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Remarks on the elliptic cohomology of finite groups

By

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1. Elliptic character

Let $Ell^*(?)$ be the elliptic cohomology based on the Weierstrass cubic

$$y^2=4x^3-g_2x-g_3$$

over

$$Ell^*=\mathbf{Z}[1/6][g_2, g_3, \Delta^{-1}](\Delta=g_2^3-27g_3^2)$$

(see [3], [11]). The coefficient ring $Ell^*=Ell^*(\mathrm{pt})$ can be viewed as the ring of modular forms on $\Gamma(1)=SL_2(\mathbf{Z})$ over $\mathbf{Z}[1/6]$. (The grading on Ell^* is given by $-2\times$weight.) In other words Ell^* is the ring which represents the functor

$$\{\mathbf{Z}[1/6]\text{-algebras } A\}\to\{\text{isomorphism classes of } \Gamma(1)\text{-test objects over } A\}$$

with universal test object

$$(E_{\mathrm{univ}}, \omega_{\mathrm{univ}})=(y^2=4x^3-g_2x-g_3,\ dx/y),$$

where a $\Gamma(1)$-test object over A means a pair (E, ω) consisisting of an elliptic curve E/A and a nowhere-vanishing invariant differential ω on E (see [10, Chapter II]). This identification is *natural* in the sense that the formal group law associated to Ell with canonical orientation is the formal group $\widehat{E}_{\mathrm{univ}}$ associated to E_{univ}, with parameter $T=-2x/y$.

For $n\geq 2$ let $E_{2n}\in Ell^{-4n}\otimes\mathbf{Q}$ be the Eisenstein series given by the q-expansion

$$E_{2n}(q)=1-(4n/B_{2n})\sum_{k\geq 1}\sigma_{2n-1}(k)q^k,$$

where

$$z/(e^z-1)=1-z/2+\sum_{n\geq 1}B_{2n}z^{2n}/(2n)!$$

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and

$$\sigma_n(k)=\sum_{d|k} d^n.$$

Then the Tate elliptic curve Tate(q) is given over $\mathbf{Z}[1/6]((q))$ by

$$y^2=4x^3-\frac{1}{12}E_4(q)x+\frac{1}{216}E_6(q)$$

and there is a unique ring homomorphism

$$\lambda: Ell^*\to\mathbf{Z}[1/6]((q))$$

classifying (Tate(q), dx/y). (This λ is nothing but a q-expansion homomorphism which is injective.) Since the formal group associated to Tate(q), viewed as $\mathbf{G}_m/q^{\mathbf{Z}}$, is canonically isomorphic to formal multiplicative group $\hat{\mathbf{G}}_m$ we have a canonical isomorphism of formal groups over $\mathbf{Z}[1/6]((q))$:

$$\theta: \hat{\mathbf{G}}_m \xrightarrow{\cong} \lambda_* \hat{E}_{\text{univ}}.$$

This isomorphism θ is actually a strict isomorphism of formal group laws, where we take local parameter $T=t-1$ for $\mathbf{G}_m=\operatorname{Spec}\mathbf{Z}[t, t^{-1}]$. Therefore, by using the theory of Landweber-Novikov operations, we have

Theorem 1.1 ([13]). *There is a unique natural transformation of multiplicative cohomology theories on finite CW-complexes*

$$\lambda(X): Ell^*(X)\to K^*(X)[1/6]((q))$$

such that :

(1) $\lambda(pt)=\lambda$.

(2) *Let* $\lambda(CP^\infty)=\lim_n \lambda(CP^n)$ *and denote by* x^{Ell} (*resp.* x^K) *the canonical orientation for* Ell (*resp.* K) *then*

$$\lambda(CP^\infty)(x^{Ell})=\theta(x^K).$$

Here $K^*(?)$ *is* $\mathbf{Z}/(2)$*-graded complex K-theory.*

The above $\lambda(?)$ is called elliptic character.

