambda_1+\Lambda_3+M_0$	
	(M_0, α_4^*) (M_0, α_6^*)	
	0 0	2
	+ 0	3
	0 +	3
	+ +	4

[EVII] $E_7/E_6 \cdot SO(2)$



	Λ			$a(\Lambda)$
(I)	M_0			
	(M_0, α_1^*)	(M_0, α_6^*)	(M_0, α_7^*)	
	0	0	0	0
	+	0	0	2
	0	+	0	2
	0	0	+	2
	+	+	0	3
	+	0	+	3
	0	+	+	3
	+	+	+	4
(II)	Λ_3+M_0			2
	$\Lambda_2+\Lambda_7+M_0$			2
	$\Lambda_5+\Lambda_7+M_0$			2

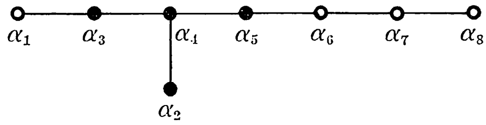
[EVIII] $E_8/SO(16)$



	Λ	$a(\Lambda)$
(I)	M_0	$m(M_0)$
(II)	$\Lambda_3+2\Lambda_1+M_0$ $\Lambda_4+2\Lambda_2+M_0$ $\Lambda_1+\Lambda_4+2\Lambda_3+M_0$ $\Lambda_2+\Lambda_3+\Lambda_5+2\Lambda_4+M_0$ $\Lambda_4+\Lambda_6+2\Lambda_5+M_0$ $\Lambda_5+\Lambda_7+2\Lambda_6+M_0$ $\Lambda_6+\Lambda_8+2\Lambda_7+M_0$ $\Lambda_7+2\Lambda_8+M_0$	1
(III)	$\|\alpha\|+M_0 \quad (\alpha \ni \Delta,\ \alpha>0,\ \alpha \notin \Pi)$	1(*****)

Remark (*****). $|\alpha|=\sum_{i=1}^{8}|m_i|\Lambda_i \ \left(\alpha=\sum_{i=1}^{8}m_i\Lambda_i\right)$.

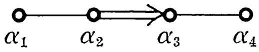
[EIX] $E_8/E_7\cdot SU(2)$



	Λ	$a(\Lambda)$
(I)	M_0	$m(M_0)$
(II)	Λ_2+M_0	
	$\Lambda_2+\Lambda_7+M_0$	1
	$\Lambda_2+\Lambda_8+M_0$	
	$\Lambda_2+\Lambda_7+\Lambda_8+M_0$	
	Λ_3+M_0	
	$\Lambda_3+\Lambda_7+M_0$	1
	$\Lambda_3+\Lambda_8+M_0$	
	$\Lambda_3+\Lambda_7+\Lambda_8+M_0$	
	Λ_5+M_0	
	$\Lambda_5+\Lambda_7+M_0$	1
	$\Lambda_5+\Lambda_8+M_0$	
	$\Lambda_5+\Lambda_7+\Lambda_8+M_0$	
(III)	Λ_7+M_0	
	(M_0, α_1^*) (M_0, α_6^*) (M_0, α_8^*)	
	0 0 0	1
	+ 0 0	2
	0 + 0	2
	0 0 +	2
	+ + 0	3
	+ 0 +	3
	0 + +	3
	+ + +	4

(IV)	Λ_8+M_0			
	(M_0, α_1^*)	(M_0, α_6^*)	(M_0, α_7^*)	
	0	0	0	1
	+	0	0	2
	0	+	0	2
	0	0	+	2
	+	+	0	3
	+	0	+	3
	0	+	+	3
	+	+	+	4
(V)	$\Lambda_7+\Lambda_8+M_0$			
	(M_0, α_1^*)	(M_0, α_6^*)		
	0	0		2
	+	0		3
	0	+		3
	+	+		4

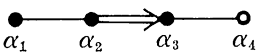
[FI] $F_4/Sp(3)\cdot SU(2)$



	Λ			$a(\Lambda)$
(I)	M_0			$m(M_0)$
(II)	$\Lambda_3+2\Lambda_4+M_0$			
	$\Lambda_1+\Lambda_3+2\Lambda_4+M_0$			1
	$\Lambda_2+\Lambda_3+2\Lambda_4+M_0$			
	$\Lambda_1+\Lambda_2+\Lambda_3+2\Lambda_4+M_0$			
	$\Lambda_4+2\Lambda_3+M_0$			
	$\Lambda_1+\Lambda_4+2\Lambda_3+M_0$			1
	$\Lambda_2+\Lambda_4+2\Lambda_3+M_0$			
	$\Lambda_1+\Lambda_2+\Lambda_4+2\Lambda_3+M_0$			
	$\Lambda_3+\Lambda_4+M_0$			
	$\Lambda_1+\Lambda_3+\Lambda_4+M_0$			1
	$\Lambda_2+\Lambda_3+\Lambda_4+M_0$			
	$\Lambda_1+\Lambda_2+\Lambda_3+\Lambda_4+M_0$			
(III)	Λ_1+M_0			
	(M_0, α_2^*)	(M_0, α_3^*)	(M_0, α_4^*)	
	0	0	0	1
	+	0	0	2
	0	+	0	2
	0	0	+	2
	+	+	0	3
	+	0	+	3
	0	+	+	3
	+	+	+	4

(IV)	Λ_2+M_0	
	(M_0, α_1^*) (M_0, α_3^*) (M_0, α_4^*)	
	0 0 0	1
	+ 0 0	2
	0 + 0	2
	0 0 +	2
	+ + 0	3
	+ 0 +	3
	0 + +	3
	+ + +	4
(V)	$\Lambda_1+\Lambda_2+M_0$	
	(M_0, α_3^*) (M_0, α_4^*)	
	0 0	2
	+ 0	3
	0 +	3
	+ +	4

[FII] $F_4/Spin(9)$



	Λ	$a(\Lambda)$
(I)	Λ_4+M_0	1
(II)	Λ_1+M_0	1
	Λ_3+M_0	1

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