2. Modularity of elliptic character for finite groups

Let G be a finite group and BG be its classifying space. Since $\lim^1 K^*(BG_i)=0$ for a filtration $\{BG_i\}$ on BG consisting of finite subcomplexes we have elliptic character

$$\lambda(BG): Ell^*(BG)\to K^*(BG)[1/6]((q)).$$

Using Atiyah's isomorphism $K^*(BG)\cong\hat{R}(G)$ ([1]), we have a natural ring homomorphism

$$Ell^*(BG)\to\hat{R}(G)[1/6]((q)),$$

where $\hat{R}(G)$ denotes the completion of the complex representation ring of G at the ideal consisting of the virtual representations of dimension 0.

For a prime p let $G_{1,p}=\{g\in G:\ g^{p^N}=1,\ N\gg 0\}$, $R_p(G)=(\hat{R}(G))_p^\wedge$ and denote by $\mathbf{C}_p$ the completion of the algebraic closure of $\mathbf{Q}_p$. Then there is a character map

$$\chi_p(G):\ R_p(G)\to\mathrm{Map}_G(G_{1,p},\ \mathbf{C}_p)$$

which is a p-adic analogue of the usual group character (see [6]). Thus for a prime $p\geq 5$ (from now on we fix a prime $p\geq 5$) we have a natural ring homomorphism

$$\lambda_p(G):\ Ell^*(BG)\to\mathrm{Map}_G(G_{1,p},\ \mathbf{C}_p((q))).$$

We shall study modularity property of this $\lambda_p(G)$. Before stating our result we give a brief account of p-adic theory of modular forms.

Let $V(n)=V(\mathbf{Z}_p[\zeta_{p^n}],\ \Gamma(1))$ be the ring of $\Gamma(1)$-generalized p-adic modular functions over $B_n=\mathbf{Z}_p[\zeta_{p^n}]$, where ζ_{p^n} is a primitive p^n-th root of unity (see [10, Chapter V], [5, Chapter I]). The ring $V(n)$ represents the functor

$$\{p\text{-adic } B_n\text{-algebras } A\}\to\{\text{isomorphism classes of trivialized elliptic curves over } A\}.$$

Here a p-adic B_n-algebra means a B_n-algebra which is complete Hausdorff in its p-adic topology and a trivialized elliptic curve over A means a pair $(E,\ \varphi)$ consisting of an elliptic curve E/A and a trivialization φ of it, i.e., an isomorphism of formal groups over A:

$$\varphi:\ \hat{E}\xrightarrow{\cong}\hat{\mathbf{G}}_m.$$

There is an obvious inclusion

$$V(n)\hookrightarrow V(n+1)$$

for every $n\geq 0$.

The Tate curve Tate(q), viewed over $\widehat{B_n((q))}=(B_n((q)))_p^\wedge$, has a canonical trivialization

$$\varphi_{\mathrm{can}}=\theta^{-1}:\ \widehat{\mathrm{Tate}(q)}\xrightarrow{\cong}\hat{\mathbf{G}}_m.$$

Then evaluation on $(\mathrm{Tate}(q),\ \varphi_{\mathrm{can}})$ gives an injective q-expansion homomorphism

$$\tilde{\lambda}:\ V(n)\to\widehat{B_n((q))}.$$

For any $a \in \mathbf{Z}_p^\times$ and $f \in V(n)$ we define $[a]f \in V(n)$ by the formula

$$[a]f(E, \varphi) = f(E, a^{-1}\varphi),$$

where a^{-1} acts on φ via an automorphism of $\hat{\mathbf{G}}_m$. This gives the action of $\mathbf{Z}_p^\times$ on $V(n)$. Let

$$V^k(n) = \{f \in V(n) : [a]f = a^k f (\forall a \in \Gamma_n)\}$$

and

$$V^*(n) = \bigoplus_k V^k(n),$$

where $\Gamma_n = \{a \in \mathbf{Z}_p^\times : a \equiv 1(p^n)\}$. Note that there is an obvious inclusion

$$V^k(n) \hookrightarrow V^k(n+1)$$

since $\Gamma_{n+1} \subset \Gamma_n$.

Let $dT/(1+T)$ be the standard invariant differential on $\hat{\mathbf{G}}_m$. Then for any trivialized elliptic curve (E, φ) we have an invariant differential $\varphi^*(dT/(1+T))$ on $\hat{E}$ which uniquely extends to a nowhere vanishing invariant differential on E. Thus for any $f \in Ell^{2k}$ we get an element $\tilde{f} \in V^{-k} = V^{-k}(0)$ defined by

$$\tilde{f}(E, \varphi) = f(E, \varphi^*(dT/(1+T))).$$

Therefore there is a ring homomorphism

$$Ell^* \to V$$

which preserves q-expansions and hence is injective. When we regard Ell^* as a subring of V via this homomorphism $E_{\mathrm{univ}} \otimes V$ admits a trivialization φ_{univ} given by

$$\varphi_{\mathrm{univ}}(T) = \exp_{\hat{\mathbf{G}}_m}(\log_{\hat{E}_{\mathrm{univ}}}(T)) \in V[[T]].$$

For $(E_{\mathrm{univ}} \otimes V, dx/y)$ is clearly isomorphic to $(E, \varphi^*(dT/(1+T)))$ as $\Gamma(1)$-test objects for any universal trivialized elliptic curve (E, φ) over V and

$$dx/y = d\log_{\hat{E}_{\mathrm{univ}}}(T)(T = -2x/y)$$

and

$$\varphi^*(dT/(1+T)) = d\log_{\hat{\mathbf{G}}_m}(\varphi(T)).$$

Theorem 2.1. *For any $x \in Ell^{2k}(BG)$ and $g \in G$ of order p^n there is a (necessarily unique) element $f \in V^{-k}(n)$ such that*

$$[\lambda_p(G)(x)](g) = f(\mathrm{Tate}(q), \varphi_{can}).$$

Proof. First consider the case $G=\mathbf{Z}/p^n\mathbf{Z}$ and $g=g_n$ (the canonical generator of $\mathbf{Z}/p^n\mathbf{Z}$).

Let

$$f_n=\varphi_{\mathrm{univ}}^{-1}(\zeta_{p^n}-1)\in pV(n).$$

Then f_n has weight -1 over Γ_n, i.e.,

$$f_n\in pV(n)\cap V^{-1}(n)=pV^{-1}(n)$$

since, for any $a\in\Gamma_n$,

$$\begin{aligned}[a]f_n&=[a](\varphi_{\mathrm{univ}}^{-1}(\zeta_{p^n}-1))\\&=[a]((\exp_{\hat{E}_{\mathrm{univ}}}\log_{\hat{G}_m})(\zeta_{p^n}-1))\\&=(\exp_{[a]_*\hat{E}_{\mathrm{univ}}}\log_{\hat{G}_m})(\zeta_{p^n}-1)\\&=((a^{-1}\exp_{\hat{E}_{\mathrm{univ}}}a)\log_{\hat{G}_m})(\zeta_{p^n}-1)\\&=a^{-1}(\exp_{\hat{E}_{\mathrm{univ}}}\log_{\hat{G}_m})(\zeta_{p^n}^a-1)\\&=a^{-1}f_n.\end{aligned}$$

Now the q-expansion of f_n is given by

$$\begin{aligned}f_n(\mathrm{Take}\,(q),\ \varphi_{\mathrm{can}})&=\tilde{\lambda}(f_n)\\&=\tilde{\lambda}((\exp_{\hat{E}_{\mathrm{univ}}}\log_{\hat{G}_m})(\zeta_{p^n}-1))\\&=(\exp_{\lambda_*\hat{E}_{\mathrm{univ}}}\log_{\hat{G}_m})(\zeta_{p^n}-1)\\&=\theta(\zeta_{p^n}-1).\end{aligned}$$

Therefore for any $x=\sum_i a_i(x^{Ell})^i$ of degree $2k$ in

$$Ell^*(B\mathbf{Z}/p^n\mathbf{Z})=Ell^*[[x^{Ell}]]/([p^n]_{\hat{E}_{\mathrm{univ}}}(x^{Ell})),$$

$$\begin{aligned}[\lambda_p(\mathbf{Z}/p^n\mathbf{Z})(x)](g_n)&=\textstyle\sum_i\lambda(a_i)[\theta(\chi_p(\mathbf{Z}/p^n\mathbf{Z})(x^K))^i](g_n)\\&=\textstyle\sum_i\lambda(a_i)\theta(\zeta_{p^n}-1)^i\\&=(\mathrm{Take}\,(q),\ \varphi_{\mathrm{can}})\end{aligned}$$

for $f=\sum_i a_i f_n^i\in V^{-k}(n)$. This proves the result for $G=\mathbf{Z}/p^n\mathbf{Z}$ and $g=g_n$.

Now for a general G and $g\in G$ of order p^n there is a unique homomorphism

$$\alpha:\ \mathbf{Z}/p^n\mathbf{Z}\rightarrow G$$

which sends g_n to g. Hence for any $x\in Ell^{2k}(BG)$

$$[\lambda_p(G)(x)](g)=[\lambda_p(G)(x)]\alpha(g_n))$$
$$=[\lambda_p(\mathbf{Z}/p^n\mathbf{Z})(\alpha^*x)](g_n)$$
$$=f(\mathrm{Tate}(q),\ \varphi_{\mathrm{can}})$$

for some $f\in V^{-k}(n)$.

Remark 2.2. The above result asserts that every element in Im $\lambda_p(G)$ is p-adically Thompson series.

3. Relations between elliptic character and HKR character

Throughout this section we denote simply by *Ell* the p-adic completion of *Ell*. (Recal that p is a fixed prime≥ 5.)

Let $\mathcal{O}_p$ be the valuation ring of $\mathbf{C}_p$ and $\mathfrak{p}_p$ be the valuation ideal. Let $\mathfrak{m}$ be a maximal homogeneous ideal of Ell^* $(=\mathbf{Z}_p[g_2,\ g_3,\ \Delta^{-1}])$ containing $(p,\ E_{p-1})$ and choose a local homomorphism

$$\psi:\ (Ell^*)^{\wedge}_{\mathfrak{m}}\rightarrow\mathcal{O}_p.$$

The restriction of ψ on Ell^* classifies an elliptic curve over $\mathcal{O}_p$ whose mod $\mathfrak{p}_p$ reduction is supersingular (see [18, Chapter V §§3-4] and [9, §§2.0-1]).

Theorem 3.1[7], [8], [6]. *Let* $G_{2,p}=\{(g,\ h)\in G^2:\ [g,\ h]=1,\ g^{p^N}=h^{p^N}=1,\ N\gg 0\}$. *Then there is a generalized character map*

$$(Ell^{\wedge}_{\mathfrak{m}})^*(BG)\rightarrow\mathrm{Map}_G(G_{2,p},\ \mathbf{C}_p)$$

which extends to the isomorphism

$$(Ell^{\wedge}_{\mathfrak{m}})^*(BG)\otimes_{(Ell^{\wedge}_{\mathfrak{m}})^*}\mathbf{C}_p\xrightarrow{\cong}\mathrm{Map}_G(G_{2,p},\ \mathbf{C}_p).$$

To relate the above character map to elliptic character we need to specify an exponential isomorphism

$$(\mathbf{Q}_p/\mathbf{Z}_p)^2\xrightarrow{\cong}\hat{E}_{\mathrm{univ}}(\mathfrak{p}_p)_{\mathrm{tors}}$$

as described in [6]. This requires some facts about elliptic curves and modular forms.

Let $M^*(n)=M^*(B_n,\ \Gamma(p^n)^{\mathrm{arith}})$ (resp. $M_1^*(n)=M^*(B_n,\ \Gamma_1(p^n)^{\mathrm{arith}})$) be the ring of $\Gamma(p^n)^{\mathrm{arith}}$ (resp. $\Gamma_1(p^n)^{\mathrm{arith}}$)-modular forms over $B_n=\mathbf{Z}_p[\zeta_{p^n}]$ (see [10, Chapter II]). These rings represent the functors

$$\{B_n\text{-algebras } A\}\rightarrow\{\text{isomorphism classes of } \Gamma(p^n)^{\mathrm{arith}}\text{-test objects over } A\}$$

and

$$\{B_n\text{-algebras } A\}\rightarrow\{\text{isomorphism classes of } \Gamma_1(p^n)^{\mathrm{arith}}\text{-test objects over } A\}$$

respectively. Here a $\Gamma(p^n)^{\mathrm{arith}}$ (resp. $\Gamma_1(p^n)^{\mathrm{arith}}$)-test object over A means a triple (E, ω, β) (resp. (E, ω, ι)) consisting of an elliptic curve E/A, a nowhere vanishing invariant differential ω on E, and a $\Gamma(p^n)^{\mathrm{arith}}$ (resp. $\Gamma_1(p^n)^{\mathrm{arith}}$)-structure β (resp. ι) on E which is an isomorphism (resp. inclusion) of A-group schemes:

$$\beta:\ \mu_{p^n}\times\mathbf{Z}/p^n\mathbf{Z}\xrightarrow{\cong}E[p^n]$$

and

$$\iota:\ \mu_{p^n}\hookrightarrow E[p^n],$$

where $E[p^n]$ denotes the kernel of multiplication by p^n on E.

For every $n\geq 0$ there are natural inclusions, which do *not* preserve q-expansions,

$$M^*(n)\hookrightarrow M^*(n+1)$$

and

$$M_1^*(n)\hookrightarrow M^*(n)$$

since a $\Gamma(p^n)^{\mathrm{arith}}$-structure β gives a $\Gamma(p^{n-1})^{\mathrm{arith}}$-structure $\beta|\mu_{p^{n-1}}\times p\mathbf{Z}/p^n\mathbf{Z}$ and a $\Gamma_1(p^n)^{\mathrm{arith}}$-structure $\beta|\mu_{p^n}$. The first inclusion has the effect $q\mapsto q^p$ on the q-expansions and the second one has the effect $q\mapsto q^{p^n}$. There is also a q-expansion preserving natural inclusion

$$M_1^*(n)\hookrightarrow M_1^*(n+1)$$

which is compatible with the above ones.

Any trivialized elliptic curve (E, φ) over A has a $\Gamma_1(p^n)^{\mathrm{arith}}$-structure given by the trivialization

$$\varphi^{-1}|\mu_{p^n}:\ \mu_{p^n}\hookrightarrow E[p^n].$$

Therefore we have a $\Gamma_1(p^n)^{\mathrm{arith}}$-test object $(E, \varphi^*(dT/(1+T)), \varphi^{-1}|\mu_{p^n})$ over A and a q-expansion preserving ring homomorphism

$$M_1^*(n)\rightarrow V^*(n)$$

which is necessarily injective.

Let

$$\beta(n)_{\mathrm{univ}}:\ \mu_{p^n}\times\mathbf{Z}/p^n\mathbf{Z}\xrightarrow{\cong}(E_{\mathrm{univ}}\otimes M^*(n))[p^n]$$

be a universal $\Gamma(p^n)^{\mathrm{arith}}$-structure. Let $Ell^*(n)$ be the integral closure of Ell^* in $M^*(n)$ ($M^*(n)[1/p]$ is itself integral over $Ell^*[1/p]$), $\mathfrak{m}_n$ be a maximal ideal of $Ell^*(n)$ containing $\mathfrak{m}Ell^*(n)$, and $\hat{\mathfrak{m}}_n=\mathfrak{m}_n(Ell^*(n))^{\wedge}_{\mathfrak{m}_n}$. Then the isomorphism

$$(\mathbf{Z}/p^n\mathbf{Z})^2 \xrightarrow{\cong} E_{\text{univ}}[p^n](M^*(n))$$

given by

$$(a, b) \mapsto \beta(n)_{\text{univ}}(\zeta_{p^n}^a, b)$$

induces an isomorphism

$$\widehat{\beta(n)}_{\text{univ}} : (\mathbf{Z}/p^n\mathbf{Z})^2 \xrightarrow{\cong} \hat{E}_{\text{univ}}[p^n](\hat{\mathfrak{m}}_n)$$

since $E_{\text{univ}} \otimes Ell^*/\mathfrak{m}$ is supersingular (cf. [18, Theorem V.3.1 and Proposition VII.2.2]). Here we may assume that

$$\widehat{\beta(n)}_{\text{univ}}(a, 0) = \varphi_{\text{univ}}^{-1}(\zeta_{p^n}^a - 1)(\forall a \in \mathbf{Z}/p^n\mathbf{Z}).$$

Therefore we have an exponential isomorphism given by

$$(a, b) \mapsto \tilde{\psi}(\widehat{\beta(n)}_{\text{univ}}(p^n a, p^n b))\left(\forall (a, b) \in \left(\frac{1}{p^n}\mathbf{Z}/\mathbf{Z}\right)^2\right)$$

and the generalized character map factors as

$$(Ell_{\mathfrak{m}}^{\wedge})^*(BG) \to \mathrm{Map}_G(G_{2,p}, \widehat{Ell^*(\infty)}) \to \mathrm{Map}_G(G_{2,p}, \mathbf{C}_p),$$

where $\widehat{Ell^*(\infty)} = \bigcup_n Ell^*(n)_{\mathfrak{m}_n}^{\wedge}$ and

$$\tilde{\psi} : \widehat{Ell^*(\infty)} \to \mathcal{O}_p$$

is an extension of ψ. (We assume that $\mathfrak{m}_n \supset \mathfrak{m}_{n-1} Ell^*(n)(\forall n \geq 1)$.)

Let

$$\chi_{2,p}(G) : Ell^*(BG) \to \mathrm{Map}_G(G_{2,p}, \widehat{Ell(\infty)})$$

denote the composition

$$Ell^*(BG) \to (Ell_{\mathfrak{m}}^{\wedge})^*(BG) \to \mathrm{Map}_G(G_{2,p}, \widehat{Ell^*(\infty)}).$$

Theorem 3.2. *For any $x \in Ell^*(BG)$ and $g \in G$ of order p^n, $\chi_{2,p}(x)(g, 1)$ lies in $(Ell^*(n) \cap M_1^*(n))_p^{\wedge}(\hookrightarrow V(n))$ and*

$$\lambda_p(x)(g) = [\chi_{2,p}(x)(g, 1)](\mathrm{Tate}(q), \varphi_{\text{can}}).$$

Proof. By using naturality, as in the proof of 2.1, it is enough to show the result for $G = \mathbf{Z}/p^n\mathbf{Z}$ and $g = g_n$. By the construction of HKR character and our choice of an exponential isomorphism, for any $x = \sum_i a_i (x^{Ell})^i \in Ell^*(B\mathbf{Z}/p^n\mathbf{Z})$,

$$\begin{aligned}\chi_{2,p}(x)(g_n, 1) &= \textstyle\sum_i a_i \widehat{\beta(n)}_{\text{univ}}(1, 0)^i \\ &= \textstyle\sum_i a_i \varphi_{\text{univ}}^{-1}(\zeta_{p^n} - 1)^i\end{aligned}$$

$=\sum_i a_i f_n^i.$

Therefore we have

$$\lambda_p(x)(g_n)=\chi_{2,p}(x)(g_n, 1)(\mathrm{Tate}(q), \varphi_{\mathrm{can}}).$$

Remark 3.3. The above relations between λ_p and $\chi_{2,p}$ are analogous to relations between moonshine and generalized moonshine.

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